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THE COLLECTED WORKS OF IRVING LANGMUIR

VOLUME 5

PLASMA AND OSCILLATIONS

With a Contribution in Memoriam
by
DR. J. D. COBINE

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Irving Langmuir — Research Scientist
General Electric Research Laboratory — 1909–1957
Nobel Prize Winner–1932

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THE COLLECTED WORKS OF
IRVING LANGMUIR

WITH CONTRIBUTIONS IN MEMORIAM
INCLUDING
A COMPLETE BIBLIOGRAPHY OF HIS WORKS

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Volume 5

PLASMA AND OSCILLATIONS

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FOREWORD

THE PLAN to publish a memorial to Irving Langmuir, including all of the scientific output of his brilliant career in research, was announced to me by Captain I. R. Maxwell, managing director of Pergamon Press, late in 1958. My associates and I were asked to aid the venture by providing editorial advice and counsel, by enlisting the cooperation of scientific friends and acquaintances, and by assisting in the collection and identification of material. Our enthusiasm for the project and our willingness to cooperate sprang from two important considerations.

First, Langmuir's career provides an outstanding example of how free, but discriminating, inquiry in pure science may yield not only vital new knowledge and understanding of nature, but also a great bounty of practical usefulness for society. Secondly, Langmuir's associates hold him not only in great respect, but in very great affection as well. Hence the preparation of these volumes has been more than a service; it has been a labor of love.

The original plan was to publish Langmuir's works in three or four volumes, but for very good reasons, which developed during the course of the project, the series has grown to twelve volumes. The quantity of Langmuir's published scientific work proved to be far greater than we had estimated, and some previously unpublished wartime research and reports on meteorological studies were of such importance that their inclusion in the volumes was mandatory. Moreover, some exceptionally interesting philosophical papers and publications served to round out the literary portrait of Langmuir as a man and as a scientist.

My associate editors, Sir Eric Rideal and Professor P. W. Bridgman, have contributed generously from their great wealth of knowledge and their intimate acquaintance with Dr. Langmuir. It is a pleasant duty to acknowledge that the many members of the Honorary Editorial Advisory Board have participated in this venture with enthusiasm, and that their editorial contributions to the separate volumes have added tremendously to the appraisal and interpretation of Langmuir's collected works. I particularly want to acknowledge with gratitude the valuable work of Professor Harold E. Way of Union College who, in the capacity of Executive Editor, has carried the major task of assuring that our responsibilities and commitments were fulfilled.

I first met Irving Langmuir in the General Electric Research Laboratory when I joined the research staff in 1930, but our first meeting might equally well have taken place on a ski hill in the Adirondacks, at Lake George where

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he liked to spend the summer, or on a climb on Mt. Marcy, for he had a prevailing love of the out-of-doors. Whether in the Laboratory or in the mountains, an intense curiosity about natural phenomena constantly pervaded his thoughts. In fact, I have never met anyone else who was *so well coupled to nature.*

I am sure that, like all observant people, Langmuir perceived the beauty of nature as portrayed by the qualities of form, color, mass, movement, and perspective. In addition, however, Langmuir was delighted and entranced even more by the challenge to understand the working of nature as portrayed in the phenomena of everyday life — clouds, ripples on water, bubbles in ice, the temperature fluctuations of air and of water, the plastic quality of snow, the flight of a deer fly, and the thousands of "simple" phenomena which nearly everyone takes for granted. These manifestations of nature held endless fascination for Langmuir, and he constantly challenged himself to explain basic phenomena in terms of known laws of science. Of course, the same curiosity characterized his work in the Laboratory, and hence, provided the unifying motivation for his career, whether at "work" or at "play".

Langmuir's scientific work is so completely and perceptively described and appraised in the separate volumes of this work that only a few general comments and observations are appropriate, or indeed possible, at this point.

One striking feature of his research method was its instrumental simplicity. Although his career extended into the glamour age of science, characterized by large, impressive, and expensive machinery such as the cyclotron, the synchrotron, and particle and radiation diffraction equipment, his own experiments were almost invariably simple and uncluttered. He seemed positively attracted to simple experimental techniques, in refreshing contrast to what sometimes appears to be a fashionable reliance on impressive and expensive complexity of research equipment. His work with heat transfer in gases, and later with electron emission phenomena from metals, employed laboratory glassware of stark simplicity. His studies of surface films, especially films on water, employed beautifully simple experimental equipment. The Laboratory work on aerosols and smokes, and later on the nucleation of supercooled clouds, was all carried on with apparatus that could be assembled from the equipment of a typical home. His classical experiments on the "speed of deer fly" came about as close as possible to the string, wax, and paperclip approach to science; yet they sufficed to establish the essential facts sought by the investigation. Probably few scientists, before or since Langmuir, have gained so much important new knowledge of nature with such simple research equipment.

Similarly, Langmuir preferred to work with a few collaborators, rather than a large group or team of researchers, for this favoured a close contact with the work on a participating basis. His ability to apply mathematical analysis to physical problems was of a high order, and he divided his time about equally between experimental work and theoretical work. The combination

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of outstanding experimental and analytical ability which he possessed occurs but rarely in a single individual; most scientists have somewhat greater interests, aptitudes, and hence accomplishment in one area or the other.

Langmuir almost invariably worked on an intense basis and was generally completely preoccupied with his current problems. His concentration was exceptional, and he might pass you in the hall without seeing you. If you reminded him of it, he would smile and acknowledge that he was highly excited about some experiments that were in progress, or about some calculation that was presenting some puzzling aspects.

We spend a good deal of time and thought nowadays on the question of motivation for scientists, seeking to understand the source and character of their drive. In Langmuir's case, one needs to inquire no further than his curiosity. This pronounced trait provided an intense internal source of motivation, which constantly drove him to inquire and probe and test hypotheses until a pattern of understanding was developed. When he was on the trail of an exciting mystery, which was usually the case, his intense concentration was remarkable to behold.

Langmuir's career contributes much to our understanding of creative output in research. For example, on the perennial question of creativity and age, it has been held by some that the bulk of human creative work is accomplished in early adult life, say in the age bracket between 25 and 35 years. It is probable that some purely statistical information might support this view. However, I would disagree strongly with the corollary conclusion that creative ability is characteristic of this age bracket. In the Laboratory, it is not unusual for creative young workers to acquire a greater span of research guidance, counselling, and even management responsibility as their career matures, and hence their creative contribution will, to a corresponding degree, appear in the work of others. I believe that in such cases scientists are generally not less, but more creative with advancing age, frequently up to and even through retirement. It is clear that purely statistical information would not readily reveal this fact.

It is interesting to examine Langmuir's career as an example of a scientist who remained in active research up to and through retirement, to see what role age played in his output. In Volume XII we have depicted Langmuir's achievements as a function of his age, using his scientific *publications* as evidence of his gross scientific output, and his *principal accomplishments* as evidence of his creative output. The resultant charts show remarkably constant productivity throughout his scientific career, and even through retirement. Throughout this period Langmuir published an average of five to six scientific papers per year. His principal accomplishments, both scientific and practical, took place almost uniformly over the period of his researches. Certainly no "creative age" can be identified in his career. The example of Langmuir's scientific history does not prove the general thesis, but from the observation

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of many research careers, I am persuaded that human creativity in science is not a significant function of age.

Creative output, however, *is* a function of many other factors that comprise the research environment. One important factor is the changing field of research. Some of the most creative scientists in the history of the General Electric Research Laboratory have worked intensively in one field for a period of some years, and have then changed quite abruptly to a new field as a source of fresh stimulation and new challenge. It is evident that in a period of 5 years, or so, one can bring a fresh point of view to a new field, make a major contribution, and perhaps exhaust one's ideas on the subject. At that point of fruition, there is a great temptation to sit back and bask in a reputation for eminence which has been established in a specialized field of science. The more courageous scientist, however, will be challenged, or will, like Langmuir, challenge himself to enter a new field. This requires courage, because in the new field he will be a neophyte but, at the same time, a scientific entrepreneur with a reputation at risk, and this risk may not pay off.

Langmuir's career exemplifies the courageous entrepreneur in science. It would be difficult to find a common demoninator, except curiosity, in many of the fields of science in which he made basic contributions. He never hesitated to attack new fields, such as protein monolayers, the generation of smoke, or meteorology, which were completely new and, hence, challenging territory to him. In each of these diverse fields, and in a great many others, he has made major basic contributions.

Some discussion of the very important applied aspects of Langmuir's scientific work is appropriate. It is a fact that, although his prevailing motivation in research was curiosity about all natural phenomena, he was always perceptive of the practical usefulness of research results, and he himself suggested possible practical applications of many of the new phenomena which he discovered. He was generally able to communicate his enthusiasm to applied scientists and engineers interested in the proposed application and to give them guidance in its exploration.

It is interesting to speculate on the way that Langmuir's career might have developed had he chosen an academic, rather than an industrial environment for his work in science. My personal belief is that his research would, in any environment, have resulted in a high order of scientific accomplishment. Although he evidenced little interest in teaching, he was in fact an outstanding teacher, and in a university he would have exerted a great influence on students who might have been fortunate enough to be in contact with him. But I doubt if an academic career for Langmuir would have, or could have, developed the great bounty of useful results for society which did come from his exposure to a creative industrial scientific environment. The human and economic impact of gas-filled lamps, high-vacuum electron tubes, atomic-hydrogen welding, space charge emission phenomena, techniques and discoveries in

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surface chemistry, thyratron arcs (with A. W. Hull), and cloud seeding techniques has been very great indeed, and in most of these developments the influence of his research environment has been unmistakable.

Wherever Langmuir worked, or might have worked, the world is vastly better because of him, and both his former associates and colleagues, and the public at large, bear a tremendous debt of gratitude for his genius in science and for his perception of human need.

June 15, 1960

C. GUY SUITS
Vice-President and Director of Research
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Schenectady, New York

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PREFACE TO VOLUME 5

THIS volume represents a continuation of the research work of Irving Langmuir in the field of gas discharges. Practically all of these papers were written during the 1920's and early 1930's. It is interesting to note that Langmuir originated the word *plasma* to denote the luminous discharge regions, a term that is in common use today.

Following Penning's discovery of oscillations in ionized gases, he, with Lewi Tonks as co-author, wrote one of the more comprehensive and most important papers in the field entitled "General Theory of the Plasma of an Arc".

Irving Langmuir, by his many papers, contributed greatly to the field of plasma physics which is assuming a position of major importance in the field of science today.

Dr. J. D. Cobine, research physicist at the General Electric Research Laboratory, and an outstanding man in the field of plasma and oscillations, was asked to write the contributed paper for this volume.

HAROLD E. WAY
Executive Editor

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INTRODUCTION TO VOLUME 5

A CONTRIBUTION IN MEMORIAM

BY DR. J. D. COBINE

Plasma and Oscillations

THE introduction to the previous volume has pointed out that gas discharges represented a relatively old field at the time Dr. Langmuir began devoting considerable study to the subject, i.e., about 1922. Since the beginning of the study of electricity scientists have been attracted to the discharges because "...the luminous discharges are spectacular and fascinating; they have an intrinsic appeal, which the simpler but tamer phenomena want; they challenge attention for their beauty and variety and mystery, as well as their adaptability and their danger".[1] Lightning, which is an impulse arc on a grand scale, had challenged man's attention for ages; the carbon arc and the low-pressure mercury arc had long been used for illumination; alternating-current power was being converted to direct current by means of mercury-arc rectifiers; and power arcs occurred whenever electric circuits were interrupted or insulation failed. A considerable amount of scientific literature had been devoted to the general characteristics of the various discharges and much research had been devoted to studies of the various simple processes occurring in discharges, such as: ionization, the mobility and diffusion of gaseous ions, and spark discharges. The principal gas discharge treatises in English were those of J. J. Thomson[2] and J. S. Townsend[3]. The stimulation provided by the techniques and analysis originated by Langmuir in the papers presented in Volume 4 of this collection greatly increased the studies being made of the luminous portions of gaseous discharges. These luminous discharge regions Langmuir named *plasma*, a name in general use today to characterize the phenomenon. His probe technique, generally called a "Langmuir Probe", established the fact that the plasma was pervaded by an electron gas in which the electrons had a maxwellian velocity distribution and a characteristic temperature.

Langmuir's papers presented in Volumes 4 and 5 were published during the years 1923–1932. By the time Langmuir's most significant gas discharge research was completed the literature of this field was reviewed in four new and significant treatises, two in English[1,4] and two in German[5,6]. The field has grown until at present the author's personal file lists over 12,000 references. Over a thousand of these references are to papers on the subjects of plasma, plasma oscillations and closely related phenomena. An indication of the accel-

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erated growth of research in plasma and discharges is given by the fact that more than one-half these papers have been published since 1955. The stimuli for much of the recent studies in plasma physics are to be found in the intriguing possibilities of controlled nuclear fusion; shock ionization, and ionosphere studies for rocket needs; the direct conversion of thermal energy of hot ionized gases to electrical form by the magnetohydrodynamic (MHD) generator; and the possibilities of using plasma for space travel propulsion and guidance.

In reviewing Dr. Langmuir's laboratory notebooks for the period of his greatest activity in the field of gas discharges the author was impressed by the wide range of subjects, often quite far from the main course of his work, that he saw fit to discuss in his notes. Some of these have been mentioned in the introduction to Volume 4, a few others follow. In August 1928 he became interested in a light source for color television. He noted that it was his belief that 90% of the television would be based on movies taken shortly before their transmission. Since most present day TV movies seem to have considerable age, it would appear that we have not quite caught up with Langmuir's imagination! At this point on the subject of light sources he remarks on the high brilliance of an arc in a vortex of whirling water that he saw at the Siemens-Halske Company during his visit to Germany the previous fall. Only in the last few years has this arc received much scientific attention and has it been exploited as a source of high intensity plasma for MHD studies, testing missile nose cones to simulate atmospheric re-entrant conditions, welding, etc. His early interest in aviation is indicated in a proposal to Dr. Dushman for a model airport covered by model clouds, and an investigation of the brightness of sunlight, moonlight, and starlight as well as a theory of the scattering of light by clouds. He noted that Lindbergh, during his visit to the Laboratory, had said that when in the clouds he could see the arc flashes from trolleys even when he could not see large flares of burning gasoline. On July 12, 1930 he noted that Dr. Whitney had left for Europe and that Dr. Coolidge, as acting director, wished a meeting with him to discuss the work. There followed a careful, methodical, outline of all the subjects that he thought should be discussed, research of importance, etc. In a number of places he suggested the possibility of using gas discharges as amplifiers and generators of oscillation. However, such uses have been relatively few to date except for relaxation oscillators, low frequency inverters and the "plasmatron". Some attention has been given to the plasma amplifier which is unfortunately troubled with noise. In fact, probably the largest application of the intrinsic variational phenomena, discussed in a number of the papers of this volume, is as a source of electrical noise. Gas-discharge tube noise sources developed by the author and his associates were used extensively in radar jamming transmitters as part of the countermeasure program during World War II.

Plasma-boundary phenomena are discussed in the first and third papers. The former discusses the difference in gas pressure that may develop in the

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zone near the cathode and that near the cathode, and the development of pressure at walls, probe techniques, etc.; the latter paper develops the field about a hole in a plate, such as a grid, and considers the flow of plasma through it.

The second paper discusses the use of a probe to separate the primary electron beam, that leaves the cathode zone, from the scattered electrons. Most of the scattered electrons have lost much of the initial energy and he gave them the name "ultimate", which is still applied. The mechanism whereby the complete randomizing of the velocities to give velocity components exceeding the value of the initial beam is discussed. Oscillations are suggested as a source of this "Langmuir Paradox"[7,8]. The study of these phenomena is continued in the fourth paper in considerably more detail. The reader should note that a correction to the third paper of Vol. 4 is presented in the fourth paper.

Oscillations in ionized gases, first discovered by Penning[9], were extensively studied by Tonks and Langmuir and are the subject of papers bearing that title. In these papers a theory is developed showing the high frequency ones to be compression, or sound, type waves in the electron gas. Their limiting frequency is calculated to be $\nu = (ne^2/\pi m_e)^{1/2} = 8980\, n^{1/2}$, where n is the electron density, e is the electron charge and m_e is the mass of the electron. This limiting frequency is today known as the *plasma frequency* or the *plasma resonance frequency*. Low frequency oscillations are ascribed to plasma-ion oscillations and an equation developed for them. Both groups of oscillations are present in plasma and much more complicated relations have been developed in recent years to explain them. The Tonks and Langmuir equations, however, predict oscillations in the range of frequencies observed. In the seventh paper Tonks and Langmuir note the presence of non-coherent, random disturbances which they call noise. It is this electrical noise that was referred to earlier in this introduction as having practical and rather extensive application.

The paper, "General Theory of the Plasma of an Arc", of which Tonks is the senior author is one of the most important papers in the field. Darrow[1] refers to it as "...important, but formidable paper of Tonks and Langmuir, bristling with one hundred and eleven symbols dispersed through ninety equations spread over forty-six pages of the Physical Review ..." The author, after struggling through it in order to abridge it for inclusion in his own textbook, felt the designation fully justified.

Various collision and interaction processes are considered in the eighth, tenth and eleventh papers. In the eighth paper the important point is made that even with an unlimited supply of positive ions the ion current cannot exceed the ratio $(m_e/m_p)^{1/2}$. Further it is shown that the cathode fall of potential cannot be destroyed. The last two papers in this Volume present interesting phenomena observed in a neon discharge. In these it is shown that slightly off resonance radiation from the plasma in neon has a quite long mean free path and can produce metastable atoms of neon as much as 20 to 30 cm from the discharge plasma.

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As the author indicated in the introduction to Volume 4, it is futile to attempt a review of the science of plasma physics in the brief space available here. The field is the principal concern of two divisions of the American Physical Society, namely the Division of Plasma Physics, and the Gaseous Electronics Conference of the Division of Electron Physics. Every two years an international conference meets in Europe for the presentation of a large number of papers. The new edition of the famous ***Handbuch der Physik***[10] contains two ponderous volumes, mostly in English, on the subject of Gaseous Discharges and numerous authoritative textbooks are available. In all of these, references will be found to Langmuir's pioneering research which is assembled in these volumes.

J. D. COBINE

References

1. K. K. DARROW: *Electrical Phenomena in Gases*, Williams and Wilkins Co., Baltimore, 1932, p. 337.
2. J. J. THOMSON: *Conduction of Electricity through Gases*, University Press, Cambridge, 1906.
3. J. S. TOWNSEND: *Electricity in Gases*, Clarendon Press, Oxford, 1915.
4. J. J. THOMSON and G. P. THOMSON: *Conduction of Electricity through Gases*, University Press, Cambridge, Vol. 1, 1958, Vol. 2, 1933.
5. R. SEELIGER: *Ein Führung in die Physik der Gasentfladungen*, J. A. Barth, Leipzig, 1st. Ed. 1927, 2nd Ed. 1934.
6. A. V. ENGEL and M. STEENBECK: *Elektrische Gasentladungen*. J. Springer, Berlin Vol. 1, 1932, Vol. 2, 1934.
7. D. GABOR, E. A. ASH, and D. DRACOTT: *Nature*, **176**, 916 (1955).
8. H. MARTIN and K. G. EMELIUS: *Nature*, **177**, 943 (1956).
9. F. M. PENNING: *Physica*, **6**, 241 (1926).
10. J. Springer, Berlin, 1956.

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THE PRESSURE EFFECT AND OTHER PHENOMENA IN GASEOUS DISCHARGES*

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Vol. CXCVI, No. 751 (1923).

WHEN an electric discharge of rather high current density is passed through a monatomic gas at low pressure, the gas tends to move towards the anode and causes the pressure at the anode to exceed that at the cathode. This pressure effect has been observed and studied by L. Hamburger,[1] F. Skaupy[2] and A. Rüttenauer.[3] Hamburger and Skaupy have attempted to explain the motion of the gas to the anode by assuming the presence of negative ions having a mobility higher than that of the positive ions. Skaupy,[4] in a later paper, considers that the effect may be due to the fact that electrons make elastic collisions with atoms while positive ions make inelastic collisions. Thus he assumes that the electrons deliver twice their momentum to gas molecules (because they rebound), while the positive ions cannot deliver more momentum than corresponds to their velocity. A little analysis, shows, however, that there is a fallacy in this argument since the rebounding electrons would subsequently collide with other atoms and thus deliver momentum in the opposite direction. In fact, it is readily seen that the effect is thus neutralized. Since positively and negatively charged particles must be present in equal numbers in any uniformly ionized gas, the momenta delivered by the electric force to the positive and to the negative particles in each unit of time must be equal and opposite. Thus, *as long as the particles remain in the gas*, there is no resultant force acting on the gas.

Hamburger's and Skaupy's experiments have been cited by Franck and Grotrian[5] as evidence for the existence of negative ions in discharges in such gases as argon, which have been thought to have no appreciable electron affinity.

Let n_e, n_p and n_n be respectively the number of electrons, positive ions and negative ions per unit volume and let v_e, v_p and v_n be the corresponding average velocities parallel to the axis of the discharge tube. Then, since there can be no appreciable space charge in any uniform positive column,

$$n_e + n_n = n_p, \tag{1}$$

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assuming that both the positive and negative ions are univalent. If i is the current carried by the discharge then (taking all velocities positive)

$$i/e = n_e v_e + n_n v_n + n_p v_p. \tag{2}$$

The number of molecules of gas which drift towards the anode through any given cross-section is $n_n v_n - n_p v_p$. Thus if $n_n v_n$ is greater than $n_p v_p$, the gas will be carried electrically towards the anode. As the pressure at the anode increases, gas flows back through the tube to the cathode, but the viscosity of the gas makes it necessary for the pressure at the anode to rise to a certain limiting value before a steady state is reached. Experiments have usually shown[6] that the mobility of negative ions is somewhat larger than that of positive ions, but the difference is not large, the ratio of the mobilities ranging from 1.0 to 1.6. Thus to cause an increase in pressure at the anode, n_n would have to be at least 60 per cent of n_p. It is well known that electrons attach themselves to molecules in electronegative gases such as oxygen at high pressures, but the data of Townsend,[7] Loeb[8] and others seem to indicate that there is no such tendency in the inert gases. However, in these experiments negative ions would have been detected only if the current carried by the negative ions had been appreciable in comparison with that carried by electrons. The mobility of the electrons in argon may be assumed to be of the order of magnitude of $\sqrt{1834 \times 40}$, or 271 times that of the ions. Thus we may assume as an example within the range of possibility: $v_e = 271 v_p$; $v_n = 1.6 v_p$; $n_n = 0.7 n_p$ and thus find by Equation (1) that $n_e = 0.43 n_n$ and by Equation (2) that the current carried by negative ions ($n_n e v_n$) is only 0.014 of that carried by electrons ($n_e e v_e$), even under these conditions which would cause gas to accumulate at the anode. Although it appears *probable* from the work of Townsend and others that the current carried by negative ions in argon is much less than 1 per cent and therefore cannot explain the pressure effect in this gas, yet it cannot be said that the experiments have proved this point.

In connection with a study of the positive column of the mercury arc, the writer has, however, developed a theory capable of explaining the increase of pressure at the anode without the necessity of assuming negative ions. By measuring the volt-ampere characteristics of small electrodes in the path of the discharge, at low pressures, such as ten bars, it has been found that the electrons in a given volume move nearly in random directions with an average kinetic energy which is independent of the current density in the arc (in the range from 0.1 to 1.0 ampere per square centimetre) but which depends on the mercury vapor pressure as shown in Table I.

The third column gives the potential difference through which electrons must fall to acquire a kinetic energy equal to the average kinetic energy of those in the arc. The distribution of velocities about the mean has been found to correspond to that given by Maxwell's law. The kinetic energies of the elec-

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TABLE I

Kinetic Energy of Electrons in the Mercury Arc

Temp. of mercury	Pressure of vapor (bars)	Average kinetic energy (volts)	Equivalent temp.
15°C	1.0	3.9	30,000°K
32°C	4.4	2.9	22,400°K
60°C	33.0	1.7	13,900°K

trons are thus the same as those of molecules in gases at the temperatures given in the fourth column.

A negatively charged electrode in the ionized gas of the arc repels electrons from its neighborhood, so that there is a positive ion sheath around the electrode.[9] By calculation from the space charge equations, it is found that the thickness of this sheath is usually not more than about 0.1 mm. The sheath acts as an electrostatic screen so that the field due to the negative charge on the electrode does not extend beyond the edge of the sheath. All positive ions that reach the sheath are attracted to the electrode. If the electrode is large compared to the sheath thickness, the positive ion current that flows to the electrode is thus practically independent of the applied voltage, and is a true measure of the *positive ion current density* I_p in the arc.

When the electrode is positively charged and the gas pressure is not too high, an electron sheath is formed in a similar manner and the electron current over a certain range is nearly independent of the applied voltage and is a measure of the *electron current density* I_e in the surrounding space.

The ratio $I_e:I_p$ is nearly the same at different current densities or mercury vapor pressures and averages about 340:1. The value of I_e may be too low because of the reflection of some electrons, whereas ions are apparently not reflected to an appreciable degree.

If the kinetic energy of the electron is the same as that of the ions, the ratio should be equal to $\sqrt{200 \times 1834} = 605$. The experiments prove conclusively, however, that the current carried by electrons in the mercury arc is *at least* 340 times that carried by positive ions.

The glass wall of the tube (or any isolated electrode) must acquire such a potential that it receives the same number of negative and of positive charges. Thus it must be sufficiently negatively charged to repel at least all but one electron out of every 340 that move towards it. To do this the walls need to have a negative potential with respect to the surrounding space, which, according to the Maxwell law, is roughly four times the potential corresponding to the average energy of the electrons; according to Table I the walls will thus be from 7 to 16 volts negative, depending on the mercury vapor pressure. It

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is clear that very serious errors are made by attempting to measure potentials in ionized gases by using sounding electrodes.*

There are, however, two simple methods by which the true space potentials can be measured. In the first method a tungsten filament is used as sounding electrode, so arranged that it can be heated when desired. The filament is first heated to a temperature just below that at which there is appreciable electron emission, but a key is arranged to short-circuit a part of the resistance in series with the filament and thus momentarily raise its temperature to a point where it is capable of emitting an easily measurable current. The potential of this hot sounding electrode is then varied and at each potential it is noted whether or not the current to it changes when the temperature is altered by pressing the key. It is evident that the electrons emitted from the filament can escape only when there is an accelerating field around it. The filament is at the same potential as the space when the electrons are just able to escape. Small corrections need to be made for the voltage drop along the filament and for the initial velocities of the electrons emitted.

The second method consists in measuring the volt-ampere characteristics of a cold sounding electrode, preferably of such large size that its dimensions are fairly large in comparison with the thickness of the electron sheath that forms about it when it is positively charged. As the potential of the electrode is raised, starting from negative values, if first takes up positive ions, then also takes up high velocity electrons. At some definite potential it takes up all the electrons moving toward it (except those reflected); a further increase in potential causes no increase in electron current except as a result of an increase in the sheath thickness. By plotting the current on semi-logarithmic paper against the potential, it is thus found that there is a definite kink in the curve at the point when the electrode potential is the same as that of the surrounding space. This method is found to give results in good accord with those obtained by the hot sounding electrodes and both methods prove that the wall of the tube, or any electrode to which no current flows, is negatively charged with respect to the surrounding space by an amount usually ranging from 5 to 15 volts.

All but a minute fraction of the electrons moving toward the wall are thus *elastically reflected* in the positive ion sheath near the glass surface and are unable to deliver energy to the wall. Direct experimental measurements of the number of electrons striking the wall and comparison with the power consumed by the arc prove that 99.7 per cent of the electrons moving towards the wall are reflected, and that these electrons lose less than 0.5 per cent of

* Because of this error arising from the use of sounding electrodes, measurements of the anode drop have been altogether too high. In the mercury arc the anode drop is usually about 2 to 3 volts, whereas measurements with ordinary sounding electrodes show 10 to 18 volts. In some cases the anode drop is actually a few volts negative. The conditions at the anode and cathode will be discussed in subsequent articles.

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their energy by such reflection. At first sight it would seem that the reflection should be specular, that is, that there should be no change in average momentum parallel to the axis of the tube. Experiments which will be described in a subsequent paper seem to prove, however, that the reflection is about 30 per cent diffuse, the collisions of an electron with the wall having the effect of reducing the average longitudinal velocity component by about one-third. This effect is made manifest particularly by the action of transverse and longitudinal magnetic fields, which change the number of collisions of electrons with the walls, without altering the number of collisions with gas atoms. It is perhaps possible that the diffuseness of the reflection is due to the fluctuations of the positive ion space density in the sheath resulting from statistical causes.

Both theory and experiment, however, indicate that the positive ions are absorbed by the wall and deliver to it all their energy and momentum.

Heating of the Wall by Positive Ions

Let us assume that the positive ions have an average kinetic energy corresponding to a voltage V_0. Let I_p be the current density of positive ions striking the wall (amperes per cm²). There will be an equal number of electrons striking the wall to neutralize the charges of the ions. The energy corresponding to the ionizing potential (10.4 volts) will be liberated presumably in the form of heat when the positive ion and the electron combine. Let V_s be the potential drop through the sheath, near the glass surface. The energy carried to the wall by the positive ion in the form of kinetic energy thus corresponds to $V_s+(4/3)V_0$. The factor 4/3 comes from the fact that the kinetic energy of particles passing a given plane ($2kT$) is 4/3 times as great as that of the particles in a given volume (3/2) kT. The energy of the electrons striking the walls is (4/3) V_0. The total energy corresponds to $10.4+V_s+(8/3)\,V_0$ volts.

In an experiment with a four-ampere arc flowing through a tube 3.2 cm in diameter with mercury vapor at a pressure of 1.0 bar, the positive ion current density I_p was found to be 0.0011 ampere per cm². Taking $V_0 = 3.9$ volts, from Table I, and $V_s = 16$ volts, we should get a heating effect corresponding to 37 volts. Thus 37×0.0011, or 0.042 watt, should be liberated as heat on each square centimetre of tube surface as a result of the positive ion bombardment. As the voltage drop in the arc was 0.24 volt per centimetre, the total energy per second generated in the arc per square centimetre of wall was 0.095 watt. Thus 44 per cent of the power consumed in the arc path was liberated on the walls as a result of the positive ions striking the walls.

At higher mercury vapor pressure the percentage of heat thus carried to the wall decreased and it also decreased as the arc current decreased. Thus in a six-ampere arc with a pressure of 15 bars, 23 per cent of the energy consumption was caused by positive ions, but at the same pressure with an arc current of one ampere, the percentage was only 15.

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Pressure Effect due to Positive Ion Impacts on Walls

The positive ions that strike the wall of the tube have an average velocity component v_x in the direction towards the cathode. Let m_p be the mass and e the charge of a positive ion. The force p_x exterted by the ion impacts on each unit area of the inner surface of the tube in a direction towards the cathode is thus

$$p_x = \frac{\varepsilon m_p v_x I_p}{e} \qquad (3)$$

since the number of ions striking unit area per unit time is I_p/e. The factor ε is a number less than unity which represents the degree to which the reflection of the electrons by the wall may be regarded as being specular. This factor seems to be approximately of the order of 0.3.

The reaction from this force p_x must be exerted in the gas and must tend to cause an increase in pressure at the anode. It is easy to understand the mechanism of this reactive force. We have seen that the forces due to impacts of ions and electrons against atoms must balance one another in any mass of ionized gas containing equal numbers of positive and negative charges, as long as all the accelerated particles remain within the gas. But when positive ions leave the gas the impact of the electrons against the atoms, producing a force acting towards the cathode, is no longer balanced by the impacts of the positive ions against the atoms.

If the gas pressure is so low that the normal mean free path λ_n is large compared to the radius r of the tube, the gas moves towards the anode until the increase in pressure there balances the force due to p_x, but at higher gas pressure the force p_x causes the gas near the wall to move towards the anode while the gas near the axis of the tube will move in the opposite direction, the increase in pressure at the anode being thereby decreased.

At higher pressures the reactive force due to the positive ion impacts on the wall may thus be assumed to be exerted on the gas at a distance $r-\lambda$ from the axis of the tube; the gas at this point will have the greatest velocity towards the anode. Let v be this velocity. Then between r and $r-\lambda$ the volume of gas flowing per unit time will be $\frac{1}{2}v\pi[r^2-(r-\lambda)^2]$. If no gas should flow back along the axis of the tube (because of the increased pressure near the anode), the volume of gas flowing per unit time between 0 and $r-\lambda$ would be $v\pi(r-\lambda)$.[2] But by Poiseuille's law, the volume of gas per unit time that flows back because the pressure gradient is $\frac{\pi}{8}\frac{(r-\lambda)^4}{\eta}\frac{dp}{dx}$, where η is the coefficient of viscosity. In a steady condition with a discharge tube closed at the end, the total amount of gas flowing through the tube must be zero, so that we have

$$v = \frac{r^2}{8\eta}\left(1-\frac{\lambda}{r}\right)^3\frac{dp}{dx} \qquad (4)$$

if we neglect $\frac{1}{2}\lambda^2$ in comparison with r^2.

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Besides the force p_x (per unit area) acting on the wall of the tube due to the impact of positive ions, there is also a force p_a due to the impact of neutral atoms because the gas at $r-\lambda$ is moving towards the anode with the velocity v. The number of atoms per unit area per unit time striking the wall of the tube is $p(2\pi MRT)^{-1/2}$ and of this the fraction $(r-\lambda)/r$ may be assumed to have come from the region where the velocity is v. Thus we find

$$p_a = pv\left(1-\frac{\lambda}{r}\right)\sqrt{\frac{M}{2\pi RT}}. \tag{5}$$

Here M is the atomic weight of the gas and R is the gas constant per gram molecule 8.37×10^7 ergs per degree.

The actual force per unit area acting on the wall of the tube is thus p_x-p_a and this multiplied by the circumference of the inside of the tube must be equal to the pressure gradient multiplied by the cross-section, so that

$$r\frac{dp}{dx} = 2(p_x-p_a) \tag{6}$$

Eliminating v and p_a from Equations (4), (5) and (6) and solving for dp/dx, we find

$$\frac{dp}{dx} = \frac{2p_x/r}{1+\frac{rp}{4\eta}\left(1-\frac{\lambda}{r}\right)^4\sqrt{\frac{M}{2\pi RT}}} \tag{7}$$

This equation is derived for the case that the pressure is so high that λ_n is smaller than $\frac{1}{2}r$. At lower pressure where $\lambda_n > r$, the equation reduces to

$$\frac{dp}{dx} = \frac{2p_x}{r} \tag{8}$$

To calculate p_x we need to know the velocity component v_x of the ions in the direction of the field.

The time taken by an ion to travel a free path λ with the velocity Ω is λ/Ω. If γ is the acceleration due to the field, then the velocity component in the direction of the field, acquired during the average path, is given by

$$\frac{dx}{dt} = \frac{\gamma\lambda}{\Omega} \tag{9}$$

This equation has been derived by G. Hertz[10] for the velocity of drift of electrons in a field. Because of the persistence of velocities of the ions after collisions[11] this should be multiplied by 1.26. Since the acceleration is $(e/m_p)(dV/dx)$, we have

$$v_x = 1.26\frac{e\lambda}{m_p\Omega}\frac{dV}{dx} \tag{10}$$

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The average velocity Ω of the ions in a given volume may be calculated from V_0, the potential corresponding to the average kinetic energy of the ions in a given volume by the relation

$$\Omega = \sqrt{\frac{16 V_0 e}{3\pi m_p}} \tag{11}$$

Combining Equations (10) and (11)

$$v_x = 0.96\lambda \sqrt{\frac{e}{m_p V_0}} \frac{dV}{dx} \tag{12}$$

and by Equation (3)

$$p_x = 0.96\varepsilon\lambda I_p \sqrt{\frac{m_p}{eV_0}} \frac{dV}{dx}. \tag{13}$$

If we express p in dyne cm^{-2}, x and λ in cm, I_p in amps cm^{-2}, V in volts, and insert the values $e = 1.591 \times 10^{-20}$ e.m. units and $m_p = M \div 6.06 \times 10^{23}$, the equation becomes

$$p_x = 9.80\, \varepsilon\lambda I_p \sqrt{\frac{M}{V_0}} \frac{dV}{dx} \tag{14}$$

In this equation, as well as those immediately preceding, λ is not the normal free path which would occur in a large volume, but is the average actual free path which at low pressures is limited by the wall of the tube. Where λ_n is larger than r, the value of λ should then be put equal to $2r$, the diameter of the tube.

The pressure effect may now be calculated by combining Equations (14) and (8)

$$\frac{dp}{dx} = 18.6\varepsilon I_p \frac{\lambda}{r(1+\varrho)} \sqrt{\frac{M}{V_0}} \frac{dV}{dx} \tag{15}$$

where for convenience the second term in the denominator of the second number of Equation (7) has been represented by ϱ. Putting in the numerical value of R, we thus have

$$\varrho = 1.09 \times 10^{-5} \frac{rp}{\eta} \left(1 - \frac{\lambda}{r}\right)^4 \sqrt{\frac{M}{T}}. \tag{16}$$

Rüttenauer measured the pressure effect in helium and argon at pressures ranging from 700 to 1200 bars, current densities from 0.6 to 1.6 amperes per cm^2 with an arc in a tube 0.75 centimetre in diameter. Within the relatively narrow ranges covered by these experiments, he was able to express his results for the pressure gradient by the empirical equation

$$\frac{dp}{dx} = C \frac{I\sqrt{M}}{p} \frac{dV}{dx} \tag{17}$$

where C is a constant, and I is the current density in the arc. Expressing p in bars, I in amps cm^{-2}, V in volts, x in cm, the value of the constant best fitting the results is $C = 77$.

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Comparing Equations (15) and (17) we see that they are in many respects similar. If the two equations are to give the same value for dp/dx, we must have

$$\frac{I\sqrt{V_0}}{I_p} = 0.241\,\frac{\varepsilon\lambda p}{r(1+\varrho)} \tag{18}$$

The temperature of the gas in the tube may be assumed to be roughly 1000°K. The viscosities[13] of helium and argon at this temperature are given in Table II. The free path at the average pressure (900 bars) is calculated from the viscosity in the usual way. The quantity ϱ is found from Equation (16). The values of

TABLE II

Analysis of Rüttenauer's Data on the Pressure Effect

Gas	η	λp	λ/r	ϱ	$\frac{I\sqrt{V_0}}{\varepsilon I_p}$	$\sqrt{\frac{m_p}{m}}$
He	4.25×10^{-4}	115	0.34	0.10	67	86
A	5.6×10^{-4}	48	0.14	0.71	18	273

$I\sqrt{V_0}/(\varepsilon I_p)$ which have been calculated by Equation (18) are reasonable. The experiments with the mercury vapor arc give the ratio $I_e{:}I_p = 340{:}1$, but the true value may lie close to 605:1, which is the square root of the ratio of the mass of the mercury ion to that of the electron. Because of elastic collisions between the electrons and the mercury atoms and the walls of the tube, the electron current density I is sometimes as much as twenty times as great as the apparent current density I in the arc. Therefore, in general, we may assume that I/I_p may be several times less than $\sqrt{m_p/m}$. The data of Table II show that the ratio of $\sqrt{m_p/m}$ to $I/\varepsilon I_p$ is equal to about 1.3 for helium and 1.5 for argon.

Rüttenauer finds that when the pressure is lowered dp/dx reaches a maximum and slowly decreases. An arc in argon with a current of 0.16 ampere in a tube 0.75 centimetre diameter gave a maximum pressure gradient of 1.86 bars per centimetre at $p = 600$ bars; 1.63 at 333; 1.30 at 133, and 0.97 at 75 bars. As the potential gradients were not recorded these data cannot be used directly to test the equations. It is clear, however, from Equation (15) that as the pressure decreases the gradient should increase about inversely proportional to the pressure until λ becomes about equal to the diameter of the tube. At lower pressures λ must be considered to remain approximately constant. The decrease in the gradient at still lower pressure is probably accounted for the increase in V_0 as was found for the case of mercury (Table I).

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Summary

In the positive column of the mercury arc the electrons have velocities distributed in accord with Maxwell's law, and the directions of motion are distributed nearly at random. The average velocity of the electrons is approximately independent of the arc current but depends on the pressure. The average kinetic energy of the electrons, expressed in terms of the corresponding voltage, is 3.9 volts at 1 bar pressure, 2.9 at 4.4 bars and 1.7 at 33 bars.

Isolated electrodes, or glass surfaces, in the uniformly ionized gas of the arc, become charged 5 to 15 volts negatively with respect to the gas. The wall of the tube thus absorbs all positive ions moving towards it, but repels all but a minute fraction of the electrons, so that these act largely as if specularly reflected from the wall without loss of energy or momentum.

The impact of the positive ions on the wall causes a heating effect on the wall which ranges from 15 to 50 per cent of the total energy in the positive column.

The momentum delivered to the wall by the ions is the probable cause of the pressure effect observed by Hamburger and Skaupy by which the pressure of the gas near the anode is greater than at the cathode. Equations are derived by which this pressure effect may be calculated. The results are in rough agreement with the meager experimental data available.

Two methods are described for measuring the space potential in ionized gases. The first consists in the use of a hot sounding electrode whose potential is adjusted until the electrons emitted by it are just able to escape. The second is based upon a kink observed in a semi-logarithmic plot of the current voltage curve of a cold sounding electrode, which occurs when the electrode reaches the same potential as the surrounding space.

References

1. HAMBURGER, L.: "Dissertation," Delft, 1917, and *Proc. Amsterdam Acad.* **25,** 1045 (1917).
2. SKAUPY, F.: *Verh. Deut. Phys. Ges.* **19,** 264 (1917).
3. RÜTTENAUER, A.: *Zeit. f. Physik* **10,** 269 (1922).
4. SKAUPY, F.: *Zeit. f. Physik* **2,** 213 (1920).
5. FRANCK, J., and GROTRIAN, W.: *Zeit. f. Physik* **4,** 89 (1921).
6. THOMSON, J. J.: *Cond. of Electricity through Gases* (1906), p. 80; Wellish, E. M., *Am. J. Sci.* **39,** 599 (1915).
7. TOWNSEND, J. S., and BAILEY, V. A.: *Phil. Mag.* **42,** 871 (1921).
8. LOEB, L. B.: *Phil. Mag.* **43,** 229 (1922).
9. LANGMUIR, IRVING: *Science* **58,** 290 (1923); *Gen. Elect. Rev.* **26,** 731 (1923).
10. HERTZ, G.: *Verh. Deut. Phys. Ges.* **19,** 268 (1917).
11. JEANS, J. H.: *The Dynamical Theory of Gases,* Univ. Press, Cambridge, 2nd ed., 1916, p. 292.
12. Calculated from data in Kaye's and Laby's tables.

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SCATTERING OF ELECTRONS IN IONIZED GASES

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ABSTRACT

Velocity distribution of electrons in low pressure discharges from hot cathodes.—Analysis of the current received by a collector placed opposite a hot cathode indicates that there are three groups of electrons present: (1) *primary electrons* which retain practically all the momentum they acquired in passing through the positive ion sheath around the cathode; (2) *secondary electrons* moving in random directions with a Maxwellian velocity distribution corresponding to a temperature roughly proportional to the energy of the primaries (200,000° for 100 volt primaries) and approximately independent of the nature or pressure of the gas or the current density; (3) *ultimate electrons* having a Maxwellian distribution of velocities corresponding to a much lower temperature than that of the secondaries, a temperature which is independent of the current density or the voltage of the primary electrons and which varies with the gas used and decreases slowly as the pressure is raised. The number of ultimate electrons is roughly 1000 times that of the primaries and secondaries for mercury vapor, although relatively less numerous in the case of hydrogen. In the uniform positive column of arcs only ultimate electrons are present.

The velocity distribution of the primary electrons may be resolved into a drift velocity normal to the cathode surface and a random motion with a Maxwellian distribution corresponding to a temperature which varies approximately with the square of the current and is a maximum at a mercury vapor pressure of about 0.6 bar, falling to a low value at 8 bars, and is considerably higher with 35 volts accelerating potential than with 80 volts. Under favorable con ditions with 40 milli-amp. this temperature may rise to 80,000° and there are then appreciable numbers of these primaries which can reach a collector charged to a potential as much as 40 volts below that of the cathode. Experiments with two crossed electron beams prove that the secondary and ultimate electrons, directly or indirectly, are responsible for very little if any of h e scattering.

A study of the ultimate electrons in the positive column of a low voltage mercury arc shows that the Maxwellian velocity distribution corresponding to a high temperature (30,000°) is maintained in a small tube in spite of the fact that the negatively charged walls of the tube constantly tend to disturb this distribution by selective removal of the faster electrons. The number of collisions of the electrons with each other and with atoms were far too few to maintain the observed distribution. Measurements of the mobilities of electrons in arcs and of the electron concentration differences produced by a transverse magnetic field gave values for the mean free path only 1/10 as great as the values determined by a direct method. These results all indicate that the electrons in low pressure arcs suffer many changes of momentum during the time intervals between successive collisions between atoms, ions or electrons.

Mechanism of electron scattering.— After analyzing the effects to be expected when a beam of primary electrons encounters a cloud of stationary electrons or a cloud of particles of small mass but of high temperature, it appears that the experimental results for the ratio of the scat-

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tering to the average retardation of the beam was best accounted for by collisions with a cloud of radiation quanta, a kind of Compton effect. The observed scattering is, however, about 10^{16} times greater than a normal Compton effect would give. It is suggested that the joint action of excited atoms and radiation may multiply the Compton effect sufficiently, or it may be that the presence of a free electron within a distance of a wave-length from an excited atom may greatly increase the probability of a quantum jump and that momentum is delivered to the electron in the process. In arcs the ultimate electrons may be in a kind of thermal equilibrium with radiation and excited atoms.

Scattered electrons in high vacuum.—In connection with a study of the Barkhausen-Kurz effect and in a magnetron with negatively charged end plates, evidence of the presence of electrons with abnormally high speeds has also been obtained. These have been explained as due to electric oscillations, but the effects have also been obtained when oscillations are not present. This seems to indicate that electron scattering similar to that observed in low pressure gases can also occur in high vacuum.

I. Electrons in the Positive Column of Low Voltage Arcs

Distribution of velocities.—By studying the volt-ampere characteristics of small electrodes or collectors placed in the path of an arc in mercury vapor at low pressures, it has been found[1,2] that the free electrons in the arc are moving in nearly random directions with velocities which are distributed quite accurately in accordance with Maxwell's distribution law.

When the collector is at large negative potentials with respect to the surrounding gas, it receives only positive ions, but at higher potentials (algebraically) electrons also are collected. At first only electrons of the highest velocities are able to reach the collector against the retarding potential, but as the potential is raised, making the retarding field weaker, electrons of lower velocity can be collected. If the velocities of the electrons are distributed according to Maxwell's law, the logarithm of the electron current flowing to a negatively charged collector should be a linear function of the potential of the collector, with a slope (when plotted using natural logarithms) of e/kT, where T is the temperature corresponding to the electron velocities and e/k has the value 11,600 degrees per volt.

When the collector is made positive with respect to the surrounding gas the electron current increases with the potential more slowly and according to entirely different laws. Thus the semi-logarithmic plot of the current-voltage characteristic should be a straight line only for negative voltages and a kink therefore should occur in the curve at a point at which the potential of the collector is the same as that of the surrounding gas. By this method collectors may be used to measure the true space potentials.

When a small plane collector is at the space potential is an arc in mercury vapor, the electron current is about 400 times as great as the positive ion current which flows when large negative voltages, sufficient to repel all electrons,

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[1] Langmuir, *J. Franklin Inst.* **196,** 751 (1923).

[2] Langmuir and Mott-Smith, *Gen. Elect. Rev.* **27,** 449, 538, 616, 762, 810 (1924).

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are used. Thus if the collector is disconnected (or is allowed to "float") it charges itself negatively with respect to the surrounding gas until it is able to repel 399/400 of all the electrons which move towards it, the electrons and positive ions then being collected in equal numbers. Experiments with mercury vapor, hydrogen, nitrogen, argon and neon at low pressures have shown that the semi-logarithmic plots for negative voltages are usually surprisingly straight over very wide ranges of current. The straight semi-logarithmic plot of the electron current extends to negative voltages far beyond that to which the collector charges itself when it is floating. For example, the straight plot was easily observed down to currents only 1/6000 of the total current.[3] A. F. Dittmer working with Karl Compton at Princeton has recently found, using an improved method by which positive ions and electrons are separated, that with a 40 milli-ampere arc in nitrogen at 8 bars pressure, the straight line plot extends down to current densities of less than 10^{-5} of the total current density. The electron temperature in this case was about 50,000°.

The electron temperatures corresponding to these straight plots are practically independent of the current densities in the arcs, but increase as the pressure is lowered. For example, currents of from 0.1 to 5 amperes in a tube 3 cm in diameter gave electron temperatures of about 30,000° with mercury vapor at 1 bar; 20,000° at 5 bars, and 10,000° at 30 bars. With other gases the electron temperatures are usually higher.

If thermal equilibrium existed in the ionized gas the electrons would of course have a Maxwellian distribution of velocities corresponding to the temperature of the gas. If this distribution is disturbed by any external cause it tends to re-establish itself through the agency of every mechanism by which electron velocities can be modified. For example, collisions of electrons with each other, with atoms or excited atoms, or the interaction of electrons with the black body radiation, will all tend to bring back the Maxwellian distribution of the electrons. The rate of re-establishment of this distribution would be different for each mechanism if that alone were allowed to act. Corresponding to each mechanism we may therefore conceive of a "time of relaxation" which is the time that would be needed for any disturbance in the Maxwellian distribution to fall to $1/e$ of its value, if only the one mechanism were effective.

We will consider first the interaction between atoms and electrons. The mercury atoms which strike the walls of the tube deliver their surplus energy and leave the glass with a kinetic energy corresponding to room temperature. For mercury vapor at 1 bar and a temperature of 300°K, there are 2.4×10^{18} atoms per cm³. Measurements with collectors in a 2 amp arc in a 3 cm tube have shown that there are 8×10^{10} free electrons per cm³. In this case the average distance an atom will travel before colliding with an electron is 1.5 cm and in a tube 3 cm in diameter it will make only about four collisions with

[3] See, for example, Fig. 1 in *Gen. Elect. Rev.* article (*loc. cit.*[2]), p. 451.

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electrons between consecutive collisions with the wall. Taking the temperature of the electrons as 30,000°, the mercury atom gains on the average only 0.08° at each collision. Thus the temperature of the mercury vapor which strikes the walls will be only 0.3° above the temperature of the walls. This at least represents the temperature that can be acquired by direct interaction with the electrons. Of course there will be a minute fraction of mercury atoms which collide with excited mercury atoms and thus acquire very high kinetic energy, but we may ignore these at present. The time of relaxation corresponding to the interaction between electrons and atoms by collisions is thus at least 100,000 times too great to be effective in bringing about temperature equilibrium and thus affording a means of maintaining a Maxwellian distribution for the electrons.

Since the normal free path λ_e of electrons under the conditions considered, is 30 cm, it is clear that the average distance an electron would have to travel before colliding with an excited atom would be much greater than this and therefore collisions of the second kind, in the ordinary meaning of this term, cannot be responsible for bringing about the electron velocity distribution. The same may be said of collisions of electrons with each other. The concentration of electrons is only 1/300 that of the atoms and if we assume the target area of an electron is the same as that of an atom, the average distance that an electron would move before colliding with another electron would be 6400 cm (allowing for the velocity of the electron being struck). A careful calculation has been made, based on Coulomb's law, of the effect of the electric force between the electrons in increasing the transfer of momentum by collisions. It is found that the effective free path varies with the electron velocity, but that the free path is still so great that this kind of interaction between electrons must be completely rejected as a possible mechanism for bringing about the observed Maxwellian distribution.

Effect of the charged glass walls.—The glass walls of the discharge tube are negatively charged because of the fact that the electron current density in the discharge is greater than the positive ion current density. In accordance with observation we may take the electron current in mercury vapor to be 400 times the ion current. If the electron temperature is 30,000° it can then be calculated (in agreement with experiment) that the walls will be charged to −15.5 volts with respect to the ionized gas and will thus be covered by a positive ion sheath. All electrons which have energy components normal to the surface of the glass of less than 15.5 volts will be specularly reflected from the walls without loss of energy, but electrons having energy components greater than this will pass through the sheath and be taken up by the walls except for the few that are reflected back. Thus the walls are continually disturbing the Maxwellian distribution by removing the high speed electrons. We should therefore expect distinct departures from a straight line semi-logarithmic plot at collector voltages below that to which the glass walls become charged, unless there is a mechanism which effectively re-establishes the Maxwellian

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distribution within a distance less than that which separates the collector from the opposing glass walls.

In the experiments made by Harold Mott-Smith Jr. and the writer with mercury arcs in a tube 3 cm in diameter,[4] the collector which was designated *H* consisted of a square nickel plate 1.9 cm on a side which had been bent into a cylindrical surface which conformed to the inner surface of the tube. If electrons traverse the tube in straight paths with constant velocity and are specularly reflected from the cylindrical walls, the radial velocity components in successive collisions with the walls remain constant. If then the walls of the tube are negatively charged to say — 15.5 volts *few electrons can reach a cylindrical collector such as H, with radial components greater than 15.5 volts unless these electrons have acquired this large radial velocity within the distance they have travelled since their last collision with the wall.*

The experiments showed in all cases that the straight semi-logarithmic plot extended to negative voltages far below those to which the collector *H* charged itself when it was allowed to float. This must be taken as proof that the higher velocity electrons were produced from low velocity electrons while these travelled a distance of less than 3 cm.

From the complete absence of a kink in the semi-logarithmic plot at the wall potential we must conclude that the time of relaxation corresponding to the mechanism by which the electrons acquire their Maxwellian distribution is small compared to the time taken for the electrons to traverse the tube, in other words, that the free path of the electrons corresponding to this mechanism is small compared to 3 cm.

Effect of charged conducting walls.—Recently Mott-Smith has carried out experiments with a mercury arc at low pressures passing through a metallic cylinder split lengthwise into two halves, which are insulated from one another. One half was used as a collector while the other was maintained at a series of different negative voltages to see if the velocity distribution of the electrons reaching one electrode was modified in any way as different groups of electrons were removed by means of the opposite electrode. It was found in fact that by charging one electrode so strongly negative that even the high speed electrons were reflected from it, the temperature of the electrons collected by the opposite half-cylinder was distinctly higher than if the electrode were at only —15 volts with respect to the gas so that all the higher speed electrons were removed. ***But in both cases a perfectly straight semi-logarithmic plot was obtained.*** The removal of the high speed electrons lowered the temperature, but the Maxwellian distribution was established while the electrons moved between the two collectors. Experiments are now in progress by A. F. Dittmer using plane collectors whose distance apart can be varied, to determine directly the rate at which the Maxwellian distribution is brought about.

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[4] *Loc. cit.*,[2] p. 538.

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Evidence as to the mean free path of electrons.[5]—If a small potential gradient dV/dx is allowed to act on electrons having a Maxwellian velocity distribution, the electrons drift in the field with an average velocity v_x given by the mobility equation

$$v_x = 4\gamma\lambda/\pi v \tag{1}$$

where γ, the acceleration of the electrons is

$$\gamma = \frac{e}{m}\frac{dV}{dx}, \tag{2}$$

λ is the mean free path of the electrons and v is the average velocity of the electrons (assumed to follow Maxwell's law). Expressing v in terms of T_e and inserting the value of e/m we get

$$v_x = 3.62\times10^{9}\cdot\frac{\lambda}{\sqrt{T_e}}\frac{dV}{dx} \tag{3}$$

where v_x, λ and x are expressed in cm-second units and V is in volts.

The current density I_x of the arc is related to v_1 by the equation

$$I_x = n_e e v_x$$

or if I_x is in amp per cm^2

$$I_x = 1.59\times10^{-19}\ n_e v_x. \tag{4}$$

Combining these equations

$$I_x = 5.76\times10^{-10}\frac{n_e\lambda}{\sqrt{T_e}}\frac{dV}{dx}. \tag{5}$$

With the 2 ampere arc in mercury vapor at 1 bar in a tube 3.2 cm in diameter the current density I_x was 0.25 amp per cm^2, $n_e = 8\times10^{10}$, $dV/dx = 0.25$ volts per cm, and $T_e = 30{,}000°$. Substituting these into Eq. (5) iand solving for λ we find that the free path λ of the electrons in the arc s 3.8 cm. Direct measurement of the number of 50 to 250 volt electrons which pass through ionized mercury vapor at 1 bar without collision with atoms, give a free path of at least 30 cm. The recent investigations of Ramsauer and others, show that for lower velocities (below those necessary to produce ionization), the free paths tend to increase. A large number of measurements of λ with arcs in mercury vapor at various pressures, in tubes of various diameters and with a wide range of current densities have been made in this laboratory. The free path calculated from the mobility always comes out to be 5 to 10 times lower than can be reconciled with direct determination or with values calculated by the usual methods of the kinetic theory.

[5] A brief note on the abnormally small values of the free paths of electrons obtained from the mobility and from the effects of tranverse magnetic fields was given in the last paper by Langmuir and Mott-Smith, *loc. cit.*,[3] p. 819.

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The application of transverse magnetic fields of 50 to 100 gauss to a mercury arc at low pressures causes the arc to be deflected to one side. By measuring the *ratio of the electron concentrations* at two points along a diameter of the tube which is perpendicular to the magnetic field, it is possible to calculate the free path by a method entirely independent of that involving the mobility. This method also gives free paths about 10 times smaller than the distances which electrons travel before colliding with atoms as obtained by direct measurement.

Both of these results as well as the rapid establishment of the Maxwellian distribution of electron velocities, suggest that *the electrons in a mercury arc suffer many changes in momentum during the time that elapses between consecutive collisions with atoms.*

Summary.—This discussion of the results obtained from a study of low pressure arcs shows: (1) that the free electrons have velocities with a Maxwel- distribution corresponding (in the case of mercury vapor at 1 bar) to a temperature of 30,000°; (2) that this distribution is maintained even when the walls are negatively charged and hence are constantly removing the faster electrons; (3) that the number of collisions with atoms and electrons is far too small to maintain this distribution, the mean free path being of the order of the tube diameter; (4) that mobility experiments indicate that the electrons suffer at least ten changes of momentum between consecutive collisions with atoms. A possible explanation of these results will be presented and discussed in Part IV below.

II. Electrons Accelerated from a Hot Cathode

These Maxwellian distributions occur not only in ordinary low pressure arcs but are produced in gaseous discharges under many different conditions. We will now consider some experiments which have been made with spherical bulbs or cylindrical vessels containing a hot tungsten cathode and a disk-shaped anode. With gases at low pressures, currents of the order of ten milliamperes and anode potentials of 50 to 250 volts, the space potential in the strongly ionized gas is nearly uniform and is somewhat above that of the anode. The cathode is surrounded by a positive ion sheath in which there is a sharp potential drop so that the electrons from the cathode are accelerated within a distance of a fraction of a millimeter, to a velocity exceeding that which corresponds to the potential difference between the anode and cathode.

Recently a very large number of experiments have been made by S. P. Sweetser, C. G. Found and H. A. Jones to investigate the production of these high speed electrons. A brief report of this work was made at the April meeting of the Physical Society[6] in Washington.

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[6] Langmuir, Abstract in *Phys. Rev.* **25,** 891 (1925).

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In one set of experiments (Exp. 548) a straight tungsten filament 1.1 cm long and 0.18 mm in diameter was mounted within about 2 cm of the center of a spherical bulb of 12 cm diameter. A disk-shaped collector 1.1 cm in diameter, backed by mica and movable in the direction of its axis, was mounted so that the axis passed through and was perpendicular to the filament at its center. The lead to the disk was insulated so that the collector could receive electrons only on the surface facing the filament. An anode was also placed in the bulb to one side of the line connecting the other electrodes.

By lighting the filament so that it emitted 5 milli-amperes and applying −50 volts to the cathode with respect to the anode (assumed to be at zero volts) the mercury vapor became strongly ionized and its potential was nearly uniform and equal to that of the anode. The electrons emitted by the filament acquired their full velocity within a few tenths mm and were therefore projected outwards in directions normal to the filament, thus forming a disk-shaped "beam" of electrons. Those electrons which reach the collector with a large part of the momentum which they acquired in passing through the positive ion sheath, we shall call *primary electrons*. The primary electrons in this beam which were intercepted by the collector were moving perpendicularly to its surface. To prevent these electrons from reaching the collector it was necessary to apply a potential lower than that of the cathode.

Thus as the collector passed through the potential of the cathode, the current-voltage curve of the collector showed a sudden change in current which was a measure of the number of primary electrons which reached the collector without collisions. This discontinuity was very sharp when the cathode emission was only 5 m-amp taking place within a range of only about 2 volts, which was not greater than would be expected because of such effects as the initial velocities of the emitted electrons and the voltage drop along the filament. Thus no appreciable number of electrons was present in the gas with velocities higher than those of the primaries.

Scattering of primary electrons.—When the electron emission was raised to 10 m-amp however, there were found to be unmistakable indications of the presence of electrons having velocities higher than the original primaries. Fig. 1 shows some typical data taken with 10 m-amp, with a cathode potential of −50 volts, and a distance of 3 cm between filament and collector. The mercury vapor pressure was only 0.23 bar corresponding to saturated vapor at 0°C. The curve marked I_b gives the collector current in micro-amperes as a function of the collector potential. The curves 0.1 I_b and 0.01 I_b represent the same data on a scale 10 and 100 times smaller so as to bring the upper portions of the curve within the plot.

It is seen that at collector potentials below −60 only positive ions are collected. The currents vary with the potential because of an "edge correction" which causes[7] the area of the positive ion sheath to increase as the sheath thick-

[7] See *loc. cit.*,[8] p. 540.

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ness increases. The dotted line *A* shows how this current would vary at higher potentials if no electrons were collected. At potentials above —59 volts, at point *M*, electrons being to be collected, showing the presence of appreciable numbers of electrons having energies of from 52 to 59 volts although the energy of the primaries was only 50 volts.

From the data of Fig. 1 we see that with the collector at —40 at point *N* practically all the primaries have been collected. With a cathode current of only 5 m-amp on the other hand, the primaries were all collected with

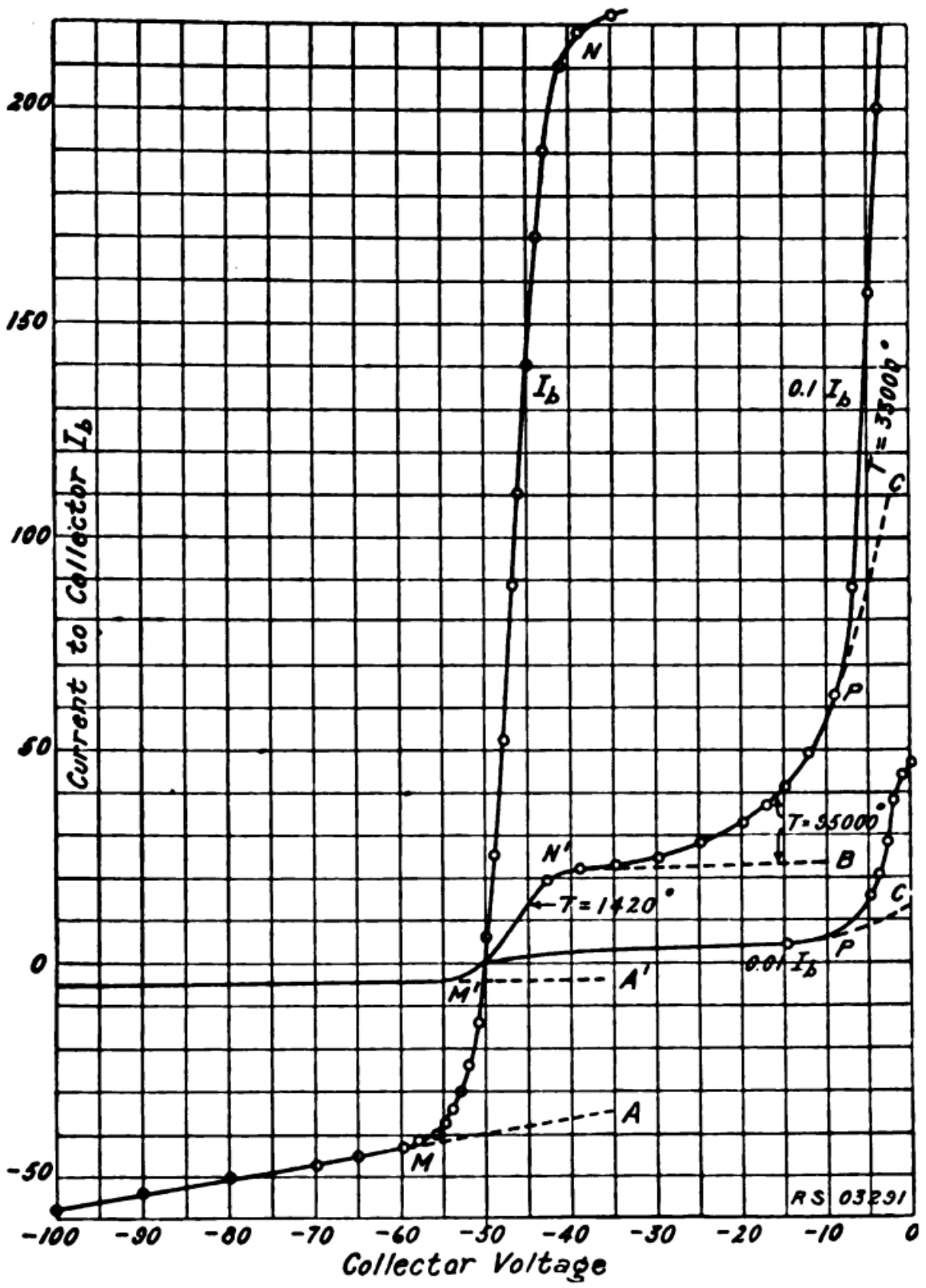

FIG. 1. Electron scattering in mercury vapor at 0.23 bar (0°C) with 10 m-amp primary current and the cathode at —50 volts.

voltages of about —47. This shows that the effect of raising the primary current from 5 to 10 m-amp was to scatter the primary electrons, causing not only an increase in the velocities of some of the primaries but a decrease in the velocity of others. The average energy of all the primaries was slightly

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lowered to about 47 volts. The total number of primary electrons collected was twice as great with 10 as with 5 m-amp cathode current, so that the effective free path of the injected electrons remained the same, if by free path we mean the distance that the electrons travel before losing the greater part of their momentum in their original direction.

By subtracting from the observed currents (Fig. 1) the positive ion currents corresponding to the dotted line A we obtain the total electron current ΔI_b to the collector. In Fig. 2 the logarithms of these currents in micro-amperes have been plotted against the collector voltage. In this way the data for from -55 to 0 volts are brought within a single convenient curve. The rise MN which occurs between -55 and -40 volts corresponds to the primary electrons. If these all had the original energy of 50 volts the rise should occur as a vertical line at an abscissa of -50 volts. The inclination of the line from the vertical thus measures the amount of electron scattering.

The line $N'B$ in Fig. 1, corresponding to the horizontal dotted line NB in Fig. 2, gives the current as it would be if no other electrons than the primaries were collected. The differences between the observed currents $N'P$ and those of line $N'B$ (Fig. 1) thus correspond to a new group of electrons which probably result from collisions of the primary electrons with gas molecules. We shall call these *secondary electrons*. This group includes primaries which have lost most of their energy and also electrons which are ejected from gas molecules by these collisions.

The distribution of velocities among the secondary electrons is found by taking the differences between the observed currents and the current corresponding to $N'B$ (Fig. 1), and by plotting the logarithms of these differences as indicated by the points along the line D (Fig. 2). Within the range from -30 to -12 volts the points lie on a straight line. This proves that the velocity distribution follows Maxwell's law. From the slope the temperature of this secondary electron group is found to be 95,000°. By extrapolation to higher voltages along the straight line D, the currents corresponding to the secondary electrons are calculated as indicated by the dotted line PC. Thus the line NPC is the curve of the secondary electrons.

The difference between the observed currents PR and the secondary currents PC (Fig. 1) gives another group of electrons which we will see have a Maxwellian velocity distribution and are in fact the same as the free electrons found in the positive column of the low voltage discharge. We shall call these the *ultimate electrons*. If the currents due to these are small compared to the current flowing to the anode the logarithms of the currents may be plotted and the slope of the resulting straight line taken to measure their temperature. But it often happens that the concentration of the ultimate electrons is so high that the current taken by the collector becomes comparable with that flowing to the anode. The collector may, in fact, rob the anode of nearly all its current and thus raise the space potential along with the collector potential. In these

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cases, as mathematical analysis shows, we should plot the logarithm of the ratio of the electron current to the collector, to the current to the anode. The slope of this line as indicated by line E measures the temperature of the ultimate electrons. In view of the logarithmic scale of Fig. 2 it will be seen that the number of ultimate electrons greatly exceeds that of the primary or secondary electrons. The relative numbers of electrons in the various groups is seen more clearly in Fig. 1 because there the currents themselves are plotted as ordinates. The dotted lines $N'B$ and PC on Fig. 1 are drawn to correspond to those in Fig. 2.

In other experiments disk-shaped collectors were mounted close to the primary electron beam but in a plane perpendicular to the axis of the cathode so as to be parallel to the direction of the electron beam. The current to such a collector gives no indication whatever of the primary electrons corresponding

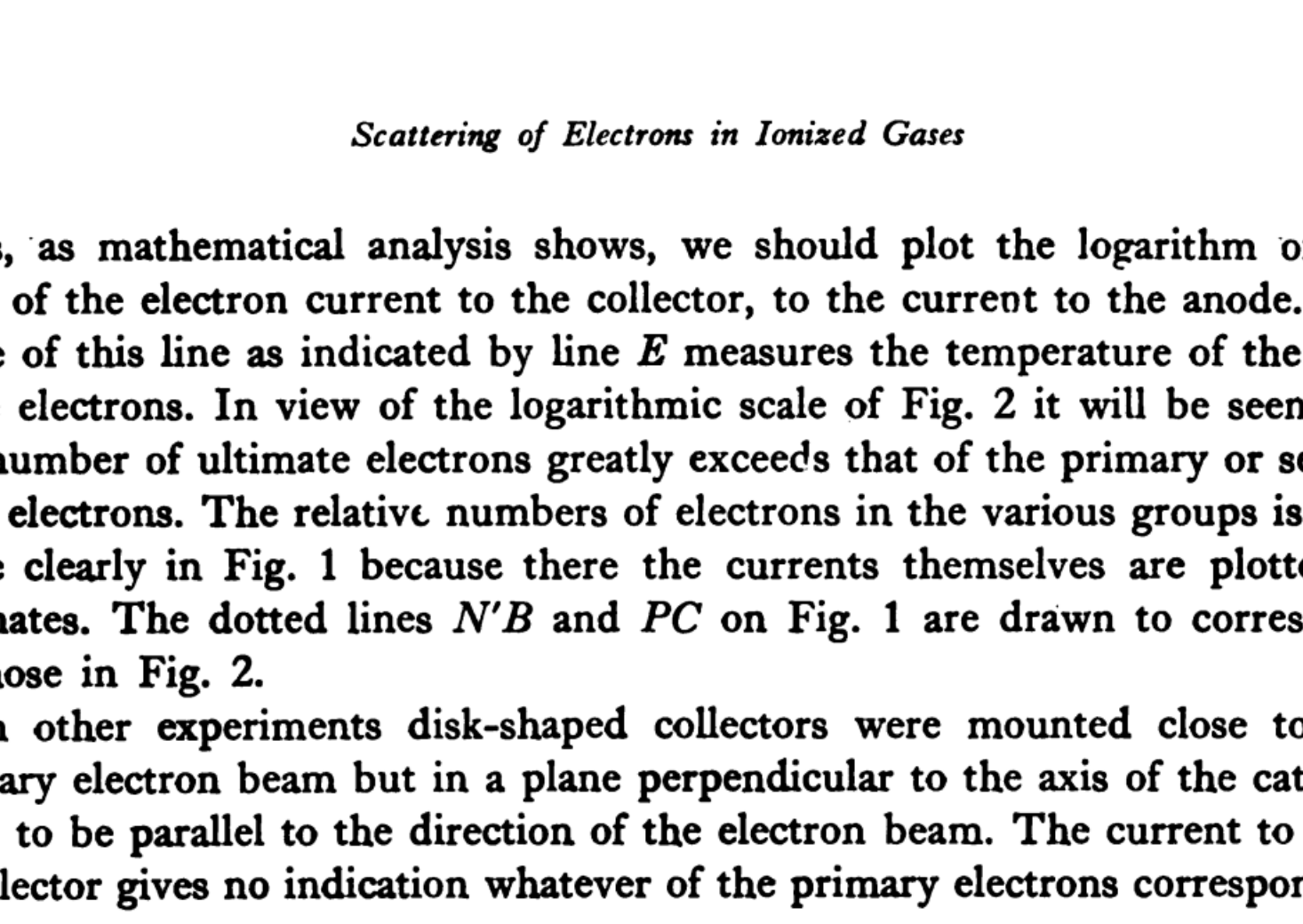

FIG. 2. Semi-logarithmic plot of the electron currents from the data of Fig. 1.

to the rise MN, but the groups of secondary and ultimate electrons corresponding to the rises between NP and PR are exactly the same as those shown in Fig. 2 for a collector mounted perpendicular to the beam. Such experiments justify our separation of the electrons into groups of primaries and secondaries and establish the fact that the primaries move mainly in one direction while the secondaries and ultimate electrons move equally in all directions.

In our present discussion our interest centers on the *distribution of the velocities of the primary electrons* after they have passed through the ionized gas. The portions of the curves MN in Figs. 1 and 2 should give us this distribution.

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In order to analyze such curves mathematically the hypothesis was made that the scattering of the primaries takes place in such a way that the resultant velocity distribution can be analyzed into a uniform translational velocity and a superposed Maxwellian distribution corresponding to a temperature T — this distribution being analogous to that of the molecules of a gas which is moving with uniform velocity.

We can now calculate what fraction of the electrons in such a beam can move against a given retarding field like that in the sheath of a negatively charged plane collector. Assuming that the velocities due to temperature motion are small compared to those of translation it can be shown that the current I of electrons which reaches a collector is given by

$$I = \tfrac{1}{2} I_0(1+\mathrm{erf}\beta) \tag{6}$$

where erf denotes the error function

$$\mathrm{erf}\, x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx. \tag{7}$$

I_0 is the current when all the primary electrons are collected and β is defined by the equation

$$\beta = (\sqrt{-V} - \sqrt{V_1})/\sqrt{V_T}\,. \tag{8}$$

Here V is the potential of the collector with respect to the ionized gas (or anode), V_1 is the potential corresponding to the translational velocity of the primary electrons and V_T is the potential corresponding to the energy kT, k being the Boltzmann constant. If V_T is expressed in volts

$$V_T = T/11{,}600. \tag{9}$$

When the collector potential is not too far from that of the cathode the value of β from Eq. (8) may be expanded giving

$$\beta = \mathrm{const} + V/(2\sqrt{V_1 V_T}). \tag{10}$$

Thus β is a linear function of the collector potential and from Eqs. (6) and (10) we then see that $\mathrm{erf}^{-1}[(2I-I_0)/I_0]$ is a linear function of V having a slope equal to $1/(2\sqrt{V_1 V_T})$. If this function of I is plotted against V and a straight line is obtained it proves the existence of the kind of velocity distribution we have assumed, and enables us to determine the temperature corresponding to the random part of the motion. These operations may be more conveniently carried out by using probability paper in which the vertical lines are uniformly spaced and the horizontal lines, which are denoted by various values of a parameter a, are spaced according to the relation

$$y = \mathrm{erf}^{-1}(2a-1)$$

Temperature of primary electrons.—In Fig. 3 the primary electron currents corresponding to the data of Figs. 1 and 2 are plotted on probability paper.

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The abscissas are the collector voltages while the ordinates are $\Delta I_b/I_0$ where I_0 has been taken as 257 micro-amperes in accordance with the data of Fig. 1. It is seen that the observed points lie very close to a straight line. The difference in voltage corresponding to the 2 points $I/I_0 = 0.1$ and 0.9 is 9.0 volts. The increment of $\text{erf}^{-1}(2a-1)$ corresponding to these values is 1.812. Thus by (10)

$$dV/d\beta = 9/1.812 = 2\sqrt{V_1 V_T}.$$

Since the energy of the primary electrons in the beam corresponds to 50 volts we place $V_1 = 50$ and thus find $V_T = 0.122$ volts or by (9) $T = 1420°$. The voltage on the probability plot (Fig. 3) which corresponds to the midpoint (i.e. where $I/I_0 = 0.5$) measures the average translational velocity of the electron beam. Thus from Fig. 3 the forward velocity of the beam corresponds to 47.2

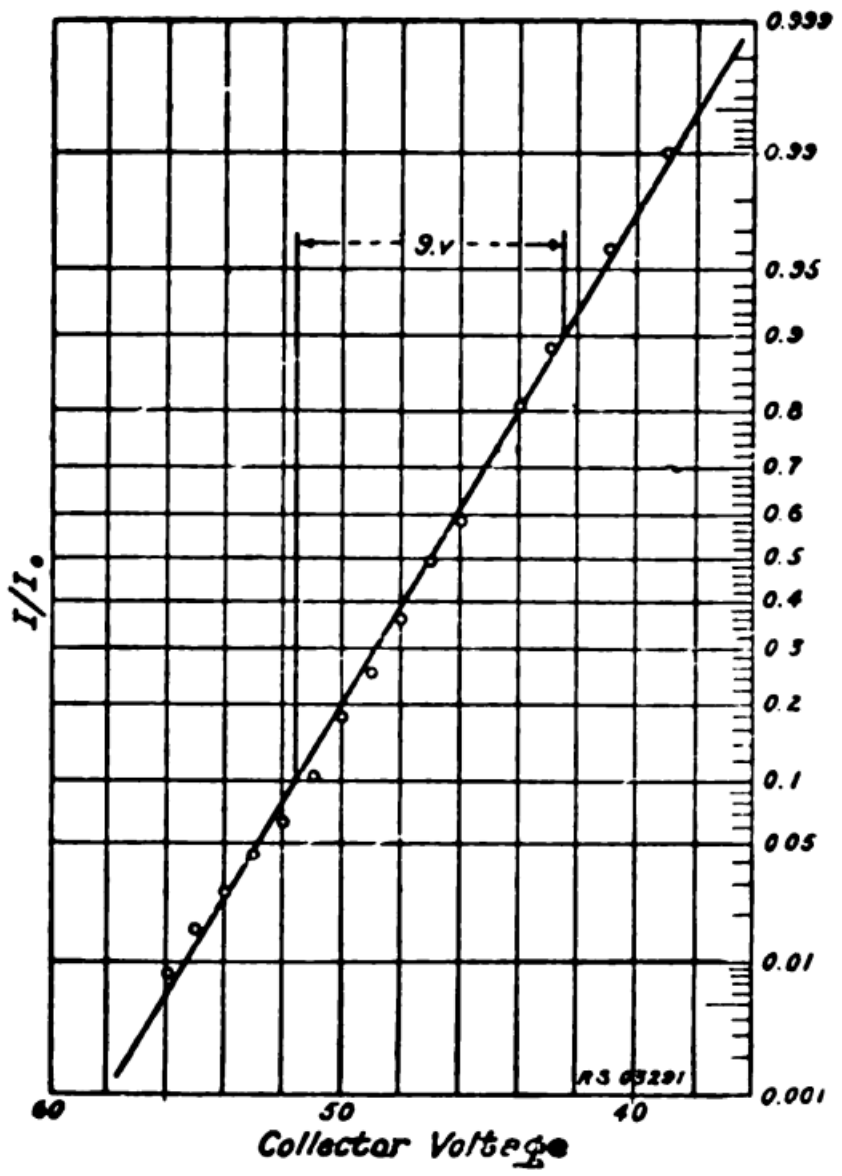

FIG. 3. Probability plot of the primary electron current from data of Fig. 1.

volts although the original velocity of the primary electrons corresponded to 50 volts. The mechanism producing the scattering therefore does not act uniformly in all directions but produces the greatest changes in momentum in a direction opposite to that of the original beam.

The data from large numbers of experiments, of which the data of Fig. 3 are typical, confirm the correctness of the hypothesis that the primary electrons have random Maxwellian velocities superposed upon the translational velocity. The degree of scattering in this case is very small as the voltage corresponding

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to the random velocities is only 0.122 while the voltage of the translational velocity is 47.

Variation with current density.—To illustrate the effects produced by modification in the experimental conditions we may consider another typical run made with the same apparatus. The pressure of the mercury vapor was 1.6 bars corresponding to saturated vapor at 20°C. The electron emission from the cathode was 30 m-amp. The cathode was at a potential of —50 volts with respect to the anode, and the distance from the collector to the cathode was 3 cm as in the run illustrated in Figs. 1 to 3. The data were plotted and analyzed as before. Appreciable electron currents were observed with collector potentials as low as —90 volts showing that some electrons (about 3 micro-amperes) having energies as high as 90 volts had been produced from the original beam of 50 volt electrons. The secondary electrons gave a straight semi-logarithmic plot corresponding to $T = 95{,}000°$ which is the same temperature as found from the data of Fig. 1. The temperature of the group of ultimate electrons was 15,600°. The plot of the primary electron current to the collector, on probability paper, gave 17 consecutive points in the voltage range from —90 to —10 volts which came very close to a straight line (maximum departure 3 volts) whose slope corresponded to $T = 56{,}000°$. The mid-point of this line was at —36 volts that the beam had been retarded by an amount corresponding to 14 volts.

The very much higher temperature of the primary electrons in this experiment as compared to the previously described run is due mainly to the higher primary current density, but is partly due to the higher pressure of mercury vapor.

Fig. 4 illustrates the current-voltage curves of the collector for potentials below that of the cathode with a series of different primary current densities. The figures written near the curves are the electron emissions from the cathode in milli-amperes. Three different cathode potentials were used, —35, —50 and —70 volts. The actual experimental points are indicated in only one curve, which being typical, illustrates the accuracy with which the observations fit the curves.

The moderately sloping parts of the curves, which are continued as dotted lines to the right, give the positive ion currents. The departures from these lines measure, as before, the primary electron currents. It will be noted that with 5 m-amp the electrons begin to be collected only when the collector comes within 1 or 2 volts of the cathode potential, showing that there are very few electrons having higher velocities than those of the original primaries. As the current density is increased to 10 or 20 m-amp, however, electrons begin to be collected at potentials of 10 or even 20 volts below that of the cathode. The effect seems to be somewhat more marked at 35 and at 50 volts than at 70 volts.

In a series of later experiments the primary electron temperatures were determined for each of a series of primary current densities corresponding

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to electron emissions ranging from 3 to 30 m-amp. The mercury vapor pressure was 0.23 bars (0°C), the collector distance 3 cm, and the cathode potential —50 volts. The primary electron temperature was found, within the experimental error, to be proportional to the 2.2 ± 0.2 power of the current.

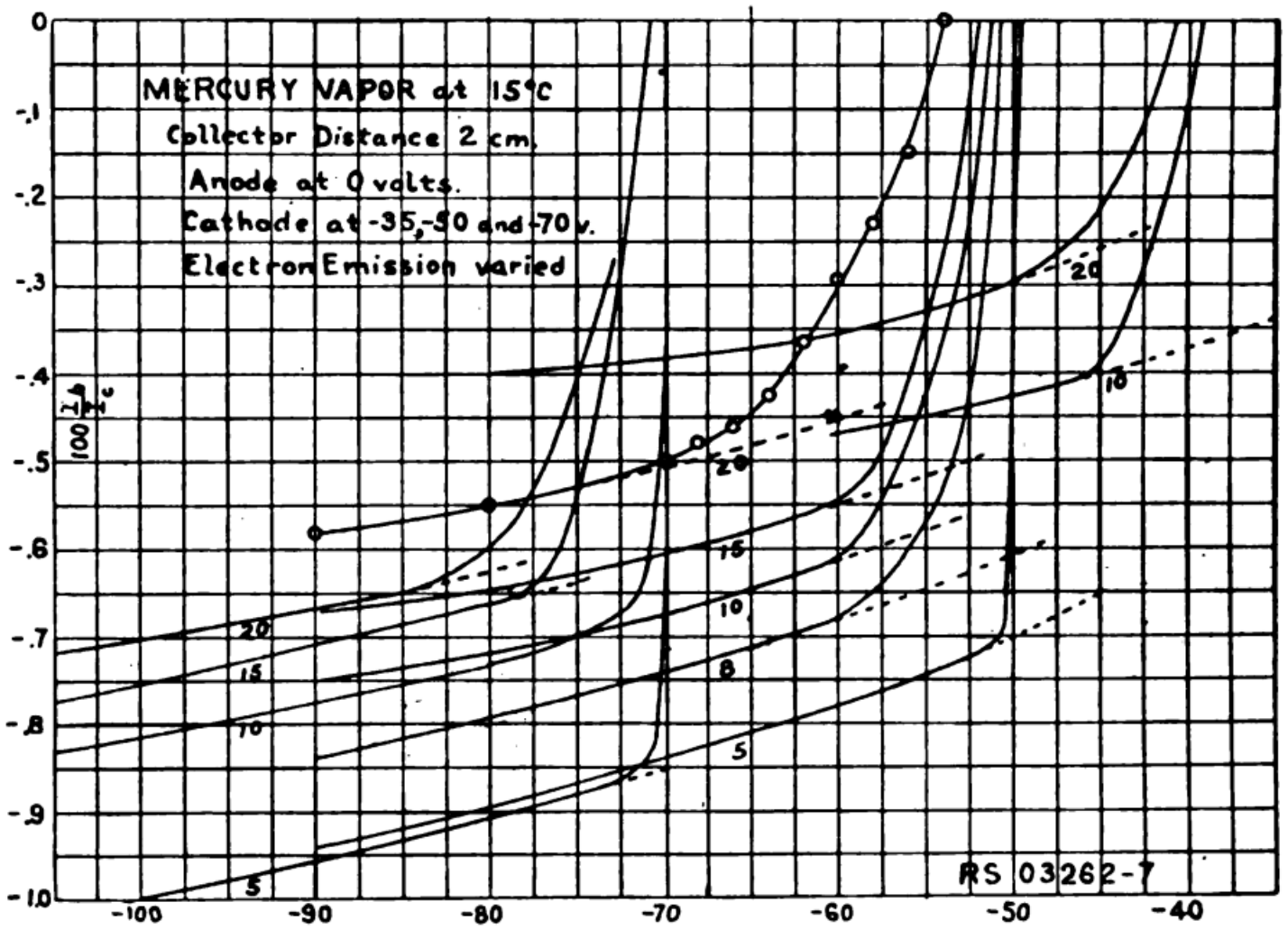

FIG. 4. The effects of cathode voltage and current density on the scattering of electrons. The ordinates give the ratio I_b/I_c where I_b is the negative current received by the collector and I_c is the electron emission from the cathode. The figures marking the curves are the values of I_c in milli-amperes.

Variation with the distance of the collector.—Experiments in which different distances between the cathode and the collector were used have shown that the primary electron temperatures increase roughly in proportion to the distance traversed by the electrons. Further experiments on this effect are in progress.

Variation with cathode voltage.—In a series of experiments with mercury vapor at 0.23 bar (0°C) with a collector distance of 3 cm and a current of 10 m-amp the primary electron temperatures varied with the voltage as follows:

voltage:	—80	—70	—50	—40	—35
T:	800°	900°	1300°	1700°	2200°

With 20 m-amp current, the other conditions being the same, the temperatures were

at —80 volts	at —50 volts
2050°	4750°

The greatest scattering of the electrons thus occurs at the lowest voltages, but, in general it is difficult and unsatisfactory to work with voltages below 50 volts because the effects of the scattering then tend to be masked by the secondary

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and ultimate electrons. As the voltage difference between cathode and anode decreases not only does the velocity of the primary electrons decrease but the character of the excitation of the ionized gas may change. For example, with higher voltage, atoms may be excited to higher energy levels. These experiments do not enable us to determine which of these factors is responsible for the change in the amount of scattering.

TABLE I

Effect of Mercury Vapor Pressure on Electron Scattering

Primary current (m-amp)	Mercury		Electron temperature	Retardation (volts)	$\frac{\Delta E_l}{\Delta E_r}$
	Temp.	Pressure (bars)			
10	0°	0.23	1360°	3.1	17.6
	20	1.6	870	2.3	20.4
	30	3.7	240	1.8	58.
	40	8.0	140	1.4	77.
20	0	0.23	5500	5.0	7.0
	5	0.37	7100	4.5	4.9
	10	0.61	9800	4.8	3.8
	15	1.0	5900	3.5	4.6
	20	1.6	4000	3.2	6.2
	35	5.5	750	1.8	18.5

Variation with mercury vapor pressure.—Table I shows the way that the electron scattering varied with the vapor pressure of mercury. The cathode was at −50 volts and the collector distance was 3 cm. The last column gives the retardation of the beam or the decrease in the translational energy of the beam as determined from the voltage of the mid-point of the straight line on the probability plot.

Experiments with crossed beams.—The fact that the electron scattering is negligible at low current density and increases rapidly at higher currents suggests that it is caused by the action of radiation or by excited atoms or ions. To gain more information as to the effect of ionization and excitation of the gas a spherical bulb (12 cm in diameter) was provided with two straight cathode filaments, A and B, each 1.1 cm long, mounted in the same plane at right angles to each other. Facing these cathodes at distances of 4 cm there were disk-shaped collectors (A' and B') of 1.1 cm diameter, the axes of these disks being in the same plane as the two cathodes.

By lighting only one cathode, A, a single disk-shaped beam of primary electrons (about 1 cm thick) was produced. The two collectors A' and B' being about the same distance from the center of the bulb, received the same number of secondary and ultimate electrons, but since B' was outside of the beam (and its surface was parallel to the direction of the beam) only the collector A' received primary electrons.

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When both cathodes were lighted the volt-ampere characteristics of the two collectors gave the amount of scattering of each of the two perpendicular electron beams, and thus made it possible to determine the influence of one electron beam on the other.

It was found that the temperature and the number of both the *secondary* and the *ultimate* electrons were always very nearly the same for both collectors whether or not the collector was in the beam of primaries. The temperatures of both these groups were independent of the currents from the cathodes. The number of electrons in each of the 2 groups was a function of the sum of the currents emitted by the two cathodes and was in fact approximately proportional to this sum. Thus it was immaterial whether one cathode emitted 20 m-amp while the other emitted none, or each of the cathodes emitted 10 m-amp.

The behavior of the primary electrons was entirely different. The total number of primaries that could be received by each of the two collectors was of course proportional to the electron emission of the opposing cathode. If the scattering of the primary electrons were due to the free electrons, or ions, or to the kind of excited atoms and radiation that is produced by the secondary and the ultimate electrons, we should expect that the temperatures of the primary electrons should be determined by the temperatures and the concentrations of these two groups of electrons. Thus the temperatures of the primaries should also be a function of the sum of the emissions from the two cathodes.

The experiments showed, however, that the primary electron temperature depended mainly on the emission of electrons from the cathode which opposed the collector and was usually only slightly influenced by the emission from the other cathode.

For example, in some runs with mercury vapor at 0.75 bar (12°C) with both cathodes at −80 volts, cathode A was heated to such a temperature that it emitted 5 m-amp while B was adjusted to emit 20 m-amp. The temperature of the primaries reaching A' was 800° while those reaching B' had a temperature 5200°. When cathodes A and B were at the same temperature and each emitted 20 m-amp, the temperatures of the electrons received by each collector was still about 5000°. By then turning off the heating current to one of the cathodes, say A, so that its emission fell to zero, the temperature of the electrons received by the collector B' facing the other filament was practically unchanged notwithstanding the fact that the concentration of the secondary and ultimate electrons and of the positive ions throughout the whole space in the bulb, fell to about half value.

With mercury vapor pressures corresponding to 20° and especially to 30°C one beam of primary electrons seemed to be able to cause a distinct increase in the temperature of the other beam (which it intersected for about 1/4 of its length). For example, at 30° and with both cathodes at −80 volts but only one cathode lighted so as to emit 20 m-amp, the electron temperature observed

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at the opposing collector was 700°. By allowing the other cathode also to emit 20 m-amp both beams of primaries gave a temperature of 2500° thus showing a marked increase in temperature caused by the other beam. However, if one cathode was heated to emit 40 m-amp while the other was cold the temperature of the beam was 20,000° or 8 times as high as when each cathode emitted 20 m-amp, although the concentrations of the secondary, and ultimate electrons was the same in the two cases. Evidently with mercury at 30° the exponent of the power of the current with which the electron temperature increases is higher than it is with vapor from mercury at 0°C.

From these results we may draw the definite conclusion that the main cause of scattering of 80 volt electrons, especially at low pressures of mercury vapor, is not the ionization, excitation or radiation produced by, or characteristic of the secondary or the ultimate electrons, but the scattering must be due to a more direct effect of the primary electrons and one which is confined to the region actually traversed by these primary electrons. This indicates that the scattering cannot be due to electrons or ions *per se*. It leaves open, however, the question as to whether radiation or excited atoms may be the cause. Thus 80 volt primary electrons are capable of exciting mercury atoms to energy levels far above those obtainable from the secondary or the ultimate electrons, and radiations of correspondingly high frequency are similarly produced. Excitation and radiation of this kind might well be confined to the region traversed by the primaries and might thus explain the results of the experiments. The distinct but slight effect of one beam of electrons on the other would then be due to the action taking place within the region (about 25 per cent of the total path) where the two beams intersect. Experiments are now in progress to study parallel beams and beams with intersect over nearly their whole lengths in order to clear up this question.*

In some other experiments an arc of 0.2 to 0.5 ampere was passed through mercury vapor in a 12 cm bulb from an external mercury cathode while the distribution of the velocities of 50 volt primary electrons from a hot cathode in the bulb was studied. It was found that the ionization due to the arc did not increase the primary electron temperatures.

III. Mechanism of Electron Scattering

Before taking up this general problem, let us enquire whether the scattering of the electron beam can be due to encounters with other electrons. Since the electrons in the beam start out from the cathode with the same velocity, they

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* *Note added Sept. 2, 1925.* In some recent experiments two straight tungsten filaments 1 cm long were mounted parallel to one another at a distance of about 1 mm, so that two electron beams could be obtained that were practically coincident. In this case the primary electron temperature depended only on the *sum* of the electron emissions from the two cathodes, showing that the scattering did not occur within the cathode sheath but resulted from some effect taking place throughout the space traversed by the primary electrons.

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have no velocity with respect to one another and therefore cannot deliver energy to one another by collision. Space charge due to the electrons in the beam is neutralized by positive ions, but even if it were not, the electric field due to space charge could produce only transverse scattering and would not change the longitudinal velocities of the electrons.

Scattering by clouds of stationary electrons.—If the scattering is to be produced by electrons, it must be due to the electrons in the ionized gas and these as we have seen have velocities much lower than those of the primaries. Let us therefore consider the scattering of a beam of electrons when it enters a cloud of stationary electrons. If we assume that the collisions follow the laws of elastic spheres we find that an electron which has made one collision with a stationary electron has on the average lost one half its momentum in the forward direction and has given half its energy to the other electron. Thus if we have a beam of electrons which are moving with a homogeneous velocity corresponding to a voltage V_0 and we let this beam encounter a cloud of stationary electrons so that each electron in the beam makes one collision with an electron, we find that the translational velocity of the beam is reduced to a value corresponding to $\frac{1}{4}V_0$. An observer, moving along with this retarded beam, will see that the electrons in the beam are now moving in random directions with velocities corresponding to the energy $\frac{1}{4}V_0$. The effect of a single collison for each electron thus causes a decrease in the longitudinal energy of the beam of ΔE_l and an increase in the *random* energy ΔE_r given by

$$-\Delta E_l = \tfrac{3}{4}V_0 e \tag{11}$$

$$\Delta E_r = \tfrac{1}{4}V_0 e \tag{12}$$

the ratio being

$$-\Delta E_l/\Delta E_r = 3 \tag{13}$$

The electrons in the retarded beam cannot, however, in this case get a Maxwellian velocity distribution. There is no mechanism by which an electron can acquire velocities exceeding those in the original beam. If the number of electrons in the cloud is so small that the average decrease in the longitudinal energy is, say, 5 per cent (about the average observed in our experiments) then 93.3 per cent of the electrons have made no collisions at all and must retain all their original translational energy while 6.7 per cent have lost 3/4 of their original energy. This would give a velocity distribution having no resemblance to a Maxwellian distribution. Thus we must conclude that the observed scattering cannot be due to the direct action of electrons upon those in the beam.

Scattering by clouds of entities of very small mass.—If the particles in the cloud have masses m_p negligibly small in comparison to those of electrons, these

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difficulties disappear. In this case the fractional decrease in the translational velocity v_l of an electron in the beam is[8]

$$-\frac{\Delta v_l}{v_0} = \tfrac{4}{3}\frac{m_p}{m}n \tag{14}$$

where n is the number of collisions, and v_0 is the original velocity of the electrons in the beam. The decrease in longitudinal energy is thus

$$-\Delta E_l = \tfrac{4}{3}m_p v_0^2 n = \tfrac{8}{3}(m_p/m)nV_0 e. \tag{15}$$

A particle of velocity v_p colliding with an electron in the beam delivers its whole momentum $m_p v_p$ to the electron and the energy E thus acquired by the electron is

$$E = (m_p/m)E_p \tag{16}$$

where E_p is the kinetic energy of the particle. Since the energy of the particles which *strike* the electron is $2kT_p$ we have for the average random energy after n collisions

$$\Delta E_r = 2nkT_p(m_p/m). \tag{17}$$

This average energy corresponds to $\frac{3}{2}kT$ so that the temperature of the primary electrons after n collisions is

$$T = \tfrac{4}{3}nT_p(m_p/m). \tag{18}$$

By comparing Eqs. (15) and (17) we find for the ratio of the decrease in longitudinal energy to the increase in random energy

$$\frac{-\Delta E_l}{\Delta E_r} = \tfrac{4}{3}\frac{V_0 e}{kT_p}. \tag{19}$$

For convenience we may express T_p in terms of the equivalent voltage V_p defined by

$$V_p e = kT_p \tag{20}$$

so that (19) becomes

$$\frac{-\Delta E_l}{\Delta E_r} = \tfrac{4}{3}\frac{V_0}{V_p} \tag{21}$$

Scattering by quanta.—The momentum of a quantum of energy $h\nu$ is $h\nu/c$. If this quantum acts on an electron it may deliver to the electron any momentum between 0 and $2h\nu/c$ according to the direction taken by the scattered quantum. If the quanta are scattered with equal probability in all directions, as if by elastic spheres, the average momentum transferred may be taken to be $h\nu/c$. The average energy delivered to the electron by n collisions is

$$E_r = \tfrac{2}{3}n(h\nu)^2/mc^2. \tag{22}$$

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[8] See Eq. (789) in Jeans' *Dynamical Theory of Gases*, 2nd edn., p 282. Cambridge, 1916.

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Thus after n collisions with quanta the electron temperature is

$$T = (4/9)\, n (h\nu)^2 / kmc^2. \tag{23}$$

We may conveniently express the energy of the quanta $h\nu$ in terms of the equivalent voltage V_q according to the relation

$$h\nu = V_q e. \tag{24}$$

Introducing the numerical values of k, m, c and e and expressing V_q in volts, Eq. (23) thus reduces to

$$T = 0.0102 n V_q^2. \tag{25}$$

Thus, for example, with quanta of a frequency corresponding to 50 volts, the temperature of the electrons increase by 25° for each collision with quanta.

The fractional decrease in drift velocity of the electrons after n collisions with quanta is

$$-\Delta v_l / v_0 = 4nh\nu / 3mc^2 = 2.62 \times 10^{-6} n V_q \tag{26}$$

and the decrease in longitudinal energy is thus

$$-\Delta E_l = 4 v_0^2 nh\nu / 3c^2 = \tfrac{4}{3} (v_0/c)^2 n V_q e. \tag{27}$$

Comparing this with (22) we find

$$-\Delta E_l / \Delta E_r = 4 V_0 / V_q. \tag{28}$$

Relation between the retardation and the scattering.—Let us compare these calculated values of $\Delta E_l / \Delta E_r$ with the results of the experiments. In the experiment for which data were given in Fig. 1 the retardation corresponded to 2.8 volts and the electron temperature, due to scattering, was 1420°. The average energy in random direction E_r corresponds to $\frac{3}{2}kT$, so to express E_r in volts we divide the temperature by 2/3 of 11,600, or 7730. Thus we find $-\Delta E_l / \Delta E_r = 15.3$; i.e., the loss of energy by retardation is 15.3 times as great as the thermal energy of the scattered electrons.

Comparing the result with Eq. (28), we find $V_0 / V_q = 3.8$, and therefore $V_q = 13$ volts since $V_0 = 50$ volts. Thus quanta of a frequency corresponding to 13 volts would produce the ratio of retardation to scattering which was found by the experiments.

Rather variable results are obtained from other experiments. For example, the last column of Table I gives values of $-\Delta E_l / \Delta E_r$ ranging from 3.8 to 77, which would correspond to quanta having frequencies corresponding to voltages V_q from 53 down to 2.7. These values are all of reasonable order of magnitude, since radiation of such frequencies may well be present in gases excited by electrons of 50 volts energy.

If we assume that the scattering occurs according to the laws of the kinetic theory, by Eq. (21), the voltages corresponding to the energies of the particles will be 1/3 of those just found for V_q. These values also seem possible, although less probable, and we therefore cannot decide between the two theories from

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these data. Since, however, no entities other than quanta are known having masses less than those of electrons, we will confine our attention at present to scattering by radiation.

Magnitude of Compton effect.—According to the classical electromagnetic theory[9] the scattering coefficient of light by free electrons is

$$\alpha = 6\pi a^2 n_e \tag{29}$$

where α is the fraction of energy scattered per unit length of path through a region occupied by n_e electrons per unit volume and a is the radius of the electron by Lorentz's theory ($a = 1.88 \times 10^{-13}$ cm). Thus we find

$$\alpha = 6.7 \times 10^{-25} n_e \text{ per cm.} \tag{30}$$

According to the correspondence principle, we should expect approximately this amount of scattering even if the scattering takes place by quanta. For metallic aluminum $n_e = 8 \times 10^{23}$ electrons per cm^3, so we obtain $\alpha = 0.53$ per cm, which gives a scattering of the order of magnitude found for the Compton effect.[10] For a mercury arc n_e is of the order of 10^{10} electrons per cm^3, so that α should be about 10^{-14}. This is in accord with the fact[11] that no Tyndall cone is observed when a strong beam of sunlight is focussed by a large lens into a nearly non-luminous 2-ampere arc in argon at a few millimeters pressure.

With a primary electron current of 10 m-amp and 50 volts, the total energy escaping from the disk-shaped electron beam is about 0.02 watt per cm^2, or about 5×10^{15} quanta ($V_q = 25$ volts) per cm^2 per second. With a scattering coefficient of only 10^{-14} there would be 50 quanta scattered per cm^3 per second. To give the observed electron temperatures of 1420° each electron would have to make (according to Eq. 25) 230 collisions with quanta. Since there are 6×10^{16} electrons per second in the primary beam, the number of collisions per second would need to be 1.5×10^{19} or about 3×10^{17} per cm^3 per second, instead of the 50 collisions calculated above.

Thus the observed scattering is about 10^{16} times greater than that to be expected from a normal Compton effect.

This difficulty seems to be very similar to that met with in connection with the theory of chemical reaction velocity[12] where it is found that a chemical molecule must become activated millions of times more frequently than it collides with other molecules. Yet the energy that it can receive by radiation, according to the usual radiation laws, is millions of times too small to cause this activation.[13]

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[9] Leigh Page, *Astrophysical J.* **52,** 67 (1920).

[10] Since the frequencies of X-rays are sufficient to remove most of the electrons in aluminium, we may consider the electrons as free electrons in regard to the scattering of this kind of radiation.

[11] According to rough experiments in this laboratory.

[12] Langmuir, *J. Am. Chem. Soc.* **42,** 2190 (1920).

[13] See also a recent paper by R. C. Tolman, *J. Am. Chem. Soc.* **47,** 1524 (1925).

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Scattering by the joint action of radiation and excited atoms.—Because of the small magnitude of the Compton effect we must conclude that the scattering of electrons is not due to any direct interaction between them and radiation. It may well be, however, that the effect is due to an *interaction between radiation, excited atoms and electrons.*

Resonance radiation is enormously strongly scattered by excited atoms, practically all the radiation coming within a distance of about one-half wave-length from an excited atom being scattered, whereas the radiation scattered by an electron is only 6 times that falling on an area equal to the cross-section of the electron. For radiation corresponding to 25 volts the wave-length is 490A. The radiation scattered by an excited atom of a 25 volt energy level is thus about 10^{14} times greater than that scattered by an electron. This is about the order of magnitude of the ratio (10^{16}) of the observed scattering to that calculated for a normal Compton effect. If resonance radiation is trapped within the region traversed by the primary electrons, the ratio 10^{16} would be reduced, so that it might well be brought down to the value 10^{14} corresponding to the radiation scattered by excited atoms.

When a quantum of resonance radiation is scattered by an atom or excited atom the direction of the quantum is changed and the difference of momentum is available to deliver an impulse to some particle. Although we would expect in general that the momentum would be delivered to the atom or excited atom, it may well be that if an electron is close enough, the momentum may be delivered to the electron. In some such way as this we may suppose that the electrons can receive momenta whose magnitude is in accord with Compton's theory, although the probability that an electron may receive this momentum is 10^{14} times greater than according to the normal Compton effect.

Consider the excitation of a mercury atom by a quantum of resonance radiation (λ2537). The difference in the angular momentum of the atom before and after excitation must be a multiple of $h/2\pi$. Before the encounter, the two-body system, consisting of atom and quantum, will also contain an angular momentum equal to the product of the momentum $h\nu/c$ of the quantum, by the distance between the center of the atom and the path of the quantum. This angular momentum in general will not be a multiple of $h/2\pi$. After the collision we have a one-body system having a fixed angular momentum. Thus it would seem that a mercury atom can be excited by a quantum only if a third body is present which can take up the residual angular momentum. An electron within a distance comparable to a wave-length may serve as such a third body and may thus increase the ease with which excited atoms are formed. Perhaps without such third bodies, resonance radiation cannot even be scattered, although it is not necessary to draw this conclusion, since scattering involves a two-body problem both before and after the scattering. Experiments to determine whether the scattering coefficient of resonance radiation is dependent on the

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presence of free electrons are in progress by Mr. Whitten in this laboratory. An attempt will also be made to determine whether low velocity electrons in mercury vapor or sodium vapor are scattered when resonance radiation is allowed to act on the vapor.

The loss of energy by retardation of a beam of primary electrons is always much greater than the energy that appears as thermal energy of the scattered electrons. A beam of primary electrons may thus deliver energy directly to the radiation, or to excited atoms, by the reverse of the process that causes scattering of the electrons. Thus the average frequency of the radiation in the primary beam probably increases as the current density of the beam is raised. This may be the reason that the primary electron temperature increases with the square or even with a higher power of the current density.

As an alternative hypothesis to account for the large amount of electron scattering, we may assume that the presence of an electron within a distance comparable to a wave-length from an excited atom, greatly increases the probability that the excited atom will emit a quantum of radiation. Thus the average intensity of radiation near the electron would be much greater than that in the space as a whole and the magnitude of the Compton effect would thereby be increased. The momentum delivered to the electron in this case may, however, be very much greater than that due to a Compton effect. Of course, if an electron increases the probability of emission of radiation by an excited atom it must also increase the probability of the absorption of a quantum by an atom according to the reverse process. If scattering of resonance radiation is due to quantum jumps then the scattering should be increased by the presence of electrons.

IV. Explanation of Maxwellian Distribution of Ultimate Electrons

We have seen that the primary electrons from a cathode at -50 or -80 volts are scattered mainly by some direct effect of the primary electrons and not by the action of the ionization or excitation caused by the secondary or the ultimate electrons. In the positive column of an arc primary and secondary electrons are absent. It might seem, therefore, that the mechanism by which the electrons in an arc acquire their Maxwellian distribution must be different from that which causes the scattering of primary electrons. It seems more probable, however, that the mechanism is essentially similar, the difference being merely one of degree.

For example, in an arc in mercury vapor at 5 bars pressure the temperature of the ultimate electrons is about 20,000°, corresponding to an average energy ($\frac{3}{2}kT$) of 2.6 volts. According to Eq. (25) the corresponding radiation ($\lambda 4700$) would increase the temperature of electrons by only 0.08° for each quantum scattered, whereas 50 volt radiation which might exist in the primary beam could raise the temperature by 25° per collision.

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In the case of the ultimate electrons in an arc it seems probable that there is a kind of thermal equilibirum between the electrons and the radiation and excited atoms. That is, the electrons lose as much energy to the radiation on the average, as they gain from it. Thus there is probably a definite relation between the temperature of the ultimate electrons and the frequency of the ultraviolet radiation or the average energy levels of the excited atoms.

The effective electron free-path determined from the mobility of electrons in an arc is not the average distance the electron moves before being acted on by a quantum, but is the distance it must move before losing most of its momentum in a given direction. By Eq. (26) we see that an electron will lose about half its forward momentum when it has made 190,000 collisions with quanta of a frequency corresponding to 2.6 volts. The experiments have given an effective free path of about 3 cm at one bar so the distance between successive interactions with quanta would average 1.5×10^{-5} cm.

It will be understood that the foregoing theoretical considerations are purely tentative. The fundamental cause of electron scattering can be established only after further experimental studies of the laws governing it.

V. Scattered Electrons in High Vacuum Tubes

One of the most striking characteristics of electron scattering is the ability of some electrons to pass to an electrode which is more negatively charged than the cathode from which the electrons originate. A similar phenomenon is observed in vacuum tubes having apparently the highest vacuum if the anode consists of one or more wires of small size at high positive potential while a surrounding cylindrical electrode is negatively charged with respect to the cathode. A current of electrons flows to this cylinder. This effect, discovered by Barkhausen and Kurz[14] was explained by them as being due to electrical oscillations of electrons in the evacuated space. These oscillations were actually detected by Barkhausen and Kurz. Gill and Morell[15] have outlined a theory which suggests a cause of the oscillations.

Experiments in this laboratory, however, have shown that although oscillations frequently exist in such a tube they do not seem to be the sole cause of the currents to the negative cylinder. For example, in some experiments the logarithm of the current to the cylinder varied linearly with voltage on the cylinder, suggesting a Maxwellian distribution of the high speed electrons in the space. The temperature of these electrons was found to be proportional to the potential on the anode. With cylinder potentials close to those of the cathode electrical oscillations were observed, but when the cylinder was made more strongly negative the oscillations stopped abruptly and completely as

[14] Barkhausen and Kurz, *Phys. Zeits.* **21**, 1 (1920).
[15] Gill and Morell, *Phil. Mag.* **44**, 161 (1922).

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far as could be determined, and simultaneously the current to the negative cylinder *increased* several fold.

Similar effects are observed in a magnetron having end plates charged negatively with respect to the cathode. A longitudinal magnetic field tends to prevent electrons from reaching the cylindrical anode and the negative potentials on the end plates tend to prevent their escaping at the ends of the cylinder. Under these conditions, small currents of electrons do flow to the cylinder in spite of the magnetic field and flow to the end plates even if these are 20 to 50 volts negative with respect to the cathode. Yet oscillations can very rarely be detected. These experiments have been repeated by Mr. C. G. Found with the very best vacuum conditions using an anode of deposited magnesium on the glass wall and immersing the whole apparatus in liquid air, but the currents to the negative electrodes were not decreased.

It will be noted that these effects might be explained if the trapped electrons in the Barkhausen-Kurz tube or in the magnetron were scattered so as to obtain a Maxwellian distribution. However, it is hard to see how this distribution can be brought about with so few excited atoms and so little radiation as must be present in these tubes.

VI. Oscillations as a Possible Cause of Scattering in Ionized Gases

Since oscillations occur so often in tubes of this character and have been suggested as a cause of the currents to negative electrodes, it has been felt throughout the experiments on the scattering of electrons in ionized gases, that it is important to determine whether oscillations are present. For this reason we have frequently used radio detectors capable of detecting oscillations of 0.5 meter wave-length and have introduced small capacities and inductances in various parts of the circuits to see if any of the currents to collectors could be modified in this way. In none of the experiments on electron scattering in low pressure gases using hot cathodes have oscillations been observed. In some experiments with mercury arcs oscillations occurred at first until high resistances or choke coils were used in the anode circuits. The effects produced by the oscillations were not at all like those characteristic of electron scattering.

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FLOW OF IONS THROUGH A SMALL ORIFICE IN A CHARGED PLATE

With Lewi Tonks and Harold M. Mott-Smith Jr. as co-authors

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ABSTRACT

Measurements of ion and electron current densities and temperatures in a low pressure arc may be made with a collector which is screened by a close parallel electrode pierced with uniform circular holes. *The electrostatic potential distribution in the neighborhood of a pierced electrode,* pierced with either slit or hole, has been worked out. *The volt-ampere characteristic for random electrons* is exponential for the higher retarding voltages and the exponent gives a temperature slightly too high. When the anode drop is known the current density of electrons can be calculated. *The volt-ampere characteristic for positive ions* is linear for small accelerating voltages. For small retarding voltages the characterisitc is not a pure exponential. The characteristic gives anode drop, ion current density, "transverse" ion temperature and "longitudinal" ion temperature. The limitations and possible errors are discussed.

In order to separate the ionic and electronic parts of the electric current to an electrode immersed in an electric discharge in a gas, A. F. Dittmer suggested allowing these carriers to pass through small holes in a plane electrode and collecting them on a second electrode parallel to and behind the first, keeping the gas pressure low enough so that no collisions occurred between the sheath edge, *A*, and the second electrode, *C*, cf. Fig. 4. By adjusting the potentials on these electrodes it can be so arranged that either ions alone or electrons alone reach electrode *C*. The necessary variation in electrode potentials has a negligible effect on the discharge because of the presence of the sheath before the pierced electrode *B*. The thickness of this sheath must be greater than the hole diameter to prevent the discharge from penetrating through the hole. Other conditions, also, must be fulfilled, as will appear when the different cases are analyzed.

The carriers, which it is desired to keep from reaching *C*, can be prevented from doing so either by the field between *B* and *C*, or by the field between sheath edge and *B*. Thus there are two fundamental potential arrangements; in the first the carriers to be collected on *C* are initially retarded then accelerated, in the second they are initially accelerated then retarded.

The first arrangement is unsuited to measurements on positive ions (hereafter called simply ions) because it requires that *B* be made positive with respect to the anode. This situation cannot usually be realized for it is the most posi-

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tive electrode of any considerable size which acts as anode in the discharge. The second arrangement, on the other hand, is unsuited to measurements on electrons. They should be collected in an accelerating field because of electron reflection. On account of this specific application of each arrangement, we shall hereafter speak of positive and negative voltages, rather than accelerating and retarding voltages.

The present paper treats first, the measurement of the current density in a high velocity beam of electrons; second, the volt-ampere characteristic of electrons having a Maxwellian distribution of velocities in the discharge; and third, the volt-ampere characteristic of ions having Maxwellian and other velocity distributions in the discharge. Preliminary to this, the equations for the electric potential field in the neighborhood of both a slit and a circular hole in an infinite charged plane are derived, and the transverse velocity and displacement acquired by a fast electron (ion) in traversing such a field are calculated.

I. The Electrostatic Field about a Slit

The potential distribution about a slit in an infinite plane charged electrode[1] is found by applying a Schwartz transformation[2] to the polygon $ABC \ldots H$ of Fig. 1. Proceeding to the limit by allowing

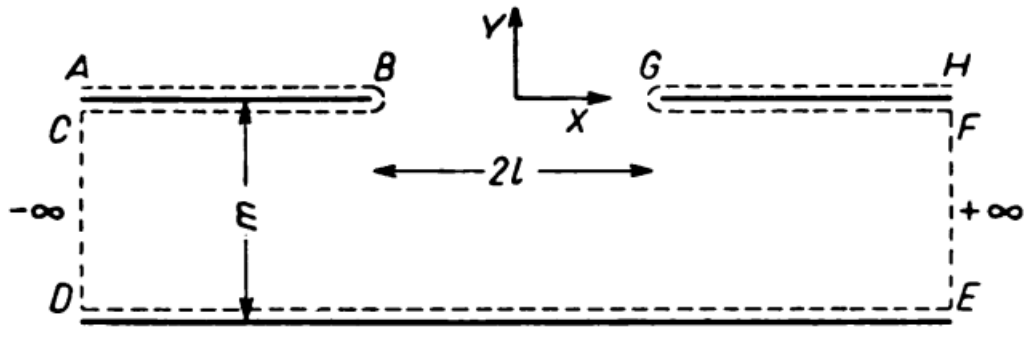

FIG. 1.

m to approach ∞ as $\partial V/\partial y$ at $y = +\infty$ remains finite and equal to $-E$, the potential V near a slit is found to be given by the parametric equations:

$$x = -\frac{U}{E} - \frac{El^2}{4}\frac{U}{U^2+V^2} \tag{1}$$

$$y = -\frac{V}{E} + \frac{El^2}{4}\frac{V}{U^2+V^2}, \quad V \geqslant 0 \tag{2}$$

and the boundary conditions of the field reduce to those shown in Fig. 2. To realize the general case where $\partial V/\partial y$ is different from zero on both sides of the electrode, it is only necessary to superpose a uniform electric field pa-

[1] The hydrodynamic case corresponding to this problem and also to the problem of the hole in a plate has been treated by Lamb, *Hydrodynamics*, 3rd edition, p. 77 and p. 137 respectively, but it has been thought worth while to sketch derivations of the potential using more usual coordinate systems.

[2] Jeans, *Electricity and Magnetism*, 2nd edition, p. 271.

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rallel to the y-axis modifying U and V in Eqs. (1) and (2) in such a way as to retain their conjugate properties.

U, the parameter, and function conjugate to V, has a very simple physical meaning. Keeping U constant and taking V as parameter in Eqs. (1) and (2) (or these equations modified as contemplated above) gives the lines of electric force just as $V =$ constant gives the equipotentials, and the value of U associated with a line of force is proportional to the total charge (per unit length) between the foot of that line of force and the foot of the line of force for which $U = 0$. Along any line $x =$ constant, in Fig. 2, the change in U between

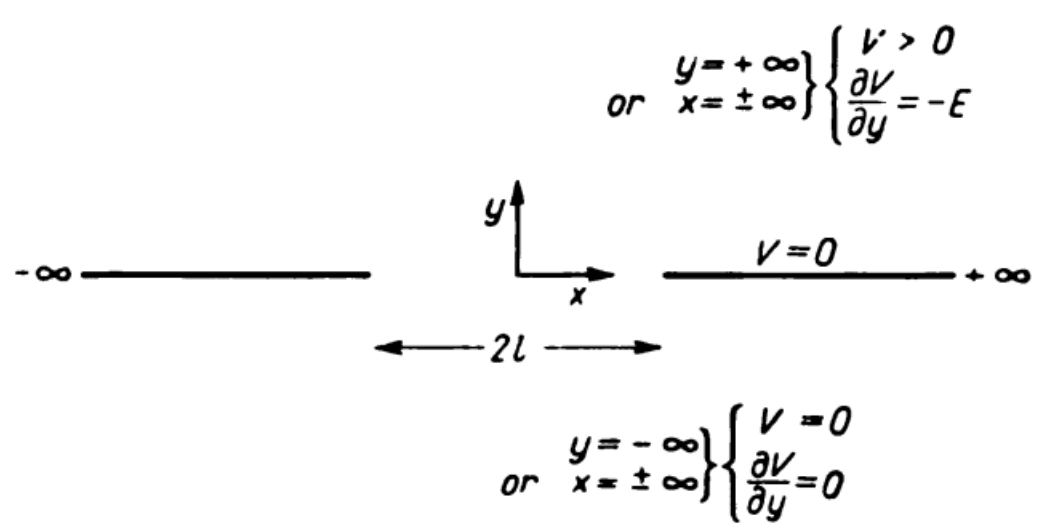

FIG. 2.

any two fixed points, as given by Eqs. (1) and (2), is the same as the change in U, given by these equations modified to include the superposed uniform field. Therefore, Eqs. (1) and (2), rather than the more complicated modifications, can be used for the general case whenever, as in the next section, changes in U along a line parallel to the y-axis are involved.

The equipotentials given by Eqs. (1) and (2) are sufficiently plane when y is somewhat greater than zero for these equations to be good approximations even when the parallel electrode, C, is near. For instance, Eq. (2) shows that the equipotential $V = -6lE$ lies at $y = 6l$ at some distance from the slit (x large, U large,) and lies at $y = 6l(1-1/144)$ at $x = 0$. In terms of potential, the potential of a conducting plane at $y = 6l$ is too low at $x = 0$ by $El/24$. When y is somewhat less than zero, the electric field is so weak and the potential is so nearly zero that the plane equipotential, A, may also be used there without altering the field for our purpose.

II. Transverse Velocity Acquired by a Charged Particle Passing Through the Slit

A particle of charge ϵ and mass m moving with high velocity parallel to the y-axis is subject to a transverse acceleration due to the non-uniformity of the field.

This acceleration is

$$\frac{d^2x}{dt^2} = -\frac{\epsilon}{m}\left(\frac{\partial V}{\partial x}\right)_{x=x'}$$

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where V is the potential in the field and x' is the distance of the particle path from the center line of the slit. Now the discussion above shows that such non-uniformity is quite local, so that the particle acquires the same transverse velocity in traveling from $y = -\infty$ to some point near electrode B as in going from electrode A, the sheath edge of Fig. 4, to the same point. Hence the transverse velocity acquired will be

$$\dot{x} = -\frac{\epsilon}{m}\int_{-\infty}^{t_1}\frac{\partial V}{\partial x}\,dt = -\frac{\epsilon}{m}\int_{-\infty}^{y}\frac{1}{\dot{y}}\frac{\partial V}{\partial x}\,dy \tag{3}$$

If $\dot{y}$ is large it changes but slightly in the neighborhood of the slit, so $\dot{y}$ will be supposed constant and

$$\dot{x} = -\frac{\epsilon}{m\dot{y}}\int_{-\infty}^{y}\frac{\partial V}{\partial x}\,dy = \frac{\epsilon}{m\dot{y}}\int_{-\infty}^{y}\frac{\partial U}{\partial y}\,dy = \frac{\epsilon}{m\dot{y}}\,|\,U\,|_{-\infty}^{y} \tag{4}$$

where U is the function conjugate to V.

Eqs. (1) and (2) are applicable, as has been remarked above. From them

$$U = -Ex', \quad \text{when } y = +\infty, \quad x = x'$$
$$U = 0, \quad \text{when } y = -\infty$$

Accordingly the total transverse velocity acquired in passing from electrode A to electrode C ($y = -\infty$ to $y = +\infty$) is

$$\dot{x}_f = -\varepsilon Ex'/m\dot{y} \tag{5A}$$

Now E is the electric field in the simple case corresponding to Eqs. (1) and (2). It is equal to the difference of the actual fields on the two sides of B so that

$$\dot{x}_f = \frac{\epsilon(E_1+E_2)x'}{m\dot{y}} \tag{5B}$$

where E_1 and E_2 are the two fields, each being counted positive when directed towards B. Expressing velocities as potentials and using the mean field strength $\overline{E} = \frac{1}{2}(E_1+E_2)$

$$V_x V_y = \overline{E}^2 x'^2 \tag{6}$$

One conclusion from the preceding is that the particle has acquired one-half its transverse velocity when it reaches the plane of B. This is obviously true for a symmetrical arrangement of voltages; any unsymmetrical arrangement can be realized by superposition of a uniform field, hence it holds also for any other arrangement. In general, however, this conclusion is not valid if the slit has finite thickness.

On the other hand, it is true that $\dot{x}_f$ is given by Eq. (5B) no matter what the thickness of electrode B and what the shape of the sides of the slit so long as the slit's cross-section is constant. This follows immediately from

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the fact that U at A and C does not depend on the shape of the slit. Of course there is the difficulty of assigning an origin of coordinates when the slit is unsymmetrical.

A further integration of Eq. (4) would give the lateral displacement of the particle. Inasmuch as the calculation for the case of the hole is simpler and the displacement in the present case is of no interest here, that result is not calculated.

III. The Electrostatic Field about a Hole[1]

The problem is to find the potential distribution due to an infinite conducting plane with a circular hole of radius a charged to potential $+V$ and placed symmetrically between two parallel infinite grounded plates at a great distance away.

The limiting value of surface charge density on the central plate far from the hole is taken to be σ_0; then the uniform density on each grounded plate is $-\frac{1}{2}\sigma_0$. A uniform density $-\sigma_0$ is then superimposed over the central plane

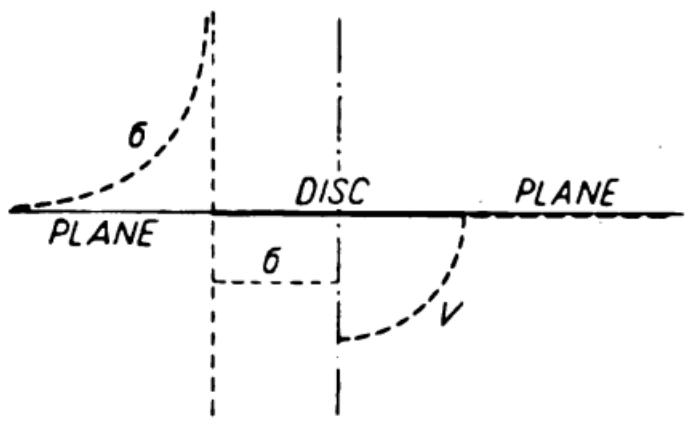

FIG. 3.

and hole in addition to the charge already existing, and to compensate for this $-\frac{1}{2}\sigma_0$ is removed from each distant grounded plate. The central conducting plane is thereby brought to zero potential, and the problem is changed to: an infinite grounded conducting plane has a circular hole in which is placed a disk charged with a uniform density of negative electricity; find the potential distribution.

The general nature of the distributions of potential V and charge σ are indicated in Fig. 3. It can be proved by direct substitution that a solution of Laplace's equation in cylindrical coordinates is

$$V(r, z) = \int_0^\infty e^{-uz} u\,du \int_0^\infty f(s) J_0(ur) J_0(us)\, s\,ds$$

where $f(s)$ is arbitrary. V vanishes for $z = +\infty$, and for $z = 0$ is

$$V(r, 0) = \int_0^\infty u\,du \int_0^\infty f(s) J_0(ur) J_0(us)\, s\,ds = f(r)$$

The last equation is a consequence of the *Fourier-Bessel integral formula* analogous to the Fourier integral in the case of circular functions. Thus $f(r)$

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is simply the potential distribution in the plane $z = 0$. Taking the hole to be of unit radius, for convenience, $f(s) = 0$ for $s > 1$, and

$$V(r, z) = \int_0^\infty e^{-uz} u du \int_0^1 f(s) J_0(ur) J_0(us) s ds \tag{7}$$

Applying Gauss' theorem at the disk,

$$\left(\frac{\partial V}{\partial z}\right)_{z=0} = 2\pi\sigma \tag{8}$$

to Eq. (7) gives an integral equation to determine $f(s)$. The shape of the curve for V in Fig. 3 suggested the solution

$$f(s) = A\sqrt{1-s^2} \tag{9}$$

This is tried in the equation and found to satisfy it,[3] giving for A the value $-4\sigma_0$. Hence the potential distribution in the plane $z = 0$ is given by

$$f(r) = V(r, 0) = -4\sigma_0\sqrt{1-r^2}, \quad 0 \leqslant r \leqslant 1$$

The potential distribution for the modified problem is found by putting this in Eq. (7) and reducing slightly

$$V(r, z) = -4\sigma_0 \int_0^\infty \frac{e^{-uz}}{u}\left(\frac{\sin u}{u} - \cos u\right) J_0(ur) du, \quad z \geqslant 0$$

The potential distribution for the original problem involving the hole of radius a is then easily seen to be

$$V(r, z) = V_0 - \sigma_0\left[2\pi z + 2a\int_0^\infty \frac{e^{-uz/a}}{u}\left(\frac{\sin u}{u} - \cos u\right) J_0(ur/a) du\right], \quad z \geqslant 0 \tag{10}$$

where V_0 is the potential of the plate with the hole.

The equipotential surfaces given by this equation are more nearly plane than those for the slit. On the axis of the hole Eq. (10) reduces to

$$V(0, z) = V_0 - \sigma_0\left[2\pi z + 4a\left(1 - \frac{z}{a}\cot^{-1}\frac{z}{a}\right)\right]$$

so that the equipotential

$$V = V, \ -2\pi\sigma_0 z_1,$$

which lies at $z = z_1$ at a distance from the hole, lies at

$$z = z_1\left[1 - \frac{2}{\pi}\frac{a}{z_1}\left(1 - \frac{z_1}{a}\cot^{-1}\frac{z_1}{a}\right)\right] = z_1\left(1 - \frac{2}{3\pi}\frac{a^3}{z_1^3}\right)$$

at the axis. Thus the surface at $z_1 = 3a$ only decreases its distance from the plane of B by 0.8 per cent.

[3] Invaluable reduction formulae are found in Watson, *A Treatise on the Theory of Bessel Functions*, pp. 373, 54, and 405, Nos. 2 and 7.

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IV. Transverse Velocity Acquired by a Charged Particle Passing through the Hole

The expression for the transverse velocity can be written immediately from inspection of Eq. (3)

$$\dot{r} = -\frac{\epsilon}{m\dot{z}} \int_{-\infty}^{\infty} \frac{\partial V}{\partial r}\, dz = -\frac{2\epsilon}{m\dot{z}} \int_{0}^{\infty} \frac{\partial V}{\partial r}\, dz$$

Evaluating this[4] and introducing $\bar{E}$ gives the same results as in section II, namely

$$\dot{r}_f = -\epsilon E r'/m\dot{z} \quad \text{or} \quad V_r V_z = E^2 r'^2 \tag{11}$$

The similarity between this case and the slit case extends further. Here as there the particle acquires one-half its transverse velocity in reaching the plane of B in the ideal case of zero thickness of B. Here also the total transverse velocity acquired is independent of the electrode thickness and the hole shape as long as the hole has circular symmetry. This follows from the existence of a "current function" which has been defined by Stokes for three-dimensional problems concerning configurations having axial symmetry. This function is analogous to the function U conjugate to the potential V in two-dimensional problems.

V. Measurement of the Current Density in a Beam of High Velocity Electrons

Under certain conditions the electrons which leave the incandescent tungsten cathode in a low pressure arc travel with nearly equal velocities in straight lines radially from the filament. One way of collecting these without at the same time collecting the random low velocity electrons and without introducing reflection errors is by using the present double electrode scheme. The potential requirements here are that a low negative voltage V_B be put on B, and a low positive voltage V_C be put on C. These potentials are relative to the sheath edge which serves as electrode A, V_B must be sufficient to repel all low velocity electrons, V_C must be sufficient to repel all positive ions, and the electrons must be moving parallel to the hole axis.

All the fast electrons which are headed for the hole pass through it and reach C. In addition some electrons which are directed toward B just outside the hole are displaced inward so that they penetrate the hole. The decrease in radial distance which enables an electron to graze the edge of the hole can be calculated readily on the assumption that B is infinitely thin and with the approximation that the transverse field is the same along the straight

[4] Watson, p. 405, No. 3, used twice, once integrated with respect to b.

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line path $r = a$ as along the actual path. Thus the displacement at the hole is given by

$$\Delta r' = -\frac{\epsilon}{mz^2}\int_{\infty}^{0}\int_{\infty}^{z}\left(\frac{\partial V}{\partial r}\right)_{r=z} dz\,dz$$

Evaluating this[5] it is found that

$$\frac{\Delta r'}{a} = \frac{\sqrt{2}}{3\pi}\frac{a\epsilon\bar{E}}{m\dot{z}} = \frac{\sqrt{2}}{6\pi}\frac{a\bar{E}}{V_D - V_B} = 0.075\frac{a\bar{E}}{V_D - V_B}, \tag{12}$$

where V_D is the potential, intrinsically negative for electrons, necessary to reduce the drift velocity $\dot{z}$ to zero. The difference $V_D - V_B$ is used instead of V_D because it is the value of $\dot{z}$ at the hole which is significant. The effective area of the hole is then fractionally greater than the true area by

$$\Delta A/A = -2\Delta r'/a = 0.15\, a\bar{E}/(V_B - V_D) \tag{13}$$

It is to be noted that whereas the field, E_2, between B and C is given by $(V_C - V_B)/\delta$, where δ is the electrode separation, the field, E_1 on the sheath side of B is more nearly $-4V_B/3\sigma$, σ being the sheath thickness, on account of the space charge there. Accordingly

$$\bar{E} = \tfrac{1}{2}\left(-\frac{4}{3}\frac{V_B}{\sigma} + \frac{V_C - V_B}{\delta}\right) \tag{14}$$

As an example, consider the following case: hole radius, $a = 0.0125$ cm; sheath thickness $\sigma = 0.0375$ cm; electrode separation $\delta = 0.075$ cm; $V_B = -10$ v; $V_C = +10$ v. Then $E_1 = 356$, $E_2 = 267$ and $a\bar{E} = 3.9$ v.

Calculating $\Delta A/A$ in per cent for various values of V_D the following is found:

V_D:	-100	-50	-30	-20
$\Delta A/A$:	0.0065	0.015	0.029	0.059

The true electron current density in the beam is found by dividing the current to C by $A(1+\Delta A/A)$.

If the hole has appreciable thickness the side of B with the stronger longitudinal field has the stronger transverse field, and, therefore, contributes the major part of the transverse velocity. In the present example, the sheath-side field is the stronger on account of the space charge there. Accordingly $\Delta A/A$ as calculated is too small.

Electrode B collects all the ions which strike it but reflects some of the electrons. The ion component can be determined by the method of section VIII and in other ways. Thus it becomes possible by comparing the electron current densities to B and C to determine the reflection coefficient of B for electrons of different voltages in the presence of gas and ion bombardment.

[5] Watson, pp. 54 and 403, No. 2.

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VI. Measurements on the Random Electrons in an Arc

The random electrons, having, presumably, a Maxwellian distribution of velocities, can be investigated in the absence of drift electrons with essentially the same potential arrangement as in the previous case. When the electric fields on the two sides of B are equal, the surface of minimum potential (m.p.s.), which caps the hole, becomes plane and the potential at any point on it is given by

$$V = V_B + (2\bar{E}/\pi)\sqrt{a^2 - r^2} \tag{15}$$

A general theorem pertaining to Maxwellian distributions, which is discussed in a forthcoming paper by Langmuir and Mott-Smith, Jr., applies here in all cases where $V < 0$ over the minimum potential surface. In this case, the entire hole constitutes a collector which repels electrons, and there will be no "interior orbits" for the electrons between sheath edge and B. Accordingly the electrons in the plane m.p.s. have a Maxwellian distribution of velocities and a density which varies according to Boltzmann's equation. The current, therefore, which flows through the hole is

$$i_C = I_- \int_0^a e^{-\epsilon V/kT} 2\pi r\, dr$$

Using (15) to integrate and then (14) together with the substitutions

$$\eta_1 = -\epsilon V_B/kT, \quad \eta_2 = -\epsilon(V_C - V_B)/kT$$

$$\nu = \frac{a}{\pi}\left(-\frac{4\eta_1}{3\sigma} + \frac{\eta_2}{\delta}\right), \quad \Phi(\nu) = \frac{2}{\nu^2}[1 + e^{\nu}(\nu - 1)]$$

we obtain

$$i_C = \pi a^2 I_- e^{\eta_1} \Phi(\nu) \tag{16}$$

Since electrons are involved ϵ is negative. Besides, it has been assumed that V_B is negative and V_C positive. Hence η_2 and ν are always positive, and η_1 negative.

From the form of Eq. (16) it is seen that $\Phi(\nu)$ is the ratio of the currents per unit area to the hole and to electrode B (assuming no reflection), and since $\ln\Phi$ is an almost linear function of ν whose derivative changes only from 2/3 at $\nu = 0$ to 1 at $\nu = \infty$ it follows that the slope of the $\ln i_c$ against V_B (η_2 constant) characteristic will differ from its usual value in other cases, namely, ϵ/kT, by a nearly constant amount. It will correspond to a higher temperature.

When V_C is made rather large so that ν is large, the equation of the characteristic is

$$i_C = \frac{2\pi^2 a I_-}{-4\eta_1/3\sigma + \eta_2/\delta} e^{\alpha\eta_2/\pi\delta} e^{\eta_1(1 - 4\alpha/3\pi\delta)} \tag{17}$[6]

[6] See the discussion following Eq. (23) for a necessary modification to this equation.

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which shows that for η_2 constant, ln i_c is nearly a linear function of η_1 whose slope is

$$\frac{d \ln i_C}{d\eta_1} = 1 - \frac{4a}{3\pi} \frac{d}{d\eta_1}(\eta_1/\sigma), \text{ nearly.}$$

Since σ is proportional to $\eta_1^{3/4}$ this becomes

$$d \ln i_C/d\eta_1 = 1 - a/3\pi\sigma, \text{ nearly.} \tag{18}$$

Consequently the temperature determined from this slope is slightly greater than the true electron temperature in the ratio $1:1-a/3\pi\sigma$. Practically, η_1 and η_2 cannot be made very small because of incalculable changes in the sheath field, but it is instructive to consider the case. Eq. (16) becomes

$$i_C = \pi a^2 I_- e^{\eta_1}\left[1 + \frac{2a}{3\pi}\left(-\frac{4\eta_1}{3\sigma} + \frac{\eta_2}{\delta}\right)\right] \tag{19}^6$$

and

$$d \ln i_C/d\eta_1 = 1 - 2a/9\pi\sigma, \text{ nearly,} \tag{20}^6$$

giving again a temperature slightly greater than the true one, now in the ratio of $1:1-2a/9\pi\sigma$. Accordingly the slope of the characteristic consists of a constant corresponding to the true temperature less a correction term which is small (not over 3.5 per cent since σ must not be less than $3a$) and inversely proportional to the three-fourths power of V_B.

If now the sheath edge potential and the positive ion current density are known (from measurements on the positive ion current which will be discussed below in section VIII, for instance) so that V_B and V_C are known, and σ can be calculated, everything but I_- in Eq. (17) is known and the electron current density can be found.

Three factors affecting this method and introducing certain corrections must be treated in greater detail. They are, first, the effect of the random ion velocities on the electric field and sheath thickness; second, the condition that the fields on the two sides of B should be equal, and third, the condition that the potential everywhere over the hole shall be less than zero.

The presence of initial velocities among the ions changes the space charge equation[7] to

$$I_+ = 5.44 \times 10^{-8}(-V_B)^{3/2}(1+\mu)/\sqrt{M}\sigma^2 \tag{21}$$

where M is the molecular weight of the ion, and

$$\mu = 0.0247\sqrt{-T_+/V_B} \tag{22}$$

T_+ being the positive ion "temperature". From this it is found that

$$E_1 = -\frac{4}{3}\frac{V_B}{\sigma}\frac{1+\mu}{1+\frac{2}{3}\mu} \tag{23}$$

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[7] Langmuir, *Phys. Rev.* **21,** 419 (1923), Eq. (17).

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Thus Eqs. (17), (18), (19), and (20) should be modified by replacing $1/\sigma$ by

$$\frac{1}{\sigma}\frac{1+\mu}{1+\frac{2}{3}\mu}$$

For σ, Eq. (21) gives

$$\sigma = 7.37\times 10^{-4}(-V_B)^{3/4}\sqrt{1+\mu}/M^{1/4}I_+^{1/2} \tag{24}$$

The condition that $E_1 = E_2$ is seen to be

$$-\frac{4}{3}\frac{V_B}{\sigma}\frac{1+\mu}{1+\frac{2}{3}\mu} = \frac{V_C-V_B}{\delta} \tag{25}$$

and using (24) it becomes

$$\frac{V_C-V_B}{\delta} = \frac{4}{3}\frac{M^{1/4}I_+^{1/2}(-V_B)^{1/4}}{7.37\times 10^{-4}}\frac{\sqrt{1+\mu}}{1+\frac{2}{3}\mu} \tag{26}$$

The theory has been worked out on the basis of keeping V_C-V_B constant, so that if a run includes an N-fold variation of V_B and (26) holds for some mean value of V_B then (26) will be out each way by the factor $N^{1/8}$ at the extremes. This results in a bulging of the m.p.s. so that its center lies at a distance z_0 from the plane of B given by the equations

$$\gamma = \frac{2}{\pi}\left(\frac{z_0/a}{1+z_0^2/a^2}+\tan^{-1}(z_0/a)\right), \qquad E_2/E_1 = (1+\gamma)/(1-\gamma) \tag{27}$$

or approximately by

$$E_2/E_1 = \frac{1+1.274z_0/a-1.70z_0^3/a^3}{1-1.274z_0/a} \tag{28}$$

which makes E_2/E_1 5 per cent too large for $z_0 = .4a$ and 8 per cent too large for $z_0 = -.4a$. The potential at the center of the m.p.s. is

$$V = V_B+2aE/\pi(1+z_0^2/a^2) \tag{29}$$

A permissible error of 1 per cent in V allows an error of at least 1 per cent in the second term of the right member. This in turn allows z_0/a to be as great as 0.1 and E_2/E_1 to be 1.29 by (28). But this is $N^{1/8}$, so that V_B may have a $(1.29)^8 = 7.7$-fold variation when properly chosen without exceeding the 1 per cent limit set above.

Finally, the condition that the center of the m.p.s. shall be negative is given by Eq. (15).

$$\frac{\eta^2}{-\eta_1} = \frac{V_C-V_B}{-V_B} \leqslant \frac{1-4a/3\pi\sigma}{a/\pi\delta} = \frac{\pi\delta}{a}, \text{ nearly,}$$

the effect of initial ion velocities being neglected. Inspection shows that when condition (25) is satisfied this condition is satisfied also since $\sigma/a \geqslant 3$.

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VII. Measurements on Positive Ions: Calculation of the Volt-Ampere Characterictic of Ions Collected on C

It was noted in the preliminary remarks that to make measurements on the ions V_B must be sufficiently negative to keep any electrons from reaching C. The situation as regards the ions is illustrated in Fig. 4 and may be described as follows: an ion of a certain u, v, w velocity class (u perpendicular to the electrodes, v and w parallel to them) leaves the ionized gas, is accelerated by B,

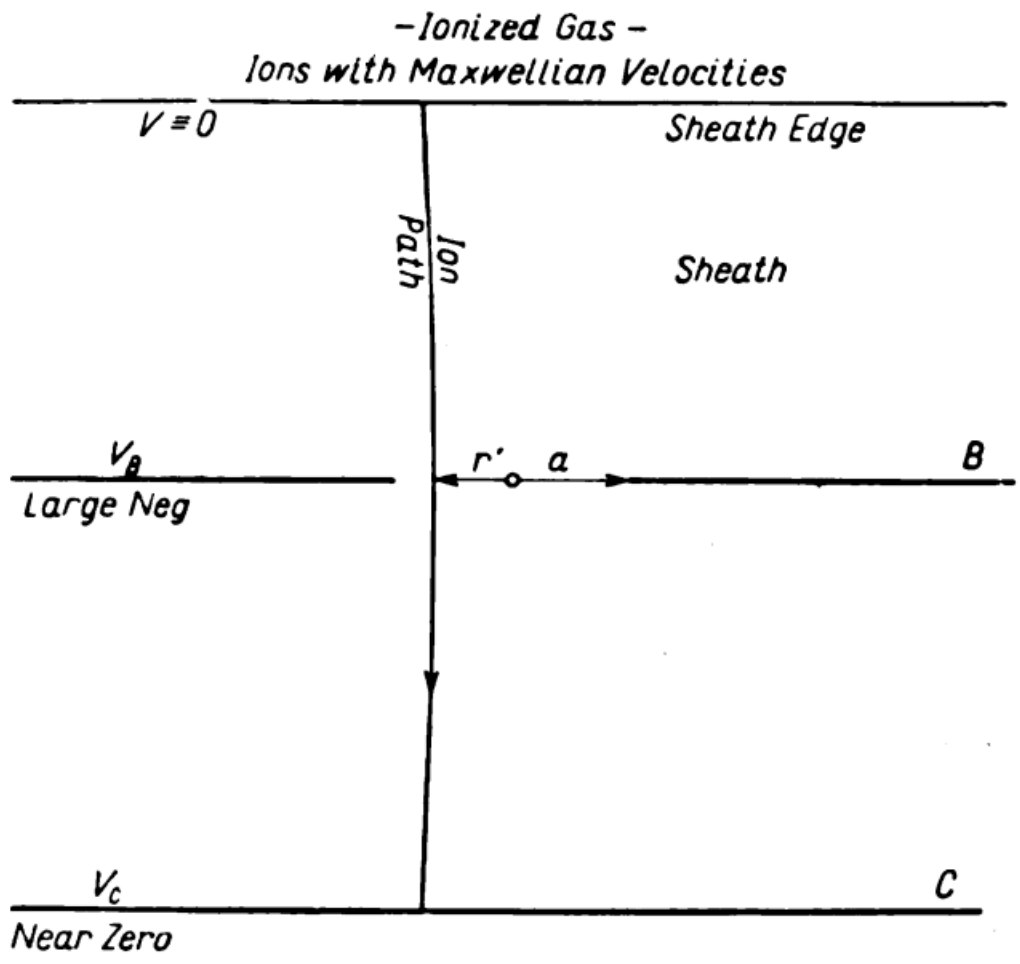

FIG. 4.

is given an added transverse velocity by the hole, and is then retarded by C so that it may not reach C. The v velocity axis is chosen in the sense and direction of the radius vector, r, from the center of the hole to the point at which the ion path cuts the plane of the hole. The transverse velocity r acquired by the ion increases its "transverse energy" by $\frac{1}{2}(r^2+2rv)$ and decreases its "longitudinal energy" by the same amount. Hence only when

$$\tfrac{1}{2}u^2-\tfrac{1}{2}r^2-rv \geqslant \varepsilon V_C/m \tag{30}$$

is the ion able to reach C, V_C being the retarding voltage on C. From Eq. (11)

$$\dot{r}_f = nr', \quad n = -\varepsilon E/m\dot{z}, \text{ a constant}$$

so that the relation (30) can be written

$$\tfrac{1}{2}n^2r'^2+nr'v-(\tfrac{1}{2}u^2-\varepsilon V_C/m) \leqslant 0 \tag{31}$$

The two roots r_1, r_2 ($r_1 < r_2$) of the equality partially fix the respective radial limits ϱ_1 and ϱ_2 between which an ion of a certain velocity class in a certain

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retarding field (V_c) must pass in order to be collected. But besides the mathematical relations

$$\varrho_1 = r_1, \quad \varrho_2 = r_2 \tag{32A}$$

there are the additional dominant physical limitations on ϱ_1 and ϱ_2

$$0 \leqslant \varrho_1 \leqslant \varrho_2 \leqslant a \tag{32B}$$

Now the effective area of the hole for (u, v, w) ions is

$$\pi(\varrho_2^2 - \varrho_1^2)$$

Accordingly the current of (u, v, w) ions is

$$i_{u,v,w} = 2h'h''m^2 I_+(\varrho_2^2 - \varrho_1^2)u \exp\left[-h'mu^2 - h''m(v^2+w^2)\right]dudvdw \tag{33}$$

the u velocity distribution being characterized by a different temperature from the v and w velocity distribution because in the body of the discharge there is a small electric field perpendicular to the electrodes which makes the longitudinal components of velocity greater. The relation between the h's and temperature are given below Eq. (36B).

Denoting the sum of $(\varrho_2^2 - \varrho_1^2)$ for a positive and an equal negative value of v by A, and using the variable

$$\bar{u}^2 = u^2 - 2\varepsilon V_C/m$$

($\bar{u}$ being the final u velocity before impact on C of an ion passing through the middle of the hole), (33) becomes

$$i_{u,v} = 2\sqrt{\pi h'' m}\, h' m I_+ e^{-2h\varepsilon V_C/m} A\bar{u} \exp\left(-h'm\bar{u}^2 - h''mv^2\right)d\bar{u}dv \tag{34}$$

after integration for all values of w. The limitations on ϱ_1 and ϱ_2 imposed by (31), (32A), (32B) necessitate breaking up the $\bar{u}$, v plane into several sections for the integration of (34) as shown in Fig. 5.

The values of the A's there given are the values appropriate to a hole all points of which are at the same potential. This is not true of the actual situation, the potential in the hole being variable and given by Eq. (15). Within the mathematical limitations of the present theory, it is sufficient, except in Eq. (39B), to write

$$A_\alpha = 4\Theta \frac{v}{n^2}\sqrt{\bar{u}^2+v^2}, \qquad A_\gamma = 2\Theta \frac{\bar{u}^2+2v^2}{n^2}$$

where

$$\Theta = 1 + 2a\bar{E}/\pi V_B = 1 - \frac{2}{\pi}\sqrt{V_t/-V_B}\,. \tag{35}$$

The integration limits are 0 and $+\infty$ for v and $-2\varepsilon V_C/m$ and $+\infty$ for $\bar{u}^2$. For accelerating potentials, $V_C < 0$, that is, the lower $\bar{u}^2$ limit lies to the right of the v-axis as indicated by the dotted line, while for retarding potentials, $V_C > 0$, that is, the lower $\bar{u}^2$ limit lies to the left of the v-axis. Practically, as long as $2\varepsilon V_C/m$ is considerably less than a^2n^2 and $h'm\, a^2n^2 >> 1$, the integration may be simplified by taking $A_\beta = A_\alpha$ and $A_\delta = A_\varepsilon = A_\gamma$.

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The integration then gives

$$i = -\Theta \frac{i_C}{V_t}(V_C + V' + V'') \qquad \text{for } -V_t << V_C \leqslant 0 \quad (36A)^8$$

$$i = -\Theta \frac{i_C}{V_t(V' - V'')}\left(V'^2 \exp\frac{V_C}{V'} - V''^2 \exp\frac{V_C}{V''}\right) \text{for } V_t >> V_C \geqslant 0 \quad (36B)^8$$

Here $i_c = \pi a^2 I_+$, is the constant ion current when all ions are collected; $V_t = m(na)^2/2\varepsilon$ is the voltage equivalent to the transverse velocity acquired by an ion grazing the edge of the hole; $V' = -1/2h'\varepsilon = -kT'/\varepsilon$ is the reciprocal

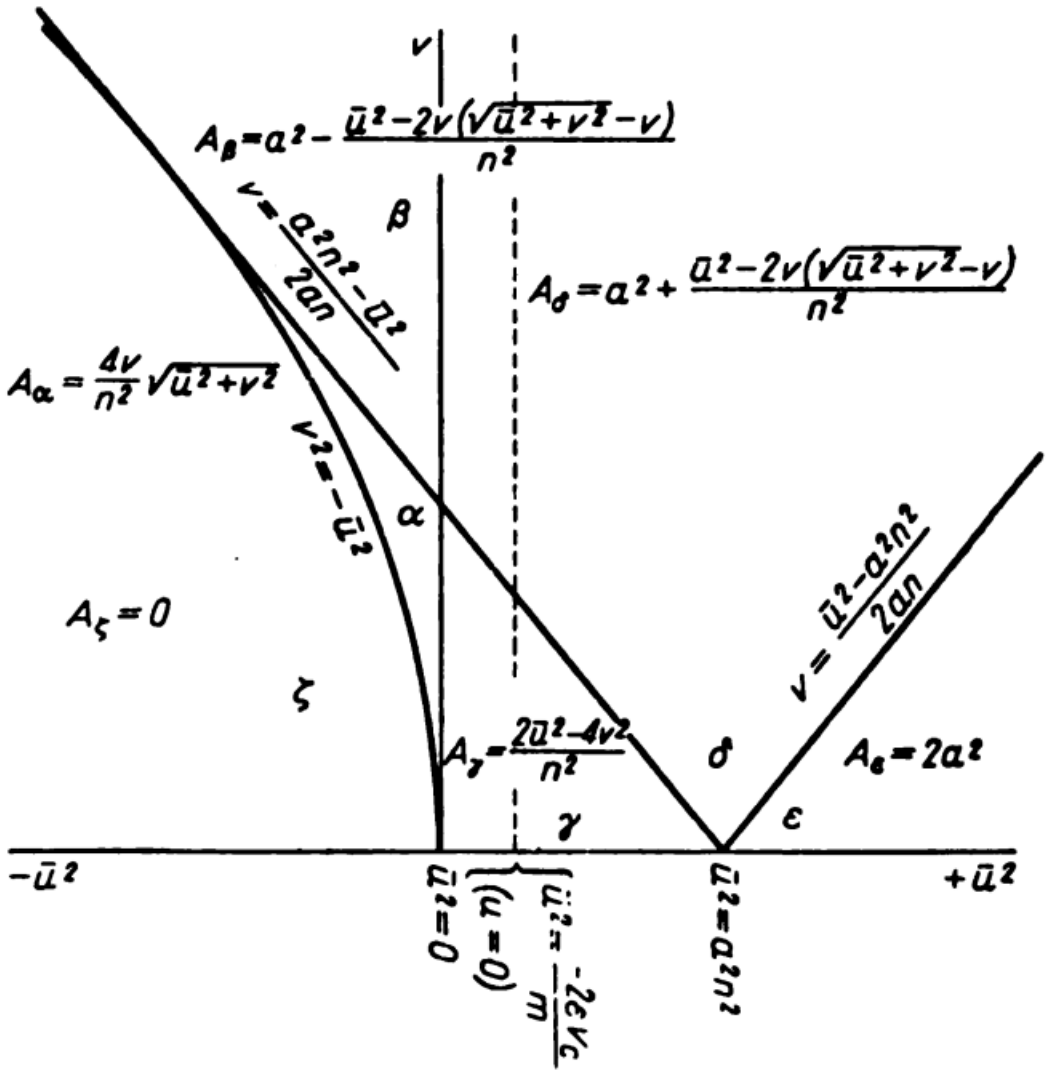

FIG. 5.

of the slope of the semi-log plot (base e) corresponding to the temperature T' of the longitudinal velocity distribution; and $V'' = -1/2h''\varepsilon = -kT''/\varepsilon$ is the same for the transverse velocity distribution.

The behavior of i for values of V_C less (more accelerating) than those covered in (36A) is very complicated except when $V'' = 0$ and therefore the general case will not be dealt with.

When transverse and longitudinal temperatures are equal (36B) reduces to

$$i = -\Theta \frac{i_C}{V_t}(2V' - V_C)\exp\frac{V_C}{V'},$$

$$\text{for } V' = V'',\ V_t >> -V',\ V_t >> V_C \geqslant 0 \quad (37)^8$$

---

[8] See section IX, §§ B and C.

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the equation of a curve on a semi-log plot which has half the slope at $V_C = 0$ that it has at $V_C = \infty$ and only approximates to a straight line of slope $1/V'$ for large values of V_C.

When the transverse temperature is zero, $V'' = 0$ and the Eq. (36B) gives a straight semi-log plot of slope $1/V'$. In this case it can also be shown that

$$i = i_C \qquad \text{for } V_C \leqslant -V_t \qquad (38A)$$

$$i = -\Theta \frac{i_C}{V_t}\left\{V_C + V'\left[1 - \exp\frac{V_C + V_t}{V'}\right]\right\} \text{ for } V_t >> -V',\ V_t < V_C < 0 \qquad (38B)$$

When both transverse and longitudinal temperatures are zero but the ions have a longitudinal drift velocity corresponding to V_D volts the equations for the volt-ampere characteristic become

$$i = i_C \qquad \text{for } V_C \leqslant V_D - V_t \qquad (39A)$$

$$i = -\Theta' \frac{i_C}{V_t}(V_C - V_D) \qquad \text{for } V_D - V_t \leqslant V_C \leqslant V_D \qquad (39B)^8$$

$$i = 0 \qquad \text{for } V_C \geqslant V_D \qquad (39C)$$

Here θ' is similar to θ except first, that the value of $\bar{E}$ to be used in it is that corresponding to $V_C = V_D$ instead of $V_C = 0$, and second, that when V_C leaves the neighborhood of V_D and approaches $V_D - V_t$, θ' increases and approaches 1.

VIII. Measurements on Positive Ions: Interpretation of Experimental Curves

A. The potential of the discharge with respect to the anode.—The voltage readings, E_C, do not give the position of $V_C = 0$ for they measure potential from the anode which is a few volts negative with respect to the discharge itself. It follows that the reading on the E_C scale of the line $V_C = 0$ is the potential of the discharge with respect to the anode. Now, Eq. (36A) shows that if a linear plot be made of the volt-ampere characteristic, a portion of it up to $V_C = 0$ will be straight. The curves of Fig. 6[9] obtained by Mr. C. G. Found using a mercury vapor arc in a tube built by Mr. A. F. Dittmer show this well. Unfortunately the deviation from a straight line is so slow at small positive values of V_C that the actual position of $V_C = 0$ cannot be found by simple inspection of the curve.

A method for finding the point $V_C = 0$ is to plot a certain root of the deviations from the straight line, against E_C. The straight line drawn through he

[9] These curves were made before any of the present theory had been developed and no particular precautions in regard to the shape or uniformity of the holes in electrode B, to the measurement of the distance between B and C, or to the accurate measurement of the small ion currents at the higher retarding voltages were made. Experiments in which these precautions are being observed are under way.

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resulting set of points cuts the E_C-axis at $V_C = 0$. The root to be used varies from the square root for $V''/V' = 0$ to the 2.7th root for $.2 \leqslant V''/V' \leqslant 1$ and this results in an uncertainty in the value of the discharge potential.

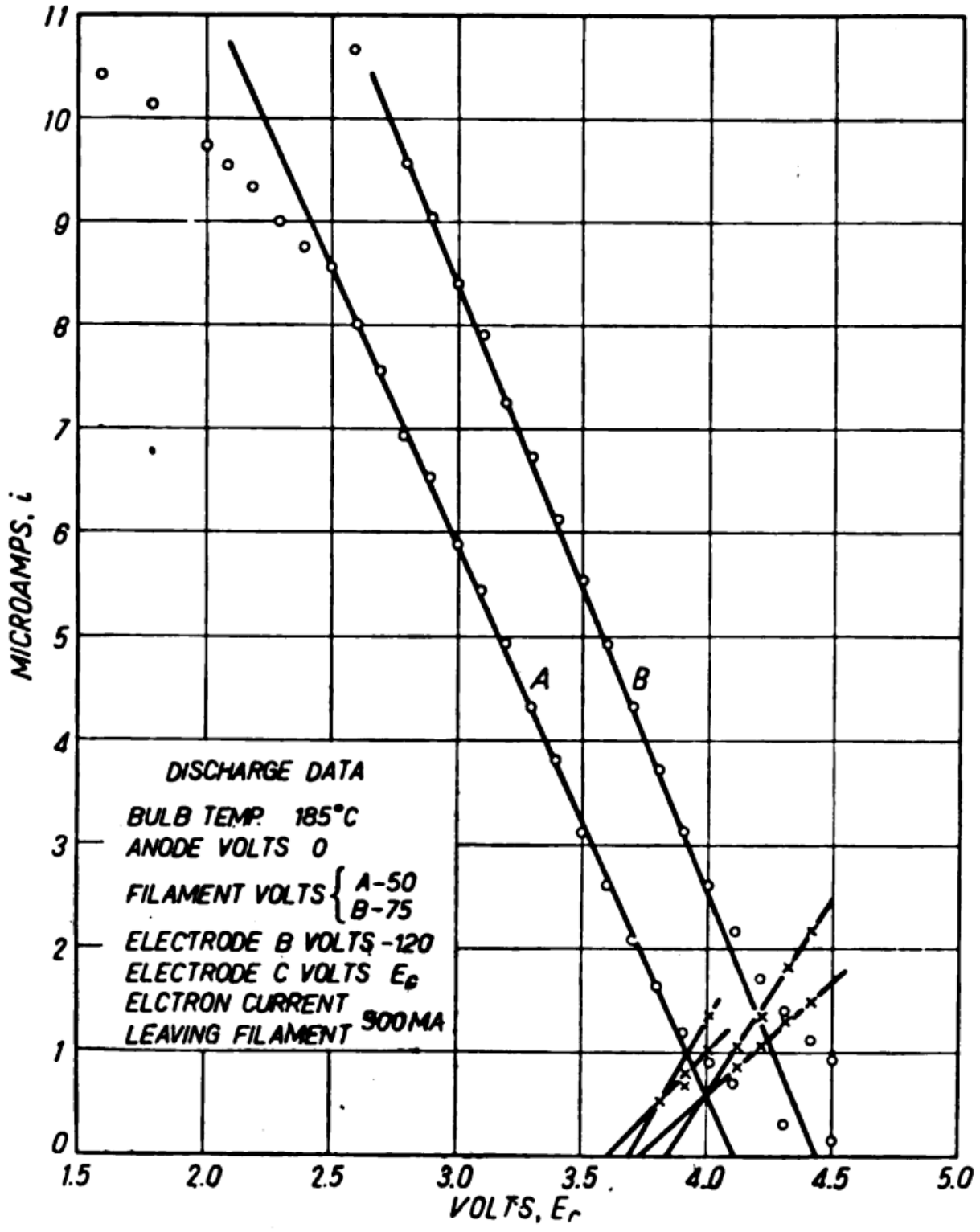

FIG. 6.

As an example, consider the curves of Fig. 6. Both the square root and 2.7th root of the deviations are plotted as crosses, the intersections of the straight lines through these with the axis are marked, and it is noted that the uncertainty is about 0.1 volt. For curve A, E_C, at $V_C = 0$, lies between 3.6 and 3.7, while for curve B, E_C at $V_C = 0$, lies between 3.7 and 3.8 volts.

B. The fundamental transverse voltage, V_t.—The slope of the linear portions of the characteristics in Fig. 6 is $-\theta i_c/V_t$. Since i_c is known in each case, V_t can be calculated by using Eq. (35) and is found to be 1.9 and 2.2 volts respectively for curves A and B.

An independent calculation of V_t can be made using Eq. (11), $\bar{E}$ being calculated by Eq. (14) with $V_c = 0$. The distance between B and C was not,

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unfortunately, accurately known, but lay between 0.1 and 0.2 cm and was estimated at 0.15 cm for the field strength calculation. On this basis it was found that:

For curve A; V_t (calc) = 2.4 v., V_t (from curve) = 1.9 v.
For curve B; V_t (calc) = 2.65 v., V_t (from curve) = 2.2 v.

a satisfactory agreement in view of the uncertainty of hole and size and electrode distance.

C. The voltage V' corresponding to the "longitudinal temperature," T' $= -11{,}600\ V'$.—Eq. (36B) shows that when a semi-log plot of the volt-ampere characteristic becomes straight at positive values of V_C, then the reciprocal of the slope is V'. The experimental difficulty is encountered that the determination of this quantity depends on the smaller and, therefore, usually the less

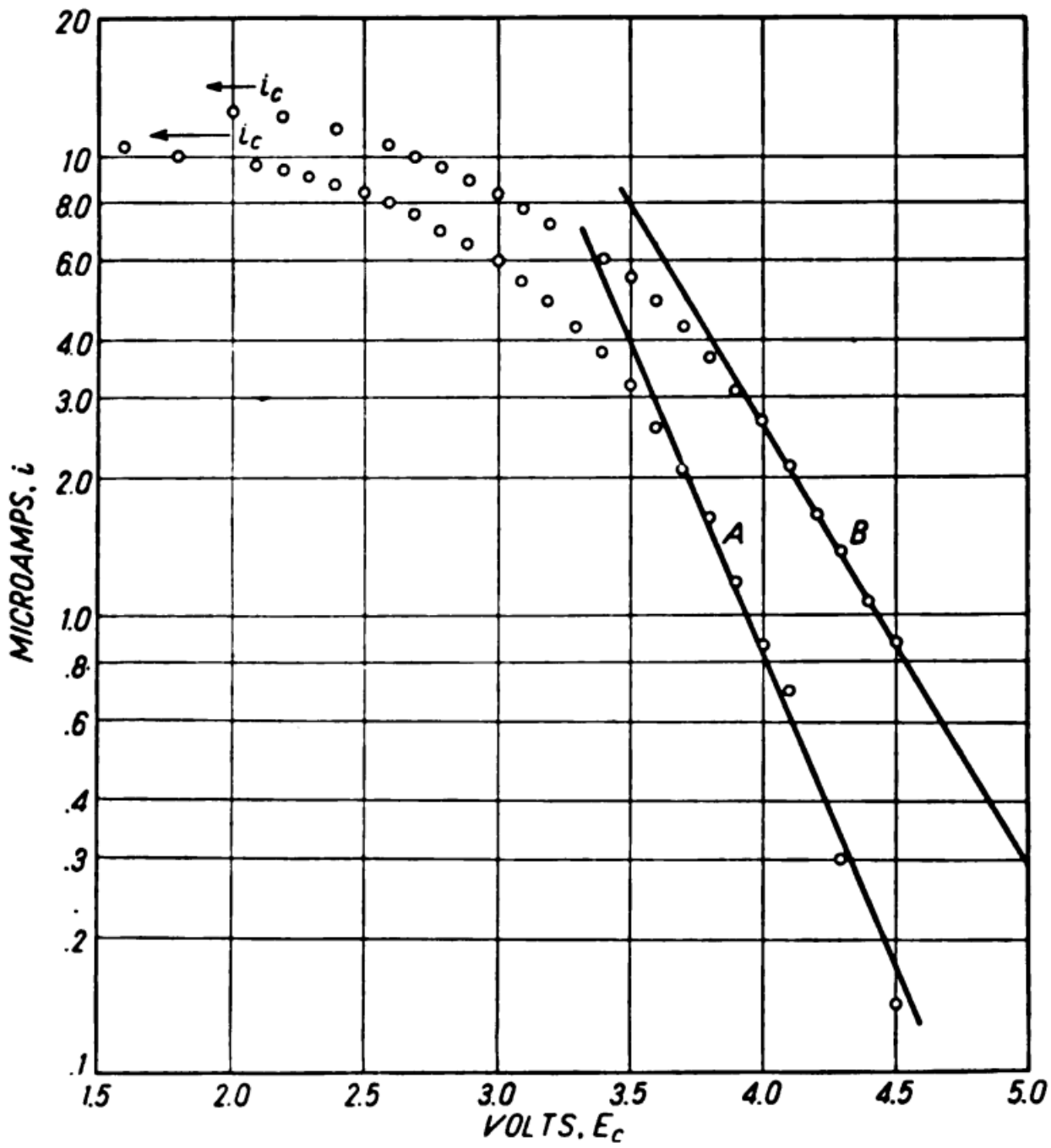

FIG. 7.

accurate current measurements. This trouble appears in Fig. 7, the semi-log plot of the observations already shown in Fig. 6. Using the straight lines drawn here as the best approximation:

For curve A; $V' = -0.32$ v, $T' = 3360°$K
For curve B; $V' = -0.45$ v, $T' = 5200°$K.

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D. The voltage V″ corresponding to the "transverse temperature," T″ $= -11{,}600\ V''$.—Eq. (36A) shows that the intercept of the E_c-axis in Fig. 6, which is terminated by $V_c = 0$ at one end, and at the other by the intersection of the straight line plot continued is the quantity $-(V'+V'')$. This relation may be used to calculate V''. Assuming that the value of E_c corresponding to $V_c = 0$ is, for curve A, 3.6 v and, for curve B, 3.7 v, and noting that curve A intersects the E_c axis at 4.1 v and curve B at 4.4 v, we have

$$\text{For curve } A;\ V'' = -0.18\ V,\ T'' = 2100°\text{K}$$

$$\text{For curve } B;\ V'' = -0.25\ V,\ T'' = 2900°\text{K}.$$

Even if the other values of E_c, namely 3.7 and 3.8 v had been used, the ratio V''/V' would still have exceeded 0.2, showing that 2.7 was the correct root of the deviations and that the proper values for E_c were used above.

A further approximation to V'' can be made using (36B) by solving it for the V'' which appears in the denominator, using an approximate value of V'' in the term involving the exponential and using a value of i at which the second exponential term is relatively small. Thus for curve B at $E_c = 4.4$ v, $i = 1.1\mu$a, it is found that $V'' = -0.24$. Accordingly $T'' = 2800°$K and $T''/T' = V''/V' = 0.53$.

E. *The decrease in i when* $V_c \leqslant -V_t$.—Eqs. (38) show that when $V'' = 0$, i is constant up to $-V_t$. But the existence of initial transverse velocities results in some ions being cut off prematurely so that a decrease in i at $-V_t$ indicates a transverse temperature different from zero. In the present case, part of the decrease must be ascribed to non-uniform hole size which affects the characteristic shape most vitally in this region. Accordingly this portion of the present characteristics is unsuited to interpretation. Even in the ideal case, however, the theoretical equation for the current is very complicated in this region and also subject to errors listed in section IX, §§B and C, so that any quantitative interpretation appears impossible.

IX. Measurements on Positive Ions: Errors and Experimental Precautions

A. *The condition* $V_t >> -V'$ *of Eqs.* (36) *and* (37).—The necessity for simplifying the integration of the ion distribution equation led to this condition. By taking $A_0 = A_e = 0$ an upper limit to the error is found to be about 4 per cent for $V_t = -5V'$, but the actual situation is much more favorable than this. Probably the best criterion of the fulfillment of this condition is the straightness of the linear plot, Fig. 6.

B. *Effect of ion paths not parallel to hole axis.*—The ions which go through the hole follow paths which deviate appreciably from parallelism with the hole axis. Insofar as this arises from the "transverse temperature," it is small, about $\sqrt{V''/V_B}$ or 0.045 radians in the case of curve B, and its effect is negligible.

The non-parallelism which arises from the acquired transverse velocity itself is only slightly larger, but its effect is not negligible. It results in a change

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in the effective area of the hole, which can be calculated from Eq. (13). $V_D = -V'$ may be neglected so that $\Delta A/A = 0.15\, a\overline{E}/V_B = -0.022$ for curve B. That is, the effective area of the hole is 2 per cent less than its true area. But cases in which this error is appreciable may arise. An examination of the integration leading to Eqs. (36A) and (36B) reveals that the hole size is not involved. This quantity enters only when the final substitutions listed under (36B) are introduced. Accordingly the difficulty can be readily analyzed by more exact definition. Retaining the definition of i_c as the total ion current through the hole it becomes necessary to append the factor $1+\Delta A/A$ to the mathematical expression for i_c which follows Eq. (36B). This introduces a correction coefficient of $1-\Delta A/A$ which is to be applied to the right members of Eqs. (36A), (36B), (37), (38B), and (39B). The values of V_t (from curve) of section 8B are thus 2 per cent low. *None of the other calculations based on the experimental curves is, however, affected by the non-parallelism of the ion path and the axis of the hole.*

C. *Thickness of the hole.*—In sections II and IV it was concluded that neither the depth of the orifice nor its shape, provided it was of the proper symmetry, has any effect on the transverse velocity acquired by an ion whose path is perpendicular to B. The path is not perpendicular to B primarily on account of acquired transverse velocity, so if the hole is cylindrical and has appreciable depth the radial distance at which the ion enters it is less than the radial distance at which the ion leaves it. Such an ion acquires a maximum transverse velocity when it grazes the edge of the hole as it leaves. The radius appropriate for calculating this velocity is the mean of the radius of entrance and the radius of the hole itself. This mean radius is seen to be $a-\frac{1}{4}t\sqrt{-V_t/V_B}$ [or $a(1+t\overline{E}/4V_B)$ by Eq. (11)]. Here again the hole size is involved, and the reasoning of §B applies. But since here the maximum acquired transverse velocity is not directly observed, the mathematical definition of V_t as it appears below Eq. (36B) will be retained. Accordingly no change has to be made in the equations because of this change in effective radius. There is, however, an accompanying decrease in the effective hole area which allows ions to pass. The complete fractional increase (algebraic) in area is by Eqs. (13) and (11)

$$\Delta A/A - t/a\sqrt{V_t/-V_B} \quad \text{or} \quad -\sqrt{V_t/-V_B}(0.15+t/a) \tag{40}$$

where t is the depth of the cylindrical part of the hole. In the new experiments which are contemplated it is planned to make the holes conical so as to reduce t to zero.

The initial transverse velocities of the ions also cause the ion paths to deviate from the perpendicular, the effect here being to reduce the effective area of the hole for ions having $v > 0$ and leave it unchanged for ions having $v < 0$. As this only involves one half the ions, and as $V_t >> -V''$, any error from this cause is quite small compared with those already dealt with.

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D. *Equivalence of mathematical and physical equipotentials.*—In order that the mathematical solution of the field about a hole (slit) should correspond to the physical conditions it has been pointed out in section 3 (section 1) that the distances, sheath edge to electrode B and electrode B to electrode C, should be at least 3 radii (3 slit widths).

E. *Return of ions through the hole.*—Any ions which are not collected by C and re-enter the sheath through the hole in B increase the space charge there and decrease the primary ion current. As the space charge contribution of the returned ions relative to the space charge contribution of the primary ions at any point is equal to the density of returned ions relative to that of primary ions at that point, an estimate of this error is easily made. Neglecting the initial transverse velocities of the ions for the moment, that is, assuming that all ion paths in the sheath are perpendicular to B, the transverse velocity acquired by an ion in passing through the hole at a distance r' from the center corresponds to an angular deflection of the path at B of $\sqrt{V_r/-V_B}$ radians, V_r being the acquired transverse voltage. The distance traveled before the ion reaches B again is 2δ (δ being the electrode separation) and consequently the lateral displacement of the ion when it again reaches the plane of B is $\Delta_1 r = 2 \cdot 2\delta\sqrt{V_r/-V_B}$, assuming that it follows a parabolic path.

The density of returning ions relative to outgoing ions is then $r'^2/(r'+\Delta_1 r)^2$ at B, and this can be evaluated using the above equation together with Eq. (11) and calculating $\bar{E}$ as in section VI, §B. Since, however, the returning ions contribute most effectively to the space charge when they are near the sheath edge the calculation of their radial distances must be carried one step further, that is, into the sheath once more, noting, of course, that in repassing the hole the ions acquire additional transverse velocity. To calculate the relative space charge contribution of these ions their relative density one quarter of the distance from sheath edge to electrode B was selected as a fair average value. As these ions have insufficient longitudinal energy to penetrate the sheath edge they are reflected from it and make a second space charge contribution as they return to B which must also be added. The computation shows that the relative space charge contribution is a function only of the ratio of electrode separation, δ, to sheath thickness, σ. It has a maximum value amounting to 0.7 per cent when $\delta/\sigma = 0.7$ and drops to 0.35 per cent when δ/σ equals either 0.2 or 2.5.

Turning now to the effect of the presence of random initial transverse velocities it is seen that this has the general tendency to decrease the returned ion density, and therefore to decrease the error. Under all circumstances, then, the return of ions through the hole may be neglected.

F. *Limiting of ion current to C by space charge.*—With an electrode separation greater than the sheath thickness it is evident that if B were perfectly transparent to ions, then theoretically no ions could reach C on account of space charge. Experimentally, however, there is only a narrow beam of ions flowing through

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a space devoid of charge, and this beam tends to diffuse on account of initial and acquired transverse velocities. It is evident that the thinner the beam is in the sheath, the greater the separation δ of the electrodes relative to the sheath thickness, σ, may be. The condition that space charge limitation shall not set in may, therefore, be roughly expressed as $\delta/\sigma \leqslant \varkappa\sigma/a$ or $\delta a/\sigma^2 \leqslant \varkappa$, where $\varkappa$ is a constant. In the present case $\delta/\sigma = 2$, $\sigma/a = 6$, hence $\delta a/\sigma^2 = 1/3$. Taking this as the maximum advisable value the condition above becomes

$$a\delta/\sigma^2 \leqslant 1/3 \tag{41}$$

G. *Reduction of potential at the center of the hole by space charge.*—If no non-uniformity is introduced into the space charge distribution in front of B by the hole, this charge gives no transverse field and cannot, therefore, change the potential at the hole center except insofar as the field E_1 is affected, a factor already dealt with in Eq. (14). But the ions which have passed through B have a non-uniform distribution and may affect the potential. Assuming, (1) a uniform field between B and C to calculate the ion velocities, (2) that the ions form a cylindrical beam behind B, and (3) that the density of ions returning from C is negligible, it is found that the change in potential sought amounts to only a few millivolts in the case of curve B. Accordingly it is most unlikely that this factor would ever become appreciable.

H. *Effect of ion repulsion.*—On account of the concentration of charge in the beam, any ion which is not on the beam's center line is subject to a transverse repulsive force which tends to diffuse the beam still more and by increasing transverse velocities, reduce longitudinal ones. No really satisfactory estimate of the magnitude of this effect has been made on account of numerous complicating factors. But in order to understand the situation better, a calculation based on certain simplifying assumptions has been made. It is easily seen that the only class of ions which is of interest is that composed of ions which just reach the plate at some fixed collector voltage. Assuming that the beam suffers no diffusion such as dealt with in §E, those ions of this class will be most affected which travel along the outside of the beam. We consider these ions only. The predominating part of the repulsion velocity acquired by such an ion will be acquired while the ion is moving either most slowly or in the vicinity of the densest space charge. Both conditions occur at the same place, namely, in the vicinity of the collector. On this basis the following assumptions were made: (1) the ion beam is bounded by the right circular cylinder which has the hole for a base, as already mentioned above, (2) an ion acquires a negligible repulsion velocity until it is within a distance of the collector of the same order as the radius of the beam (hole), (3) on account of the electrical image of the beam in the collector, the charge effective in repulsion is that in a cylinder cut from the beam which is bounded by the planes $x = x_i/4$, $x = 7x_i/4$ and of diameter $3x_i/2$, x being distance from the collector, and x_i being the x of the ion, (4) the repulsive force exerted by this charge is that of a sphere of uniform charge

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density, of the same total charge as the cylinder, of the same diameter as the cylinder, which is tangent to the cylindrical beam boundary at the ion, i.e., x_i, (5) the charge density in the beam is given by the space charge equation treating the collector as a plane cathode and using the current density to electrode B.

Then for mercury ions it is found that the voltage corresponding to the repulsion velocity is

$$V_R'' = 8.43 \times 10^3 I_+^{2/3} a^{4/3} \tag{42}$$

For curve B, Fig. 7, $I_+ = 8 \times 10^{-4}$ amps/cm², $a = 0.0125$ cm. Hence V_R''— $= 0.21$ v.

Contrary to assumption (3) the repulsion velocity acquired by the ion in travelling from $x_i = 3a$ to $x_i = a$ is not negligible, though less than this. Assuming the charge between $x = a$ and $x = 3a$ (as given by the space charge equation) to be redistributed uniformly throughout a sphere of radius a which is always tangent to the beam boundary at the ion as the ion travels from $3a$ to a, it is found that

$$V_R' = 3.88 \times 10^3 I^{2/3} a^{4/3} \tag{43}$$

whence $V_R' = 0.097$ v, and the total repulsion voltage is

$$V_R = (\sqrt{V_R'} + \sqrt{V_R''})^2 = 0.59.$$

This is very large compared, for instance, with V'' which is 0.24 for this curve. A re-examination of the assumptions shows that (1) is certainly not fulfilled and (5) is open to doubt. It is not unreasonable, however, to suppose that the actual space charge, including ions returning to B, if confined as contemplated in assumption (1), would be near enough on the average to that given by assumption (5). Accordingly, attention must be confined to (1). The considerations of §E show that three quarters of the way from B to C only a fraction, λ, say, of the ions lie within the cylindrical boundary of assumption (1). Accordingly, the actual force exerted on the ion by the charge inside the cylinder is certainly less than λ times the calculated force. λ is given by the approximate formula $\lambda = 0.29\sigma/\delta$, in the range $1.4 < \delta/\sigma < 4.0$. But the repulsive force of ions outside the cylinder balances a portion of this force, say $\sqrt{\lambda}$. Thus the actual force is $\lambda^{3/2}$ of the calculated force, and hence the repulsion voltage is λ^3 or $0.024\ (\sigma/\delta)^3$ of that calculated. In the present case V_R becomes 0.59×0.003 $= 0.002$. This, if correct, is small enough compared with V'' to be neglected, but the uncertainty involved demands that an experiment with ions or electrons of known temperature be done. With the above corrections the repulsion voltage in terms of the molecular weight, M, of the ion becomes

$$V_R = 99 M^{1/3} I_+^{2/3} a^{4/3} (\sigma/\delta)^3 \tag{44}$$

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Summarizing the factors which vitally affect the interpretation of an experimental curve in the form of conditions to be satisfied, we have, from the paragraphs noted:

A. $$V_t >> -V' \text{ (Validity of integration)} \tag{45}$$

F. $$\frac{\delta}{a} \leqslant \frac{1.82\times 10^{-8} V_B^{3/2}}{Ma^2 I_+} \text{ (Space charge limitation)} \tag{46}$$

H. $$\left.\begin{matrix} -V'' \\ -V' \end{matrix}\right\} >> 99 M^{1/3} I_+^{2/3} a^{4/3} (\sigma/\delta)^3 \text{ (Ion repulsion)} \tag{47}$$

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COLLISIONS BETWEEN ELECTRONS AND GAS MOLECULES

With HOWARD A. JONES as co-author

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ABSTRACT

Using a simple method in which positive ion sheaths serve as perfect grids we have obtained data on the *probability of various types of collisions* occurring between Hg, H_2, N_2, He, Ne and A molecules and electrons of moderate velocity, i.e., 30–250 volts. A theory has been developed and the *mean angular deflection for elastic and inelastic collisions between electrons and gas molecules has been calculated.* Evidence on the mechanism of ionization is presented. The current densities of the various Maxwellian groups of electrons and the average temperatures of these groups are calculated from the data and are shown to be in good agreement with data obtained by us using Langmuir and Mott Smith's probe wire method. The *current density of positive ions* has been measured by both plane and probe wire collectors and the *space potential* both measured and calculated. The evidence in support of negative anode drops has been pointed out and the ions flowing to the anode measured separately by means of a perforated collector. The *maximum number of positive ions produced by an electron* with a given velocity before losing its ionizing power has been measured and shown to be independent of current density and pressure being dependent only on the velocity of the electron and the nature of the gas.

The *mean free path of high velocity electrons* (i.e., above the ionizing potential) for various types of collision can be evaluated from the probability data. The mean free path for inelastic impact has been calculated separately and independently of the reflection of electrons from the collectors. The *small angular scattering of electrons* is discussed in detail. Phenomena, similar to Langmuir's high speed electrons in mercury vapor, have been observed and the curves showing the energy transfer between primary electrons in H_2 and A are given. The *resonance and ionization potentials* observed in the work are also tabulated.

MOST quantitative experiments on the ionization of gases involve the use of a stream of electrons moving with homogeneous and definitely known velocity. For this purpose the electrons from a hot cathode are usually accelerated by a positively charged grid or perforated plate into a region which is made as nearly field-free as practicable. To avoid space charge effects and the resultant uncertainties in potential distribution it is necessary to work with very low currents, which usually involve electrometer measurements. Furthermore, the grids must be of very fine mesh or the perforations in the plate of small diameter to avoid transverse velocity components in the electron stream and to make the effective potential of the grid the same as its actual potential. The customary methods are thus not adapted to the study of certain phenomena

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which appear only when rather large current densities are employed. With the high intensity of ionization accompanying larger currents it becomes possible, however, to use quite different methods for studying ionization phenomena.

In apparatus of ordinary size and with potentials above 40 volts, electron currents of a few milliamperes cause such intense ionization and high electrical conductivity that the interior of a gas at low pressure necessarily becomes nearly field-free. The potential drop between cathode and anode is then largely confined to a thin layer or sheath covering the surface of the cathode in which there is a positive ion space charge. This sheath has a well defined outer edge beyond which the field of the cathode does not extend. The sheath thus acts as an ideal grid for accelerating the primary electrons from a cathode into a field-free region.

The presence of such a homogeneous beam of electrons may be detected by a small disk-shaped collector placed in the electron stream so that the electrons move normal to its surfaces. When the collector is at a negative potential with respect to the ionized gas it is covered by a positive ion sheath. It collects all the positive ions that reach the edge of the sheath and all the electrons (expect those reflected at the collector surface) which have velocity components normal to the surface sufficient to carry them against the retarding field in the sheath.

Thus if the collector is at a potential below that of the cathode it will receive only positive ions, but as soon as its potential is higher than that of the cathode, the primary electrons, which have not lost momentum by collision or otherwise, will be able to pass through the sheath, and reach the collector. The current-voltage characteristic of the collector thus shows a break at the cathode potential. If, however, the collector is turned so that its plane is no longer perpendicular to the direction of propagation of the electrons, the collector potential will have to be raised above that of the cathode by an amount ΔE before primary electrons can be collected. This voltage displacement of the break ΔE is thus a measure of the transverse energy of the electrons, viz., the energy component parallel to the surface of the collector.

Apparatus and Method

Unless the dimensions of the collector are large compared to the thickness of the sheath, the sheath area will vary with the voltage, so that the interpretation of the current-voltage characteristic becomes more complicated. If the cathode is a straight filament the collector should be part of a cylindrical surface having the filament as axis in order that the primary electrons may impinge normally at all points of the collector surface. This suggests that the collector may advantageously be made a complete cylinder surrounding the filament and having a length equal to that of the filament. Such

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a tube is illustrated in Fig. 1 where F is the cathode filament, C is the coaxial cylinder which may be used to collect the primary electrons that reach it, A and B are end-plates which fit closely within the cylinder C and are provided at their centers with small holes through which the cathode filament passes.

In most cases one of the plates (A) is used as anode, its potential being sufficiently high to give strong ionization of the gas within the cylinder C. The electrons from the cathode are then accelerated in strictly radial direc-

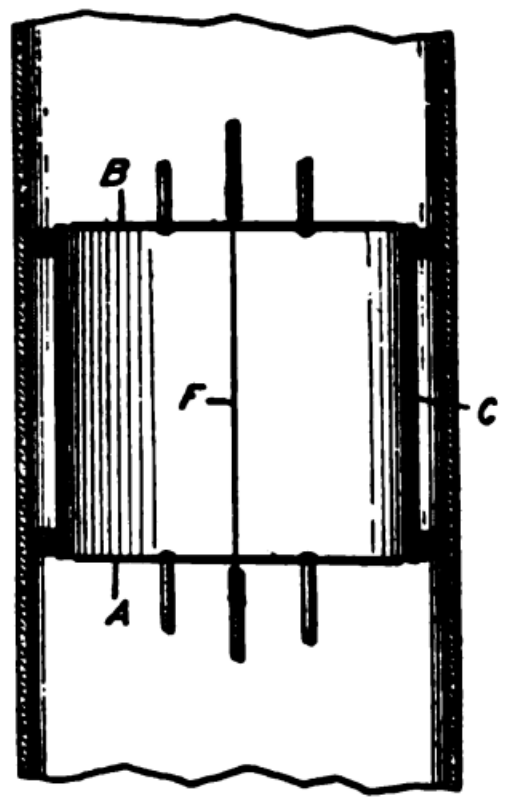

FIG. 1.

tions within the cathode sheath, and at low gas pressures a fraction of these electrons reach the cylinder C with normal incidence after having traveled a definite distance through the gas.

The potential of the ionized gas is approximately that of the anode, and is, in fact, usually a couple of volts above it (negative anode drop). We shall see that at low pressures the mobility of the electrons is so high that the ionized gas constitutes a very satisfactory field-free region.

Any electrode which is at a potential below that of the ionized gas will be covered by a positive ion sheath. If C is a few volts negative (we shall take the negative end of the cathode as of zero potential) the primary electrons which move radially across the tube are retarded within the sheath on C, are brought to rest, and are driven out of the sheath again without having lost energy, and still moving in a radial direction. All the positive ions which diffuse to the edge of the sheath on C will be collected, and the positive ion current is therefore independent[1,2] of the negative voltage on C. Illustrations of these constant currents may be seen in Figs. 2, 3, 4, etc., for negative vol-

[1] Langmuir, *Science* **58**, 290 (1923).

[2] Langmuir, *Gen. Elect. Rev.* **26**, 731 (1923).

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tages. In a similar way the plate B can be made negative and the positive ion current to this electrode measured. Thus by making corrections for the few ions collected by the filament (and by the anode), the total number of ions formed from a given number of primary electrons can be determined. One of the main objects of the investigations to be described in this paper has been to obtain quantitative data on the degree and nature of the ionization produced in a gas by a specified stream of primary electrons of known velocity. A tube of the type of Fig. 1 lends itself admirably to this purpose, not only because of its simple geometry, but also because of the fact that the ionized gas is completely enclosed[3] within metallic electrodes, any one of which may be used as a collector.

If C is made slightly positive the retarding field in its sheath will no longer be able to stop the primary electrons, so that some of these will be collected, and a break in the current voltage curve occurs at the cathode potential (zero) (see Fig. 2). This current discontinuity affords a simple and accurate means of determining electron free paths. The current-voltage curve of the collector C at higher voltages gives data on the velocity distribution (and the distribution of directions of motion) of the various other groups of electrons. Comparisons between the curves obtained with C (Fig. 2) and with B (Fig. 3) as collectors are particularly useful in giving information regarding the directions of motion of the electrons. We can thus estimate the fraction of the primary electrons which make elastic collisions and we also find that practically none of these are scattered through angles as large as 90 degrees.

In a similar way, by analysis of such curves as those of Fig. 2, we find that a certain fraction of the electrons lose an energy corresponding to a resonance potential of the gas (6.7 volts in Fig. 2), but even these electrons are not deflected through large angles. The ionized gas also contains a large number of electrons which move in random directions with velocities distributed according to Maxwell's law.[4,5]

Particularly interesting results are obtainable with tubes like that of Fig. 1 by the use of longitudinal magnetic fields. With weak fields the electron paths are curved so that the electrons reach the cylinder with oblique incidence. The current-voltage curves obtained in this way give good data for measuring the number of electrons that make elastic collisions with gas molecules.

With strong magnetic fields the distribution of electrons and positive ions between two collectors C and B is entirely changed. This furnishes a valuable

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[3] A short preliminary account of the methods of using a tube like that of Fig. 1 to study free paths and ionization phenomena in gases has been published by the present authors in *Science* **59**, 380 (1924).

[4] Langmuir, *J. Franklin Inst.* **196**, 751 (1923).

[5] Langmuir and Mott-Smith, Jr. *Gen. Elect. Rev.* **27**, 449, 538, 616, 762, 810 (1924). See also; *Phys. Rev.* **28**, 727, (1926) and *Zeit. f. Physik* **46**, 271 (1927).

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quantitative method of studying mobilities of electrons and ions. The intensity of ionization near the filament F, even at low pressures of gas, is then many times greater than at C so that relatively large positive ion currents flow to the cathode. This effect, however, is readily studied quantitatively by noting the increase in filament resistance caused by the heating effect of the positive ion bombardment.

Description of Typical Current-Voltage Characteristic Curves

The curves shown in Figs. 2, 3, and 4 were obtained with a tube like that of Fig. 1, in which the cylinder C was 3.1 cm in diameter and had an effective length of 3.8 cm (area 37.0 cm²). The plates A and B were 3.0 cm in diameter (area 7.0 cm² each), and had holes 0.35 cm in diameter through which

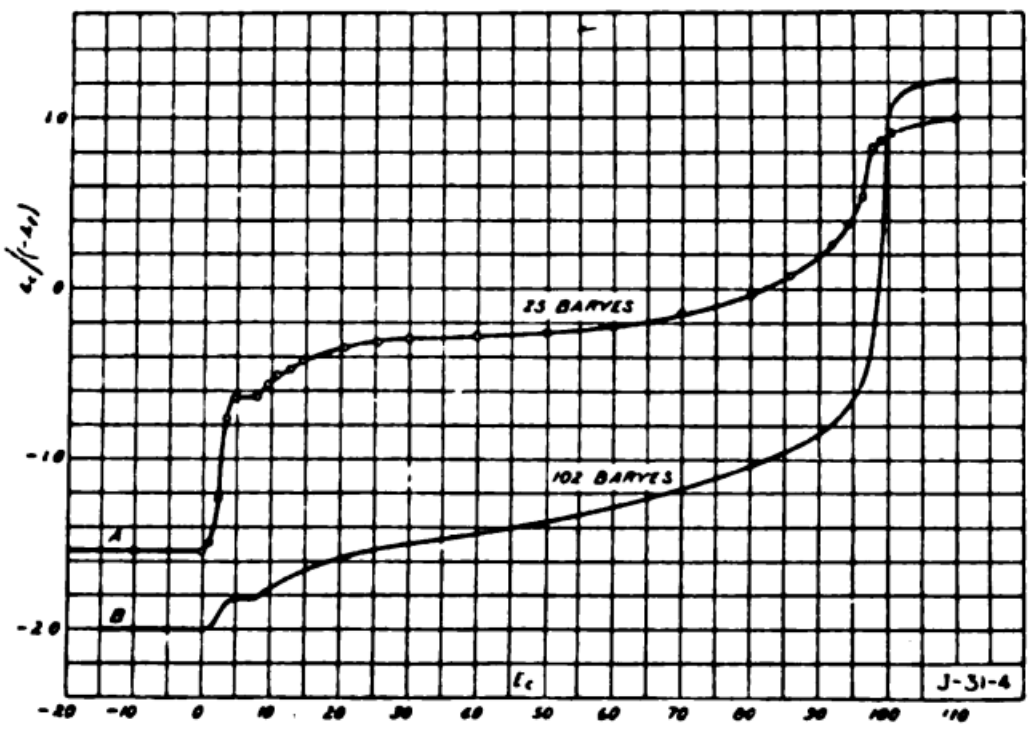

FIG. 2.

the filament (of 0.178 mm diameter) passed. To light the filament to a temperature at which 5 milliamperes of electrons were emitted required about 2.5 amperes and 3.3 volts. The electron emission was fairly uniform along the filament up to within about 5 mm of the ends of the filament and then dropped off rapidly because of the cooling effect of the leads, which extended very close to the holes in the plates.

Curve A (Fig. 2) was obtained with mercury at a pressure of 25 baryes (51.5°C), while B corresponds to 102 baryes (75°C) of mercury vapor. The two plates A and B were connected together and were used as anode at 100 volts, while the potential of the cylinder C was varied and the current i_C was measured.[6] The filament F was heated by a storage battery and the current

[6] We adopt the convention that the sign of the current to any electrode is that of the charged particles that it would have to emit in order to carry that current. Thus if F emits electrons or receives positive ions i_F is negative, while i_C is positive if C receives electrons and negative if it receives positive ions.

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i_P of electrons emitted (and positive ions received) was measured directly. This current was maintained at approximately -5 milliamperes. The ordinates in Fig. 2 represent the ratio $i_C/(-i_P)$ and the abscissas are the collector po-

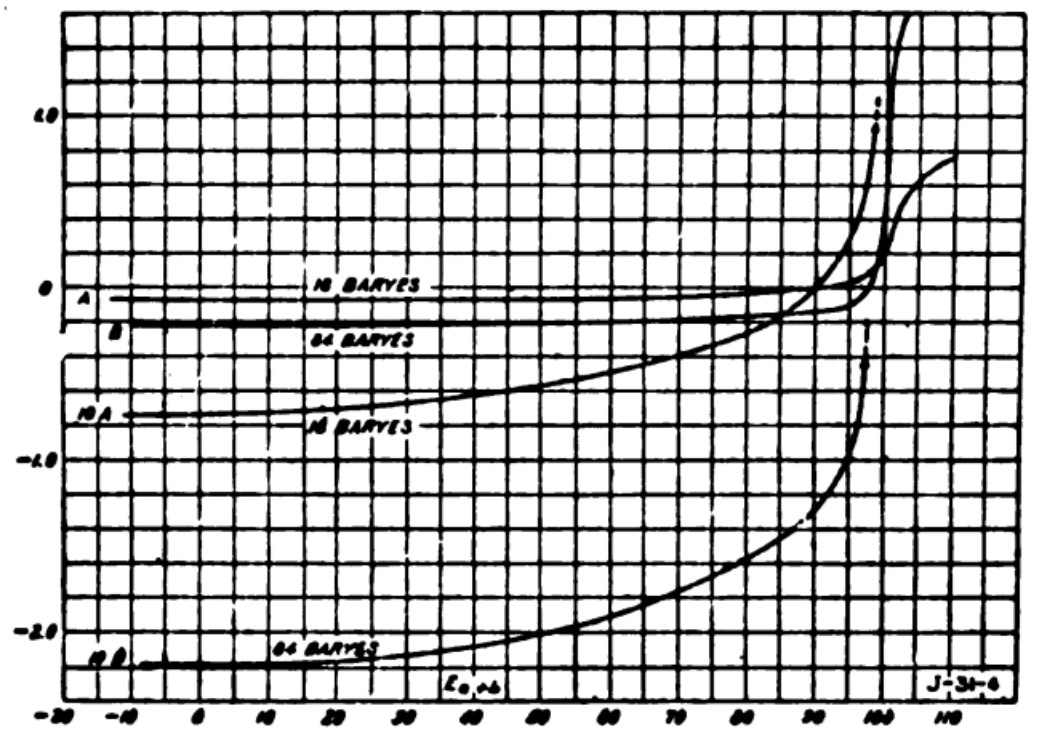

FIG. 3.

tentials E_C. It was found in nearly all cases that for moderate variations in i_P the ratio $i_C/(-i_P)$ remained nearly constant.

The curves of Fig. 3 were obtained under similar conditions to those of Fig. 2, except that the functions of the two electrodes $(A+B)$ and C were

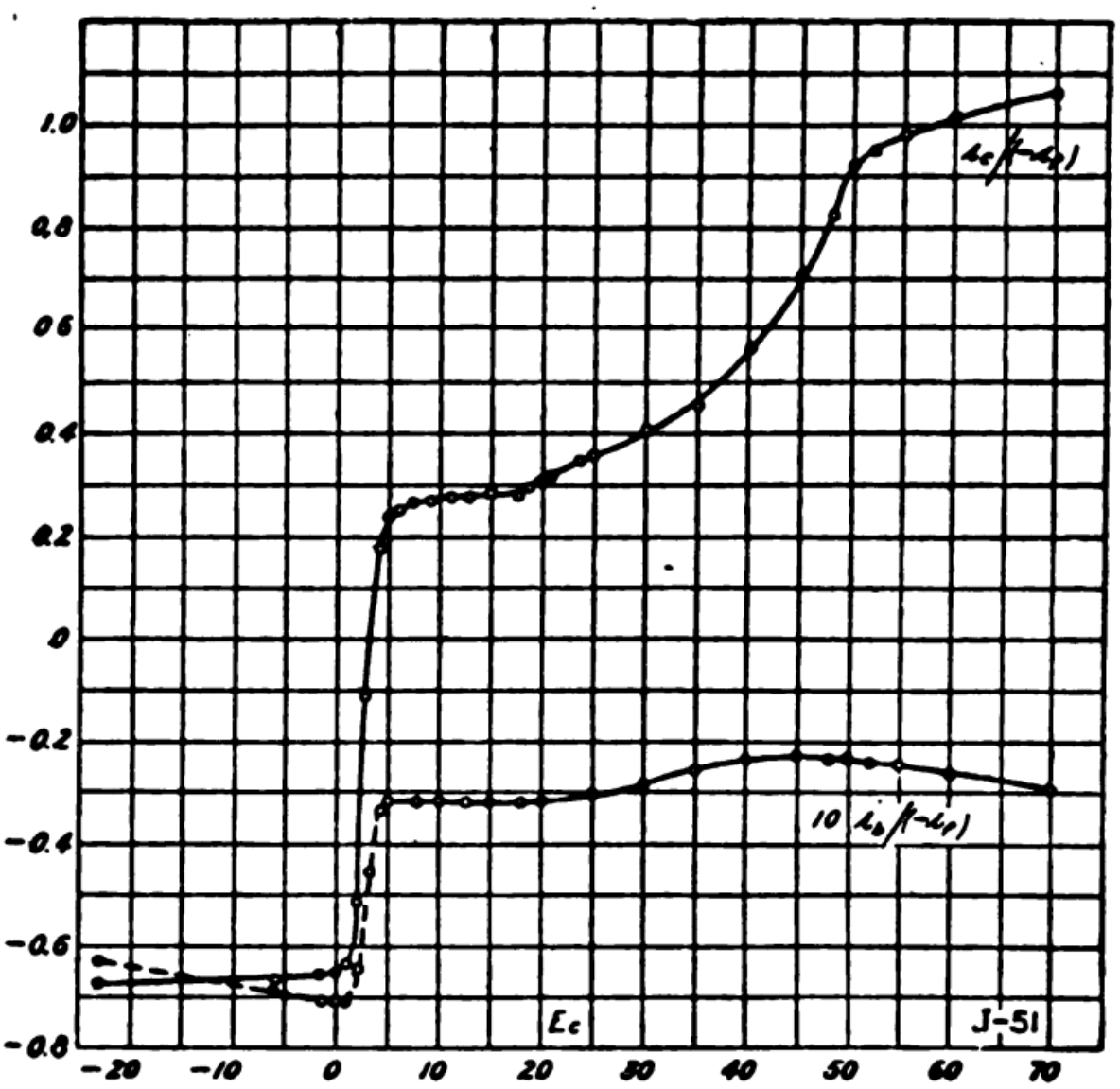

FIG. 4.

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interchanged. Thus the cylinder C was used as anode and was maintained at $+100$ volts, while the two plates A and B (connected together) were used as collector. The lower curves marked $10A$ and $10B$ are the same as the curves A and B except that the ordinates are multiplied by 10.

The data in Fig. 4 were obtained with argon at a pressure of 30 baryes. The plate A was used as anode at a potential of $+50$ volts, while B and C were used as collectors. The potential E_C of the cylinder was varied, but the plate B was maintained at a constant potential of -23 volts. The ordinates of the dotted curve representing the current i_B have been multiplied by 10.

Positive Ion Current Density I_p

The practically constant currents that are observed in Figs. 2, 3 and 4 when the collector is at negative potentials measure the rate at which ions diffuse to the edge of the sheath on the collector. These currents divided by the effective areas of the collectors give us a measure of the positive ion current densities I_p in the ionized gas. If the velocities of the positive ions are distributed according to Maxwell's laws corresponding to a temperature T_p, the number of ions per cm^3 can be calculated from the equation (reference 5, p. 455, Eq. (44))

$$n_p = 4.03 \times 10^{13} I_p (m_p/mT_p)^{1/2} \tag{1}$$

where m_p/m is the ratio of the mass of the ion and the electron, and I_p is to be measured in amps per cm^2. From data which we shall discuss later we may assume T_p is roughly constant and equal to 3000°K, so we are able to estimate the intensity of ionization, i.e., the number of ions per cc, from measurements of the positive ion current densities.

In case the thickness of the positive ion sheath on a collector is appreciable, the effective area of the collector may be different from its actual area. Let us, therefore, determine the thickness of the sheaths in the examples we have cited. In general, the sheath thickness may be calculated from the space charge equations, and these values agree excellently with those obtained by direct visual observation of the dark space surrounding the collector (reference 5, pp. 545, 622). For electrodes having the small curvature of those we are now considering, the space charge equation for parallel planes is applicable:

$$I_p = 2.34 \times 10^{-6} (m/m_p)^{1/2} V^{3/2}/x^2 \tag{2}$$

Here V represents the voltage drop through a sheath of thickness x (cm).

Table I contains a summary of the data on positive ion currents and sheath thickness from the curves of Figs. 2, 3, and 4. The first column refers to the number of the figure and the designation of the curve, and the second column gives the gas and its pressure in baryes.

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TABLE I

Positive Ion Currents and Sheath Thicknesses

The currents i_F, I_B and I_C are in milli-amperes. Data of Figs. 2, 3 and 4

1	2	3	4	5	6	7	8	9	10
Fig.	Gas	β_B	β_C	i_F	V	Sheath thickness		Positive current density	
						x_B	x_C cm	I_B	I_C
2*A*	Hg 25	—	1.54	−5.4	105	—	0.19	—	0.256
2*B*	Hg 102	—	1.99	−6.3	105	—	0.16	—	0.375
3*A*	Hg 18	0.073*	—	−4.5	105	0.63	—	0.023	—
3*B*	Hg 84	0.218*	—	−5.5	105	0.33	—	0.086	—
4	A 30	0.071	0.65	−4.9	73	0.31	0.22	0.054	0.112

* These values of β_B include the currents flowing to the electrode *A* which in this case was connected to *B*.

The sheath thicknesses in cm as calculated from Eq. (2) are given in columns 7 and 8. In the case of the data from Figs. 2 and 3 these thicknesses were calculated for an assumed collector potential of −5 volts, but for the data of Fig. 4, E_C was taken to be the same as E_B, which was kept at −23 volts. In columns 3 and 4 β_B and β_C are taken to represent the nearly constant values of $i_B/(-i_F)$ and $i_C/(-i_F)$ corresponding to the negative collector potentials.

The values of the positive ion current densities I_B and I_C in columns 9 and 10 were obtained by multiplying β_B or β_C by i_F (Column 5) and dividing by the areas of the sheaths on the collectors calculated from the values of x_B and x_C.

The intensity of ionization n_p as given by Eq. (1) may now be calculated from these values of I_p. For example, the value $I_C = 0.256$ ma. per cm² from the data of Fig. 2 corresponds to 1.1×10^{11} ions per cm³ if we assume $T_p = 3000°$.

We see from the data of Fig. 4 in Table I that I_C is about twice I_B. This effect is caused by radial electric fields which accelerate the ions outwards. Since the rate of production of ions per unit volume near the filament is greater than near the cylinder, the concentration of ions must decrease as the distance from the filament increases. The electron concentrations must everywhere in the body of the gas be nearly equal to those of the positive ions, and as the electrons have high mobilities they will set up potential differences which are related to their concentrations according to the Boltzmann equation

$$n_1/n_2 = \epsilon^{Ve/kT} \tag{3}$$

where n_1 and n_2 are the electron concentrations in two regions, T is the temperature of the electrons, and V is the potential of the first region with respect to the second. The ratio e/k is equal to 11,600 degrees per volt. Thus if the positive ion space charge is neutralized principally by electrons having a tem-

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perature of 10,000° there will be a potential difference of 0.60 volt between two regions in which the positive ion concentrations differ in the ratio of 2 to 1. Since concentration differences of this order of magnitude are to be expected even at the lowest pressure[7] there must be radial potential differences of the order of one volt. The velocity of the ions in a radial direction will therefore be greater than in the direction of the cylinder axis, and this explains the fact that I_C is greater than I_B in Fig. 4.

Comparing the values of I_C from Fig. 2 (Table I) with those of I_B from Fig. 3, we find that the ratios in these cases range from 4 to 11. It must be noted, however, that the values of I_C were measured when $A+B$ was anode, while I_B was measured when C was anode. With C as anode a certain fraction (about 50 per cent at 18 baryes) of the primary electrons were removed by this electrode, so that the total number of ions produced was less than if B or $A+B$ had been anode. However, allowing for this effect and for the differences in the pressures for the data of Figures 2 and 3, we find that the ion current densities at the cylinder are still 5 times (at 18 baryes) or 3 times (at 84 baryes) greater than those at the plates. Experiments with mercury vapor with connections like those of Fig. 4 have given ratios I_C/I_B not exceeding 2, so we must conclude that the high values for this ratio in Figures 2 and 3 are an indication that the radial electric field is greater when C is anode than when an end plate is anode. This conclusion is in accord with our theory of the cause of the radial field, for when C is negative the primary electrons that are reflected back into the gas from the sheath on C produce ions near the surface of C. The removal of these primary electrons by making C anode lowers the concentration of ions near the cylinder relatively more than near the filament and thus increases the radial field.

The currents flowing to negatively charged collectors are practically independent of the collector voltage according to the curves of Figures 2, 3, and 4. The dotted curve in Fig. 4, where the lower end of the curve shows a marked negative slope, corresponds to a case in which the current to the plate B is altered by changing the negative potential on the cylinder C. It was found, however, that if B and C were connected together so as to be at the same potential, the current to each electrode remained constant as the negative potential changed. The negative slope of $i_B/-i_F$ is very largely explained by considering the thicknesses of the sheaths on the electrodes. When the potential of C is changed from -23 to 0 volt (in Fig. 4), the sheath on C changes from 0.22 to 0.17 cm. This sheath on C partly covers the plate B and prevents positive ions from being collected near the edge of the plate. The effective diameter of the plate (actual diameter 3.1 cm) thus increases from 2.66 to 2.76 cm, as E_C changes from -23 to 0 volts.

[7] Direct measurements of the ion concentrations in a tube carrying an arc in mercury vapor at low pressures gave concentrations at the axis about 2 to 4 times greater than those at the wall. See *loc. cit.* (5) p. 769.

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The increase in area is 8 per cent, while the positive ion current ($-i_B$) increases about 14 per cent. Thus the change in collecting area accounts for only part of the change.

At higher pressures, where the free paths of the electrons become small compared to the radius of the tube, the radial concentration gradient of ions is determined by diffusion.[8] With a given concentration of ions near the axis the rate of flow to the sheath on C should increase as the sheath increases in thickness because of the decreased distance through which diffusion occurs. With a given rate of production of ions near the axis, the concentration should therefore decrease as the sheath becomes thicker. This effect, together with the decreased collector area, probably explains the negative slope of the left hand part of the curve for i_B in Fig. 4. It should be noted that the actual decrease in $i_B/(-i_F)$ from -23 to 0 volts is only 0.009, while the increase in $i_C/(-i_F)$ is 0.02.

Number of Ions Produced by Each Electron (β)

Since the ionized gas (Fig. 1) is completely surrounded by metallic electrodes at each of which the positive ion current density can be measured, we can determine the total number β of ions produced by each electron emitted from the cathode, assuming no recombination.

Let us express by β_A, β_B, etc., the ratio of the positive ion current at any electrode to the current i_F that flows from the filament. Then β_F gives the fraction of the current from F which is carried by the positive ions that strike the filament and $1-\beta_F$ is the fraction carried by electrons. If no recombination of ions and electrons occurs within the ionized gas the total ionization β per electron emitted is evidently

$$\beta = (\beta_A+\beta_B+\beta_C+\beta_F)/(1-\beta_F) \qquad (4)$$

In experiments like that which gave Fig. 4 we measure β_B and β_C. The value of β_A corresponds to the positive ion current flowing to the anode. If, as is usually taken for granted, the anode were positively charged with respect to the space near it, we should place $\beta_A = 0$ since the ions would be repelled from the anode. But in general, where the anode is of large area and especially where the gas is strongly ionized by high velocity primary electrons from a hot cathode, the anode will be at a negative potential with respect to the ionized gas. Direct measurements of the space potential by small collectors near the anode have shown these negative anode drops (reference 5, page 767). Compton and Eckart,[9] following a suggestion of one of us, have

[8] Schottky and Issendorff, *Zeits. f. Physik* **31**, 163 (1925) have developed the mathematical theory of the diffusion of electrons and ions subject to the condition that the concentrations of the ions and the electrons shall everywhere be equal.

[9] K. T. Compton and Carl Eckart, *Nature* **114**, 51 (1924); *Phys. Rev.* **25**, 139 (1925). See also; Compton, Turner and McCurdy, *Phys. Rev.* **24**, 577 (1924).

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demonstrated their existence in low voltage arcs at higher pressures.

A. F. Dittmer has suggested the use of plane collectors provided with one or more small round holes (of diameter small compared to the sheath on the collector) back of which is a Faraday cage or a second plate. By this arrangement the positive ions and electrons that pass through the holes can be separated. Measurements recently made by us with such a perforated collector have shown that it receives just as many positive ions when it is anode as when it is at a large negative potential.

We may therefore conclude that we should place $\beta_A = \beta_B$. The only remaining unknown quantity in Eq. 4 is β_P. This may be calculated from space charge equations with the degree of accuracy with which I_P in the neighborhood of the cathode may be estimated. Since β_P is usually very small, this method is often permissible. In further experiments now in progress β_P is being measured by the changes in resistance and in the electron emission of the filament due to the temperature rise that results from the positive ion bombardment.

If the positive ion current to F is limited by the rate at which positive ions reach the sheath surrounding the filament we may calculate β_F by the space charge equation

$$i_F\beta_F = 14.68\times10^{-6}L(m/m_p)^{1/2}V^{3/2}/r\beta_0^2 \tag{5}$$

where L is the length and r the radius of the filament, V is the voltage drop through the sheath of radius a, and β_0 is a function of a/r for which tables are available.[10] As we do not know a we cannot use this equation directly to obtain β_F, but since the outside surface of the sheath is the effective collecting area we may use the relation

$$i_F\beta_F = 2\pi aLI_p \tag{6}$$

and can then change Eq. (5) to the form

$$(a/r)\beta_0^2 = 2.34\times10^{-6}(m/m_p)^{1/2}V^{3/2}/(r^2I_p). \tag{7}$$

All the quantities of the right hand member are known. Thus by preparing a curve giving $(a/r)\beta_0^2$ as a function of a/r from Table III of paper in ref. 10 we may determine a and then by (6) can find β_F.

For the case of the experiment that gave Fig. 4 let us place (see Table I) $I_P = 1.12\times10^{-4}$ amps per cm². Putting $V = 73$, $(m/m_P)^{1/2} = 1/272$ and $r = 0.0089$ cm we find by (7) that $(a/r)\beta_0^2$ is 605, which corresponds to $a/r = 12$. Thus a, the radius of the sheath on the cathode, is 0.107 cm. If we take I_P near the axis of the tube to be twice as great, viz., 2.24×10^{-4}, we find $a/r = 9.4$ and $a = 0.084$ cm. By (6) we now calculated β_F, taking ($i_P = 4.9$ ma.) $L = 3.8$ and find for $I_P = 1.12\times10^{-4}$, $\beta_F = 0.058$, while for

$$I_p = 2.24\times10^{-4}, \quad \beta_F = 0.092.$$

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[10] Langmuir and Blodgett, *Phys. Rev.* **22,** 347 (1923).

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When the radius of the sheath around a cylindrical collector is as great as 10 times that of the collector itself, the tangential components of the velocities of the ions when they enter the sheath may cause a considerable fraction of the ions to describe orbits about the collector and thus pass out of the opposite side of the sheath.

Under these conditions the current flowing to a collector of area A is

$$i = AI_p f \tag{8}$$

where f is a function of a/r, V and T_p, whose value approaches a/r when Ve/kT_p is less than $2(a/r)-2$ and approaches the value $2[(1+Ve/kT_p)/\pi]^{1/2}$ when a/r and T_p are sufficiently large.[11] For the case in hand we find the following values for f

I_p	$f(T_p = 5000)$	$f(T_p = 10{,}000)$
1.12×10^{-4}	10.52	8.72
2.24×10^{-4}	8.95	7.88

We see that the radius of the sheath, which is proportional to f is comparatively little affected by doubling either I_p or T_p. Taking the higher value of I_p as the more probable, and choosing $f = 8.4$, we find from Eq. (8) by placing $i_p = 4.9$ m. amp., $A = 0.212$ cm² and $i_F\beta_F = i$, the most probable value of β_F to be

$$\beta_F = 0.081. \tag{9}$$

Inserting $\beta_C = 0.67$, $\beta_A = \beta_B = 0.062$ and $\beta_F = 0.08$ in Eq. (4) gives us for the total ionization the value $\beta = 0.95$. That is, in an experiment with argon at 30 baryes, each 50 volt electron produces 0.95 ion before losing its ionizing power.

If the conditions of the experiment are such that no recombination occurs, and all the primary electrons give up their energies to gas molecules before reaching the anode, we should expect β to be independent of pressure, the primary electron current and such geometrical factors as the diameter or length of the cylinder C.

Thus we can understand why it is, in experiments such as that of Fig. 2, that the positive ion current to the negatively charged cylinder is so nearly independent of the potential of the cylinder even when the sheath thickness varies considerably. If the number of ions per cc in the gas remained constant, the current to C should be proportional to the diameter of the sheath, so that the voltage-current curve would have a negative slope for negative cylinder voltages. But if β remains constant and if the current i_A is small compared to i_C, i_C must also be constant, and therefore the intensity of ionization of

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[11] The complete expression for f is given in the *Gen. Elect. Rev.* article reference,[5] p. 454. A discussion and derivation of these equations has been given by Mott-Smith and Langmuir.[5]

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the gas must increase as the sheath on the cylinder becomes thicker. This effect is due to the reflection of the electrons from the sheath edge, which increases the primary electron current density and makes the rate of ionization per unit volume inversely proportional to the volume.

When A and C are connected together the fact that the curves for i_A and i_C are flat (for negative voltages) is explained by the constancy of β. In the data of Fig. 4, where E_B was kept constant while E_C was varied, we have seen that the negative slope of i_B was about equal to the positive slope of i_C. This is a result of the constancy of β.

Primary Electrons Which Reach the Cylinder C

If the cylinder C is brought to a positive potential the retarding field is no longer able to prevent primary electrons from reaching it. At pressures low enough for an appreciable fraction γ of the primary electrons to reach the collector C without having collided with atoms, there is thus an abrupt change Δ_C in the value of $i_C/-i_F$ when the potential E_C is raised from negative to positive values. In Fig. 2 we see for mercury vapor at 25 baryes that Δn is 0.90, while with a pressure of 102 baryes Δn is only 0.18, indicating that as the pressure increases a larger fraction of the electrons from the cathode lose energy by collision.

The change Δ_C is due to two factors: (1) the electrons taken up by C and (2) the decrease in the positive ion current to C resulting from the removal of the primary electrons by C.

The primary electron current from the cathode is $i_F(1-\beta_F)$. Of these electrons the fraction γ reach the cylinder when it is positive, but only the fraction $1-\varrho$ of these are collected, where ϱ is the reflection coefficient for electrons. Thus the current due to primary electrons which reach the cylinder directly is

$$i_F\gamma(1-\beta_F)(1-\varrho) \tag{10}$$

and the current of electrons reflected is $i_F\gamma\varrho(1-\beta_F)$. When the potential of the cylinder is only a few volts positive, the primary electrons lose nearly all their energy in traversing the sheath on the cylinder, and for these low velocities the reflection coefficient ϱ is small (about 0.2). Although the reflected electrons probably leave the surface of the collector with a distribution of directions approximating to Lambert's cosine law, they are so strongly accelerated by the field in the sheath that they emerge from the sheath moving in a radial direction, so that we may consider the reflection to be specular.

Thus at low pressure some of the reflected electrons pass back across the tube, a fraction γ^2 of them reaching the opposite side of the cylinder, so that the fraction $\gamma^2\varrho$ of them are reflected a second time. The total electron current is thus obtained by multiplying that given by (10) by $1+\varrho\gamma^2+\varrho^2\gamma^4+\cdots = 1/(1-\varrho\gamma^2)$.

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Each primary electron emitted from the cathode produces an ionization which contributes $\beta_C/(1-\beta_F)$ to the positive ion current to C. Similarly, each primary electron removed by the collector C will cause a decrease $B'_C/(1-\beta_F)$ in the positive ion current to C. Here β'_C differs from β_C only because the ions produced near the cylinder are more likely to reach the cylinder than if they were produced near the filament, and because the radial potential gradient which draws ions away from the filament is decreased by ions produced near the cylinder. At low pressures it is probable that β'_C and β_C are practically equal.

Since the total change in current to the cylinder, when this becomes positive, is $i_F\Delta_C$ we find

$$\Delta_C = \gamma(1-\varrho)(1+\beta'_C-\beta_F)/(1-\varrho\gamma^2). \tag{11}$$

Fig. 3 shows that there is no corresponding break in the current to the end-plate if the potential of the plate is varied while C is kept negative. This means that there are no electrons in the ionized gas which move in an axial direction with the velocity of the primaries. However, the current i_B does suffer a break (as in Fig. 4) if E_C, the potential of the cylinder is changed from negative to positive values. Let us represent the change in $i_B/-i_F$ by Δ_B. Each electron from the cathode contributes a fraction of the positive ion current to B equal to $\beta_B/(1-\beta_F)$ and we may assume that each electron taken up by the cylinder C causes a decrease of $\beta'_B/(1-\beta_F)$ in the current to B. Here β'_B and β_B are probably even more closely equal than are the corresponding quantities β'_C and β_C.

By reasoning similar to that used in deriving (11) we find

$$\Delta_B = \gamma(1-\varrho)\beta'_B/(1-\varrho\gamma^2). \tag{12}$$

By division we obtain from (11) and (12)

$$\Delta_B/\Delta_C = \beta'_B/(1+\beta'_C-\beta_F). \tag{13}$$

The break Δ_B is observed with value unchanged when E_c is raised from a negative to a positive value, whether B is kept at constant negative voltage or is connected to C. While Δ_C is due to changes in both the electron and the ion currents to C, Δ_B is caused solely by the change in ion current.

The experiments show that the ratio of the observed values of Δ_B and Δ_C agrees well with that calculated by Eq. (13). For example, the data from Fig. 4 give $\beta_B = 0.071$, $\Delta_B = 0.039$, $\beta_C = 0.65$ and $\Delta_C = 0.92$ and we have already estimated the value of β_F to be 0.08. The value of Δ_B/Δ_C calculated from Eq. (13) by ignoring the distinction between β and β' is thus 0.0451, while the ratio of the observed values of Δ_B and Δ_C is 0.0424. This difference, amounting to 6 per cent, is in the direction to be expected, since β'_C should actually be slightly greater than β_C.

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Theory of Current-Voltage Characteristic for Primary Electrons Which Make Elastic Collisions

A detailed analysis of the shape of the current-voltage characteristic of $i_C/(-i_F)$, such as the curve shown in Fig. 4, will enable us to calculate the numbers of electrons which make elastic and inelastic collisions and also to determine the distribution of the angles through which the electrons are deflected or scattered by their collisions with gas molecules.

We have seen that each primary electron moves along a radius until it either reaches the sheath on the cylinder or collides with a gas molecule. Although the kinetic energy of an electron is not appreciably changed by an elastic collision, the velocity no longer remains radial so that when the electron reaches the sheath on the cylinder (used as collector) it is not able to move against so large a retarding potential as if it had not collided. In general, the greater the angle through which the electron has been deflected by the collision, the higher will be the collector potential required to collect the electron. Let us now attempt to calculate what fraction of the electrons which make collisions can reach the collector when this is at any given potential.

In Fig. 5, which shows the tube in cross-section, O is the cathode filament and the circle of radius R is the collecting cylinder. Consider an electron which is projected from the cathode sheath in the radial direction OU and let this

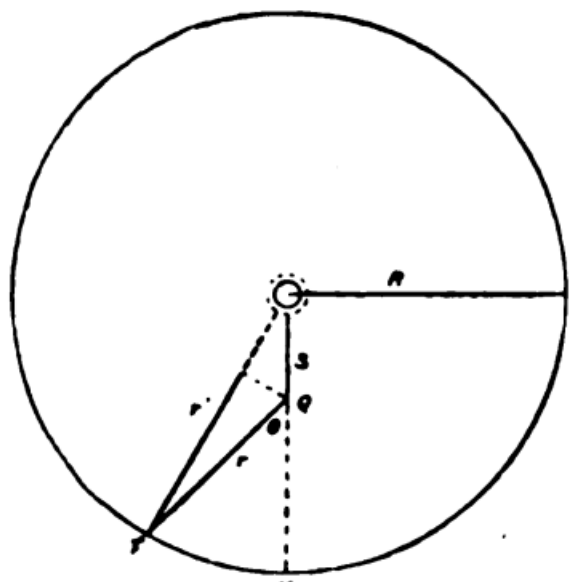

FIG. 5.

electron make an elastic collision with a molecule at Q after having moved a distance S from the cathode. After the collision the electron in general will have a velocity component along the axis of the cylinder, but this component will have little effect in determining whether or not the electron will be collected by the cylinder, provided (as we are assuming) that the cylinder is long enough. We therefore need consider only the velocity component perpendicular to the axis. We shall call r the magnitude of this component and θ the angle which its direction makes with the original direction of motion OU. When the electron arrives at the cylinder its radial velocity component

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is the projection of r on OT, and is thus equal to $r[1-(S^2/R^2)\sin^2\theta]^{1/2}$.

The electron will be collected only if this radial velocity equals or exceeds a certain value u_0 given by

$$u_0 = [2(e/m)(V_0-E)^{1/2} \tag{14}$$

where V_0 is the potential of the ionized gas (approximately anode potential) and E is that of the collector, both measured from the cathode. Thus the condition that the electron shall be collected is

$$r^2-(r^2s^2/R^2)\sin^2\theta \geqq u_0^2. \tag{15}$$

Scattering through cone of semi-angle θ_M.—Let us now consider a large number N_s of electrons which make their collisions with gas molecules in a small element of volume surrounding Q after having traveled a distance S. We will assume provisionally that the velocity vectors after collision are distributed at random through a cone of semi-angle θ_M whose axis is the original

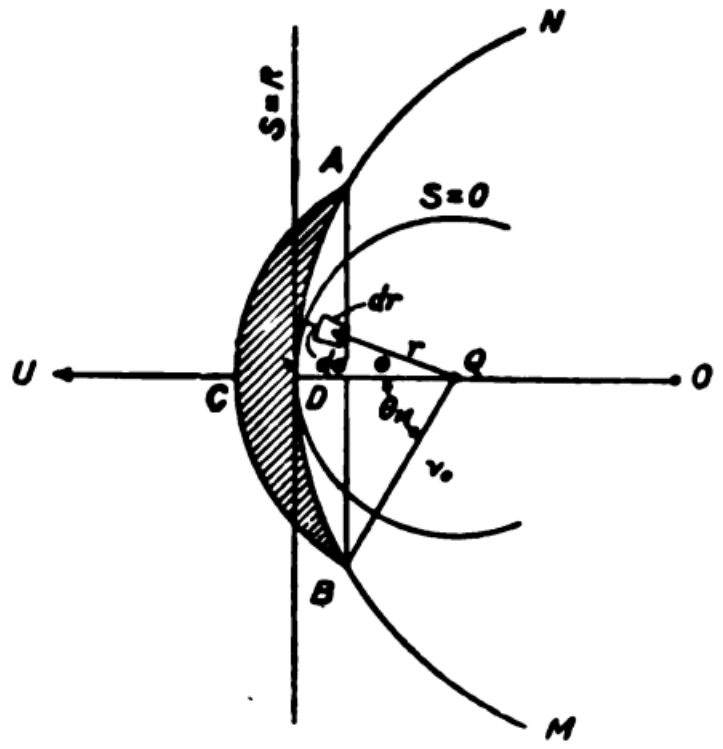

FIG. 6.

direction of motion of all the electrons (OU). In other words the end points of the vectors are distributed with uniform density on a zone of a spherical surface of radius v_0 where v_0 is the magnitude of the original velocity and is given by

$$v_0 = [2(e/m)V_0]^{1/2}. \tag{16}$$

The *projected* velocities r may therefore be represented in a diagram with polar coordinates, r and θ as in Fig. 6, by points lying in a circular segment ABC of radius v_0 and semi-angle θ_M and it is readily found that the number of points in a specified element of area $rdrd\theta$ is

$$\frac{N_s}{2\pi v_0^2(\sin\frac{1}{2}\theta_M)^2}\cdot\frac{r\,dr\,d\theta}{[1-(r^2/v_0^2)]^{1/2}}. \tag{17}$$

Not all of these representative points correspond to electrons which can be collected, but only those which satisfy the condition (15). Thus for any given

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value of S the points corresponding to the electrons that can be collected lie, in Fig. 6, to the right of a certain curve, (such as MDN) whose equation is found by taking the equality sign in (15). This curve is in general an ellipse, whose center is at Q and which, as indicated in Fig. 6, degenerates into a circle for $S = O$ and into a straight line for $S = R$. For any given distance S the number of electrons collected (N_C) is therefore found by integrating the expression (17) over the shaded region indicated in Fig. 6. This integration is best carried out by expanding the expression (17) into a series in terms of E/V_0. Taking only the first order terms and restricting ourselves to values of θ_M less than about 30° we thus find (approximately)

$$N_C = \frac{N_S R}{S} \cdot \frac{E/V_0}{\sin^2 \theta_M} \tag{18}$$

provided that S is greater than a critical value S_1 defined by

$$S_1 = R(E/V_0)^{1/2}/\sin \theta_M \tag{19}$$

For this value of S the ellipse MDN of Fig. 6 passes through the points A and B. When S is greater than S_1 the expression for N_C/N_S is the same (except for the proportionality factor $1/\sin^2 \theta_M$) as it would be if the electrons were scattered uniformly through *all* angles instead of through a cone of semi-angle θ_M. Thus Eq. (18) is relatively simple.

When, however, S is less than S_1 the limits of integration depend upon θ_M and the expression for N_C/N_S becomes very complicated, but it is found that the function can fairly well be represented by

$$N_C = N_S(E/V_0)^{1/2}k(S)/\sin\theta_M \tag{20}$$

where $k(S)$ is a quantity which is nearly independent of E and of θ_M but varies from $4/\pi$ to 1 as S increases from 0 to S_1.

For collisions taking place near the filament, that is, for $S = 0$, Eq. (20) gives for small values of θ_M and E/V_0

$$N_C = (4/\pi)N_S(E/V_0)^{1/2}/\sin\theta_M. \tag{21}$$

If the scattering occurs uniformly through all angles so that $\theta_M = 180°$ as in the case of electrons colliding with heavy elastic spheres, a rigorous expression for N_C can readily be worked out by integrating over the whole annular ring between the circles indicated in Fig. 6. We find in this case

$$N_C = N_S(E/V_0)^{1/2}. \tag{22}$$

For collisions occurring near the cylinder, that is, for $S = R$, a rigorous expression for N_C is obtainable:

$$N_C = \frac{N_S[1-(1-E/V_0)^{1/2}]}{2\sin^2(\theta_M/2)} \tag{23}$$

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provided that E/V_0 is not greater than $\sin^2\theta_M$. If E/V_0 is greater than $\sin^2\theta_M$ then we have

$$N_C = N_S \text{ for } \theta_M < 90°$$

or

$$N_C = N_S\left[1-\frac{(1-E/V_0)^{1/2}}{\sin^2(\theta_M/2)}\right] \text{for } \theta_M > 90°. \tag{24}$$

Distribution of collisions along the radius.—We must now take into account the fact that the collisions of the electrons with molecules can occur at all points along the radius. If λ is the mean distance that an electron travels through the gas before making a collision then $1/\lambda$, which we shall denote by P, is the probability of collision per unit length of path. For each particular type of collision that may occur in a gas we shall have a different value of P. We shall use the following notation for the probabilities per unit length of path; P for collisions of all kinds, P_e for elastic collisions, P_k for inelastic collisions, P_i for collisions that cause ionization, P_r for resonance collisions, i.e., those that cause the impinging electrons to lose a definite energy corresponding to a resonance potential of the gas.

The first three of these quantities are related:

$$P = P_e+P_k. \tag{25}$$

We have used γ, according to Eq. (11), to denote the fraction of the primary electrons which reach the sheath on the cylinder C after having undergone some specified change as a result of collisions. In accordance with this use we may let: γ_0 be the fraction of the primaries which reach the sheath[12] on C without colliding with gas molecules; γ_n be the fraction of the primaries which reach the sheath on C after having made n *elastic* collisions but no inelastic collisions (we have γ_1, γ_2, etc., corresponding to $n = 1$, $n = 2$, etc.); γ_e be the fraction of the primaries which reach the sheath on C after having made one or more elastic collisions, but no inelastic collisions; and γ_r be the fraction of the primaries which reach the sheath on C after making one resonance collision. This includes the electrons which may also have made one or more elastic collisions (either before or after the resonance collision). From these definitions we see that

$$\gamma_e = \gamma_1+\gamma_2+\gamma_3+\cdots \tag{26}$$

$$\gamma_0 = \epsilon^{-RP} \tag{27}$$

We calculate γ_1 as follows. If N_P primary electrons leave the cathode sheath then $N_P P_e \epsilon^{-SP} ds$, of these make elastic collisions with molecules in the distance ds after they have traveled a distance S without having previously made inelastic collisions. In accordance with experiments which will be described,

[12] In these definitions it is specified that the electrons shall reach merely the edge of the sheath, i.e., the boundary between the sheath and the field-free ionized gas, for at present we do not wish to distinguish between the electron that can reach the *cylinder* against the retarding field in the sheath and those that are not able to do so.

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we assume that the electrons are deflected by the elastic collisions only through rather small angles so that after the collision the distance the electron must travel before reaching the cylinder is $R-S$. Thus the fraction $d\gamma_1$ of the primary electrons which reach the cylinder after a single elastic collision (but no inelastic collision) is

$$d\gamma_1 = P_e\epsilon^{-SP}\epsilon^{-(R-S)P}\,ds = P_e\epsilon^{-RP}\,ds \tag{28}$$

and therefore by integration from $S = 0$ to $S = R$

$$\gamma_1 = RP_e\epsilon^{-RP}. \tag{29}$$

In general we find in a similar manner

$$\gamma_n = (1/n!)(RP_e)^n\epsilon^{-RP}. \tag{30}$$

Thus by Eqs. (26) and (27) we obtain

$$\gamma_e = \epsilon^{-R(P-Pe)} - \gamma_0 \tag{31}$$

and by (25)

$$\gamma_0+\gamma_e = \epsilon^{-RPk}. \tag{32}$$

By elimination of P between Eqs. (29) and (31) we obtain

$$\gamma_0/(\gamma_0+\gamma_e) = \epsilon^{-RPc}. \tag{33}$$

For the resonance collisions we have, by the method used for Eq. (29),

$$\gamma_r = RP_r\epsilon^{-RPk} \tag{34}$$

and thus by Eq. (32)

$$\gamma_r = RP_r(\gamma_0+\gamma_e). \tag{35}$$

Let N_e be the average number of elastic collisions that are made by the electrons which reach C after having made *at least one elastic but no inelastic collisions.*

Let N_r be the average number of *elastic* collisions that are made by the electrons which reach C after having made *one resonance* collision. We see that

$$N_e = \frac{\gamma_1+2\gamma_2+3\gamma_3+\cdots}{\gamma_1+\gamma_2+\gamma_3+\cdots} = \frac{RP_e}{1-\epsilon^{-RP_e}} \tag{36}$$

and

$$N_r = \frac{\gamma_1+2\gamma_2+3\gamma_3+\cdots}{\gamma_0+\gamma_1+\gamma_2+\gamma_3+\cdots} = RP_e. \tag{37}$$

By combining (36) and (33) we obtain the more convenient equation

$$N_e = RP_e(\gamma_0+\gamma_e)/\gamma_e \tag{38}$$

Analysis shows that the electrons which reach the cylinder after having made n elastic collisions have traveled an average distance $R/(n+1)$ since the last collision. In fact it is readily seen if we consider only electrons that reach C after having made one or more elastic collisions, that these collisions are distributed with equal probability along the radius.

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Scattering by collisions uniformly distributed along a radius.—We wish to consider the current-voltage curve due to electrons, collected by C, which have made single elastic collisions distributed at random along a radius. We have defined γ_1, as the fraction of the primary electrons that reach the sheath on C after having made single elastic collisions. In general, because of the retarding field in the sheath, only the fraction γ of the primary electrons will reach the surface of C. Evidently γ will become equal to γ_1 if the retarding field should disappear, that is if E, the potential of the collector is made equal to V_0, the potential of the gas. The value of γ is found by integrating the expression for N_C/N_s from Eqs. (18) and (20) along the radius. Thus

$$\gamma = (\gamma_1/N_s R) \int_0^R N_C \, ds. \tag{39}$$

This integration must be carried out in two steps, from $S = 0$ to $S = S_1$, by Eq. (20) and from S_1 to R by Eq. (18). Placing for convenience

$$E/V_0 = \nu \tag{40}$$

we thus find[13]

$$\gamma = \gamma_1 (\nu/\sin^2 \theta_M)[1+\log (\sin \theta_M/\nu^{1/2})]. \tag{41}$$

This equation, like (18) and (20) is based on the assumption that ν and θ_M are small quantities. However, from an examination of the second and third order terms in the expansions it appears that Eq. (41) should give a good approximation (as accurate as the experiments) up to values of ν as high as 0.6 and values of θ_M as high as 30°.

From Fig. 6 we see that all electrons which are scattered through angles of less than θ_M will be collected by the cylinder, no matter where the collisions occur along the radius, if the voltage of the cylinder is raised sufficiently to make the point D meet the line AB or pass to the left of it. This will occur if

$$u_0 \leqslant v_0 \cos \theta_M. \tag{42}$$

By Eqs. (14), (16) and (40) we find that this condition is equivalent to

$$\nu \geqslant \sin^2 \theta_M. \tag{43}$$

When ν becomes equal to $\sin^2 \theta_M$, γ should be equal to γ_1, and in fact we see that Eq. (14) gives this value under these conditions. For values of ν greater than $\sin^2 \theta_M$ of course Eq. (41) does not hold but γ remains constant and equal to γ_1.

Although Eq. (41) is not applicable when θ_M is a large angle, it is possible to derive by similar methods the following approximate equation for the case that $\theta_M = 180°$, that is when electrons are scattered equally in all directions by collisions uniformly distributed along a radius

$$\gamma = \gamma_1 \nu [0.88 + \tfrac{1}{4} \log (1/\nu)]. \tag{44}$$

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Scattering according to probability laws.—We have thus far taken the directions of the scattered electrons to be uniformly distributed within a cone of semi-angle θ_M. If we have n electrons scattered in this way, the angular density η (expressed as electrons per steradian) can be calculated from the relation

$$n = 2\pi\eta(1-\cos\theta_M) = 4\pi\eta\sin^2(\theta_M/2). \tag{45}$$

When θ_M is small this becomes

$$n = \pi\theta_M^2\eta. \tag{46}$$

The error in using (46) in place of (45) is only 2.3 per cent for $\theta_M = 30°$ and 1 per cent for 20°.

Let us now assume that the scattered electrons instead of having uniform angular density are distributed according to a type of probability law such that

$$\eta = \eta_0\epsilon^{-\theta^2/\theta_0^2} \tag{47}$$

where η_0 is the maximum angular density and θ_0 is the angle at which the angular density of the scattered electrons is $1/\epsilon$ of η_0. It can be readily proved that θ_0 is the root-mean-square deflection of the electrons. The average angle θ_{av} and the most probable angle θ_p of scattering are related to θ_0 as follows

$$\theta_{av} = \tfrac{1}{2}\pi^{1/2}\theta_0 \text{ and } \theta_p = \tfrac{1}{2}2^{1/2}\theta_0. \tag{48}$$

The total number n of electrons that are scattered according to Eq. (47) is approximately (for not too large values of θ_0) given by

$$n = \int_0^\infty 2\pi\theta\eta\,d\theta = \pi\theta_0^2\eta_0. \tag{49}$$

Comparing this result with Eq. (46) we see that the total number of electrons is the same as in a cone of semi-angle θ_0 having electrons distributed with the uniform angular density η_0.

From the results already obtained we may now by an integration process derive an equation for the current collected by C when this is at a voltage E for the case that the electrons are scattered according to probability laws by collisions uniformly distributed along the radius.

If N_F primary electrons leave the cathode $N_F\gamma_1$ of them reach the sheath on C after making single elastic collisions, and we will assume that these are scattered according to Eq. (47). We may divide these $N_F\gamma_1$ electrons into an infinite number of groups, each group consisting of $N_F\,d\gamma_1$ electrons scattered through a cone of semi-angle θ (variable for different groups) with a uniform angular density $d\eta$. The number of electrons in a group is thus

$$N_F\,d\gamma_1 = \pi\theta^2 d\eta.$$

Let the collector C be maintained at a given voltage E, then from Eqs. (43) and (40) we see that if $\theta < \nu^{1/2}$ all the electrons in the group will be collected, but if $\theta > \nu^{1/2}$ only a part will be collected, the fraction collected being, by Eq. (41), equal to

$$(\nu/\theta^2)[1+\log(\theta/\nu^{1/2})].$$

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Thus we find that the total number $N_F\gamma$ of electrons reaching C is

$$N_F\gamma = \int_{\eta_c}^{\eta_0} \pi\theta^2 d\eta + \int_0^{\eta_c} \pi\nu[1+\log(\theta/\nu^{1/2})]\, d\eta \tag{50}$$

where η_c is the value of η which, by Eq. (47), corresponds to the critical angle $\theta = \nu^{1/2}$. Thus

$$\eta_c = \eta_0 e^{-\nu/\theta_0^2}. \tag{51}$$

We may express $d\eta$ in terms of θ by differentiating Eq. (47), and by Eq. (49) we find that

$$\eta_0 = N_F\gamma_1/(\pi\theta_0^2). \tag{52}$$

If we then substitute $y = \theta^2/\theta_0^2$, Eq. (50) becomes

$$\gamma/\gamma_1 = \int_0^x y\epsilon^{-y}\, dy + x\int_x^\infty [1+\tfrac{1}{2}\log(y/x)]\epsilon^{-y}\, dy$$

where

$$x = \nu/\theta_0^2 = E/(V_0\theta_0^2). \tag{53}$$

Carrying out the integration we get

$$\gamma/\gamma_1 = 1-\epsilon^{-x}+\tfrac{1}{2}x\int_x^\infty \epsilon^{-x}\, dx/x. \tag{54}$$

The integral in Eq. (54) is equal to $-Ei(-x)$ where *Ei* stands for the "exponential integral" given for example in Glaisher's Tables (*Phil. Trans.* 140, 367 (1870)). By means of the series expansion for $Ei(-x)$ we find for small values of x that

$$\gamma/\gamma_1 = x(0.7114-\tfrac{1}{2}\log x)$$

which gives only 3 per cent error when $x = 1$. For larger values of x a good approximation is

$$\gamma/\gamma_1 = 1-\tfrac{1}{2}\epsilon^{-x}.$$

Table II gives values of γ/γ_1 as a function of x, calculated from Eq. (54).

TABLE II

The Fraction of Collectable Electrons as a Function of $x = E/(V_0\theta_0^2)$ *as given by Eq* (54)

x	γ/γ_1	x	γ/γ_1	x	γ/γ
0.001	0.0042	0.2	0.3035	1.5	0.8519
0.01	0.0301	0.3	0.3950	2.0	0.9136
0.02	0.0534	0.5	0.5334	3.0	0.9698
0.005	0.1105	0.75	0.6553	4.0	0.9892
0.10	0.1863	1.00	0.7418	5.0	0.9961

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It is of interest to compare Eq. (54) which is for electrons scattered by probability laws, with Eq. (41) which is for electrons scattered uniformly through a cone. Replacing sin θ_M by θ_M in Eq. (41) and by analogy with Eq. (53) placing

$$x' = \nu/\theta_M^2 \tag{55}$$

we have

$$\gamma/\gamma_1 = x'[1+\tfrac{1}{2}\log(1/x')]. \tag{56}$$

A double logarithmic plot of the function of the right member of Eq. (54) is given in Fig. 8 by the full line ST (also marked $\delta = 0$). The corresponding function of Eq. (56) is represented in this figure by the dotted curve. By means

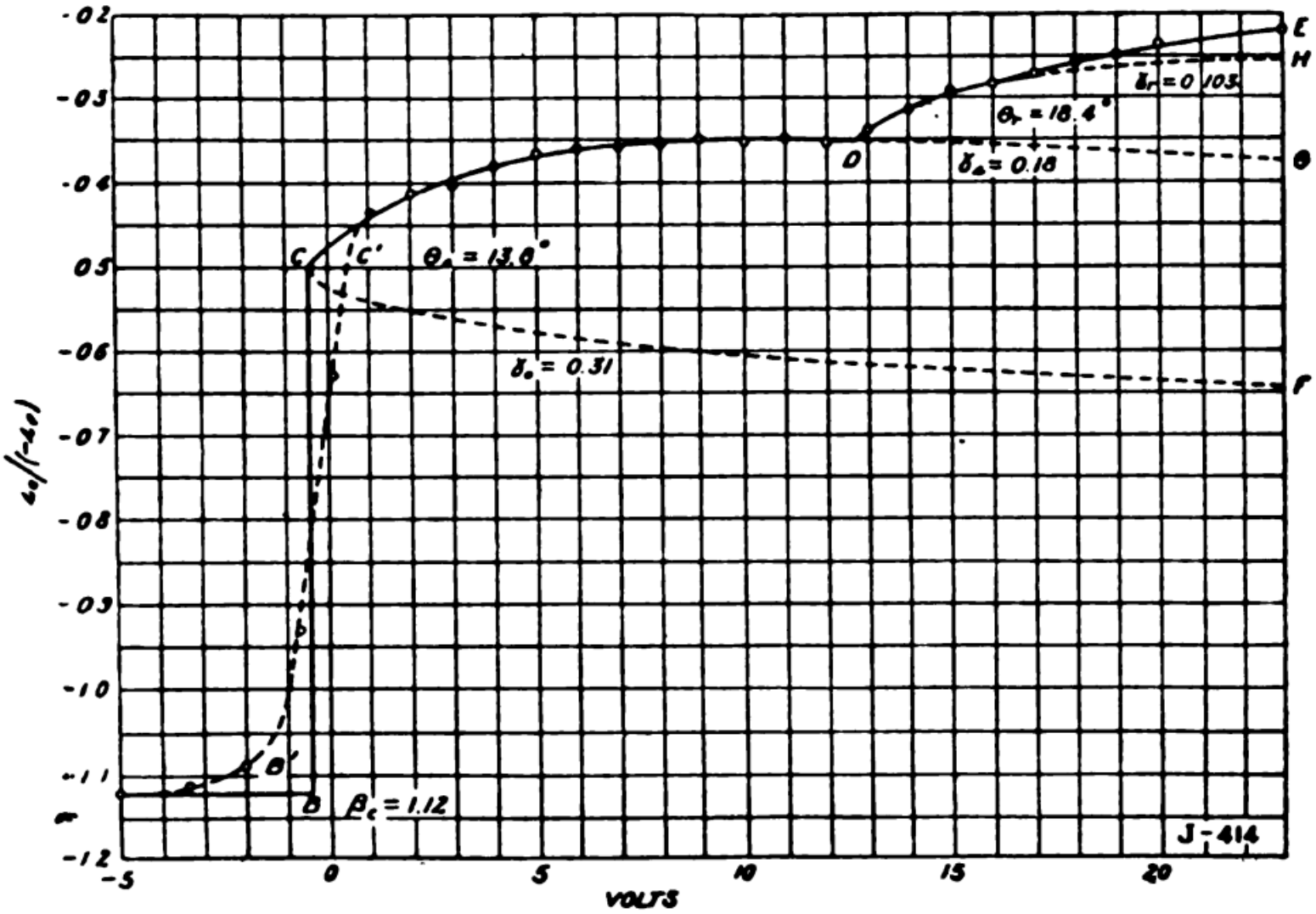

FIG. 7.

of the function given in Table II we shall be able to analyze current-voltage curves such as those of Fig. 2 and thus determine from the experiments the number of electrons that make elastic collisions and the mean angle θ_0 through which they are scattered by the collisions.

Application of the theory for electrons which have suffered collisions.—To illustrate the method of using the equations let us consider in some detail a typical example which gave the data recorded in Table III and Fig. 7. In this experiment nitrogen was used at 50 baryes and 650°K (in the cylinder C). This pressure was corrected for the effect of thermal effusion as will be described later. The cylinder C_r was of molybdenum, 5.0 cm long, 5.2 cm in diameter, and the end-plates were 4.85 cm in diameter and 5 cm apart. One of them, A, was used as anode at 100 volts while the other, B, was maintained at −6.0 volts.

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The filament F was of tungsten, 5.0 cm long, 0.0254 cm in diameter, heated to about 2300°K by an intermittent current, which read 5.95 amps on an A.C. ammeter and about 3.92 on a D.C. meter. A rotating commutator operated by a 60 cycle synchronous motor was used to produce the intermittent filament current and to connect the circuit for measuring the current i_C only in the intervals during which no current flowed through the filament. Thus the effects of voltage drop along the filament were avoided. The circuit for C was closed 43.6 per cent of the time so that the observed currents as read by a D.C. microammeter were multiplied by 1/0.436 to get the instantaneous current i_C.

Table III, in columns 1 and 2, gives the instantaneous currents to the cylinder C at various cylinder voltages. The currents are expressed as a fraction of the current i_F from the cathode. As these data are given only to illustrate the method used for calculation, alternate readings have been omitted from the table; all the observed points between −5 and +23 volts are indicated by the small circles in Fig. 7. The same data over a wider range of voltages are also given in curve V of Fig. 12.

TABLE III

Typical Current Voltage Data. Nitrogen at 50 *baryes and* 650°K; $R = 2.60$ *cm*; *anode at* 100 *volts*; $\beta_c = 1.12$; $\beta_F = 0.11$; $\gamma = 0.498\Delta/(1-\varrho)$; $\Delta = i_c/(-i_F)-1.12$

E_c	$i/-i_F$	$1/(1-\varrho)$	γ	obs. $\gamma-\gamma_0$	cal $\gamma-\gamma_0$	Diff.	γ'	E'
2	−0.413	1.100	0.387	0.077	0.076	+0.001	—	—
4	−0.384	1.135	0.416	0.106	0.107	−0.001	—	—
6	−0.362	1.163	0.439	0.129	0.128	+0.001	—	—
8	−0.354	1.187	0.453	0.143	0.142	+0.001	—	—
10	−0.352	1.208	0.462	0.152	0.152	0.000	—	—
12	−0.354	1.228	0.468	0.158	0.160	−0.002	—	—
14	−0.318	1.245	0.497	0.187	0.166	+0.021	0.0176	1.2
16	−0.286	1.262	0.524	0.214	0.171	+0.043	0.0380	3.2
18	−0.259	1.278	0.548	0.238	0.173	+0.065	0.0583	5.2
20	−0.240	1.292	0.566	0.256	0.175	+0.081	0.0735	7.2

The points in Fig. 7 lie along a broken curve $AB'C'DE$. The section AB corresponding to $\beta_C = 1.12$ gives the positive ion current. The break $B'C'$, which corresponds to γ_0, is due to electrons which reach C without collisions. Along the curved portion $C'D$ the electrons that have made elastic collisions are being collected, while in the range DE the electrons which have made resonance and ionizing collisions are also collected.

We wish now to distinguish between the electron current and the ion current. To do this we let Δ represent the increase in current[14] (at any given voltage) over the value −1.12 which corresponds to β_C. Then by Eq. (11) we

[14] For convenience we shall frequently refer to the current i_C when we really mean $i_C/(-i_F)$.

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may calculate γ the fraction of the primary electrons that are able to reach the surface of the cylinder at any given voltage. We may neglect the small quantity $\varrho\gamma^2$ and thus write Eq. (11) in the form

$$\gamma = \Delta[1/(1-\varrho)]/(1+\beta_C-\beta_F). \tag{57}$$

Reflection coefficient for electrons.—To use this equation we must know ϱ, the reflection coefficient for electrons. Since ϱ is a function of the velocity of the electrons, it might seem at first that for groups of electrons which are not all collectable at a definite voltage, the value of ϱ should enter Eq. (57) as a complicated integral. For example, if the collector is at +5 volts, the electrons collected include those that are just able to move against a retarding field of one volt as well as those that can move against five volts.

However, the electrons in these different groups, since they cannot have made inelastic collisions, must all have the same total energy as the primary electrons, and thus they differ only in their angles of incidence on the collector. There is considerable evidence, experimental and theoretical, that the reflection coefficient, at least on smooth clean surfaces, depends mainly on the total energy of the electrons rather than upon the normal component of the energy. Therefore, for electrons which have made elastic collisions, we are justified in taking ϱ the same for all those that are collected at a given voltage. Eq. (57) may thus be used if we take ϱ as the reflection coefficient for electrons which are moving with an energy corresponding to the voltage E (the energy of impact on C). Farnsworth[15] has given data for the reflection coefficient ϱ of electrons from tungsten surfaces the values 0.20 for 2 volts, 0.32 for 6 volts and 0.38 volts electrons. Hull's observations with dynatrons indicate that the reflection from molybdenum is considerably less than from tungsten. Petry[16] has published data on the reflection of electrons from molybdenum, but only relative, not absolute values of ϱ were given. Mr. Petry has kindly furnished us with the notes of his original observations and from these we have calculated values of 0.11 at 5 volts, 0.17 at 10 volts, 0.19 at 15 and 0.22 at 20 volts. Based partly on these data and partly on other data in this laboratory[17] we have adopted values for ϱ which give

$$1/(1-\varrho) = 1+0.0645E^{1/2}. \tag{58}$$

The potential E in this equation should correspond to the energy of the electrons which reach the collector. Examination of Fig. 7 shows that electrons begin to be collected when E_C is −2 volts and a large fraction are collected before C becomes positive. The initial velocities of the electrons as they leave the filament and any contact potential difference that may exist between the

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[15] H. E. Farnsworth, *Phys. Rev.* **25**, 41, (1925).

[16] R. L. Petry, *Phys. Rev.* **26**, 346, (1925).

[17] We shall see that the data from the present experiments at a series of pressures all indicate that $1(1-\varrho)$ at about 15 volts is close to 1.25 in agreement with Eq. (58).

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tungsten filament and the molybdenum cylinder tend to produce small displacements of the curves parallel to the voltage axis. Therefore we take, as our zero point of potential in calculating E, the midpoint on the steep rise $B'C'$ corresponding to the electrons that come through without collisions. Thus $E = E_C + 0.4$ volt, not only in Eq. (58) but when we calculate x or v by Eq. (53).

The data in the 3rd column of Table III are obtained from Eq. (58) in this way. The effect of possible errors in the value of ϱ will be discussed later. Taking $\beta_C = 1.12$ and assuming $\beta_F = 0.1$, $\beta_A = 0.11$ we thus find from Eq. (57) the values of γ given in the 4th column. From our theoretical study of the current-voltage curves we concluded that the relation between γ and the voltage E can be expressed by Eqs. (54) or (56) according as the electrons are scattered according to probability laws, or are scattered uniformly through a cone.

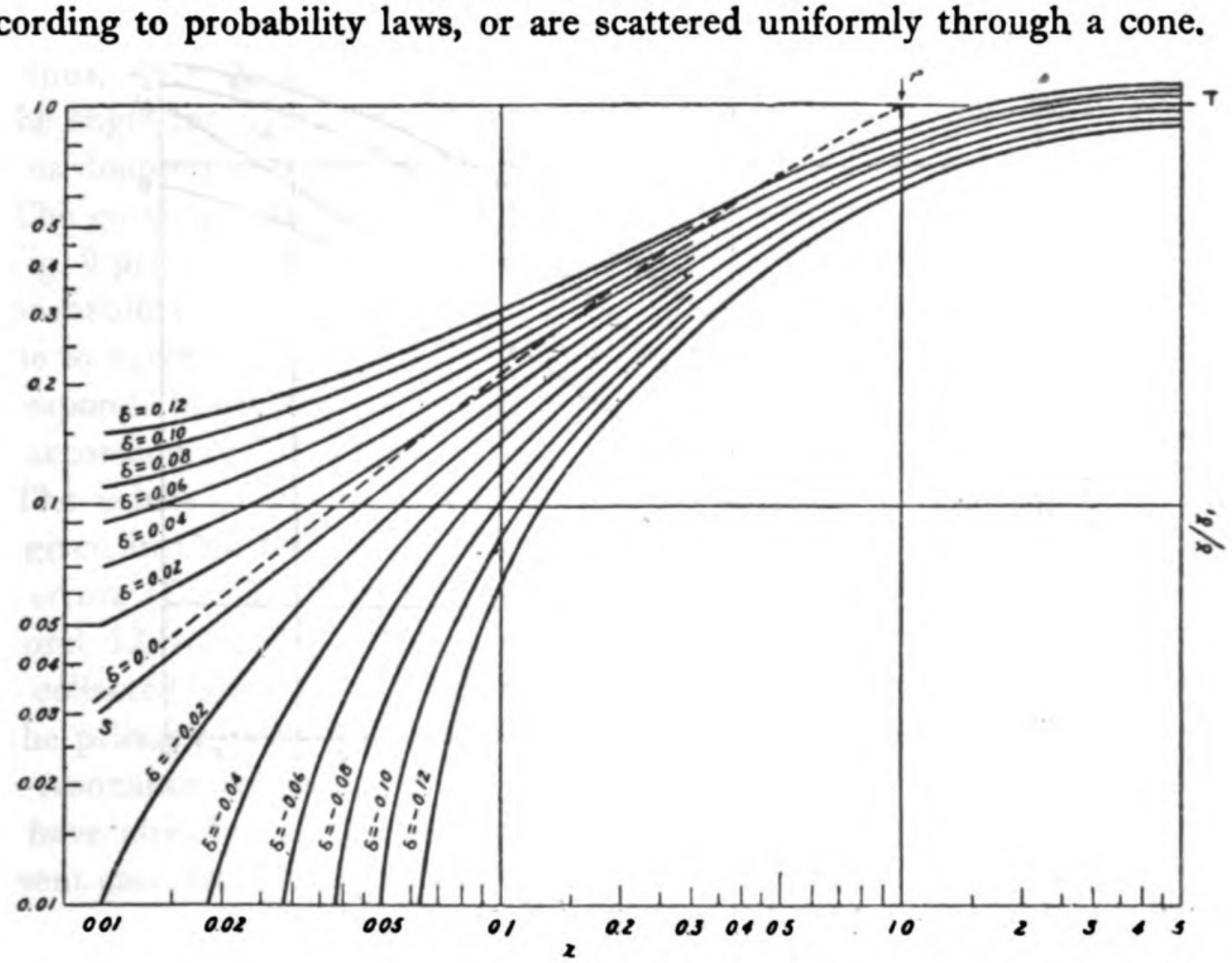

FIG. 8.

Let us now compare our experimental data for γ with those calculated from these equations.

In the range from C' to D in the curve of Fig. 7, the electrons collected, corresponding to γ consist of two parts: those that have made no collisions (γ_0) and those that have made elastic collisions. It is only this second part that corresponds to γ in Eq. (54). From Eq. (53) we see that x is proportional to E. By plotting the logarithms of γ/γ_1 and of x, as in Fig. 8, we thus see that changes in the values of the "unknowns" γ_1 and θ_0 do not alter the shape or size of the curve but result merely in vertical or horizontal displacements of the curve as a whole. On the other hand a change in the value of γ_0 corresponds to the

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addition or substraction of a quantity and this does change the shape of the curve.

These latter changes may be studied by plotting a family of curves (as in Fig. 8) from the data of Table II, corresponding to $(\gamma/\gamma_1)\pm\delta$, where δ is given successively the values 0.02, 0.04, 0.06, etc.

We may now use this plot in a very simple way, to determine from the experimental data for γ (Col. 4 of Table III) the values of γ_0, γ_e and θ_0. We proceed as follows:

By examination of a plot of γ (from Table III) against E, or directly from a plot such as Fig. 7, we make a rough preliminary estimate of the value of

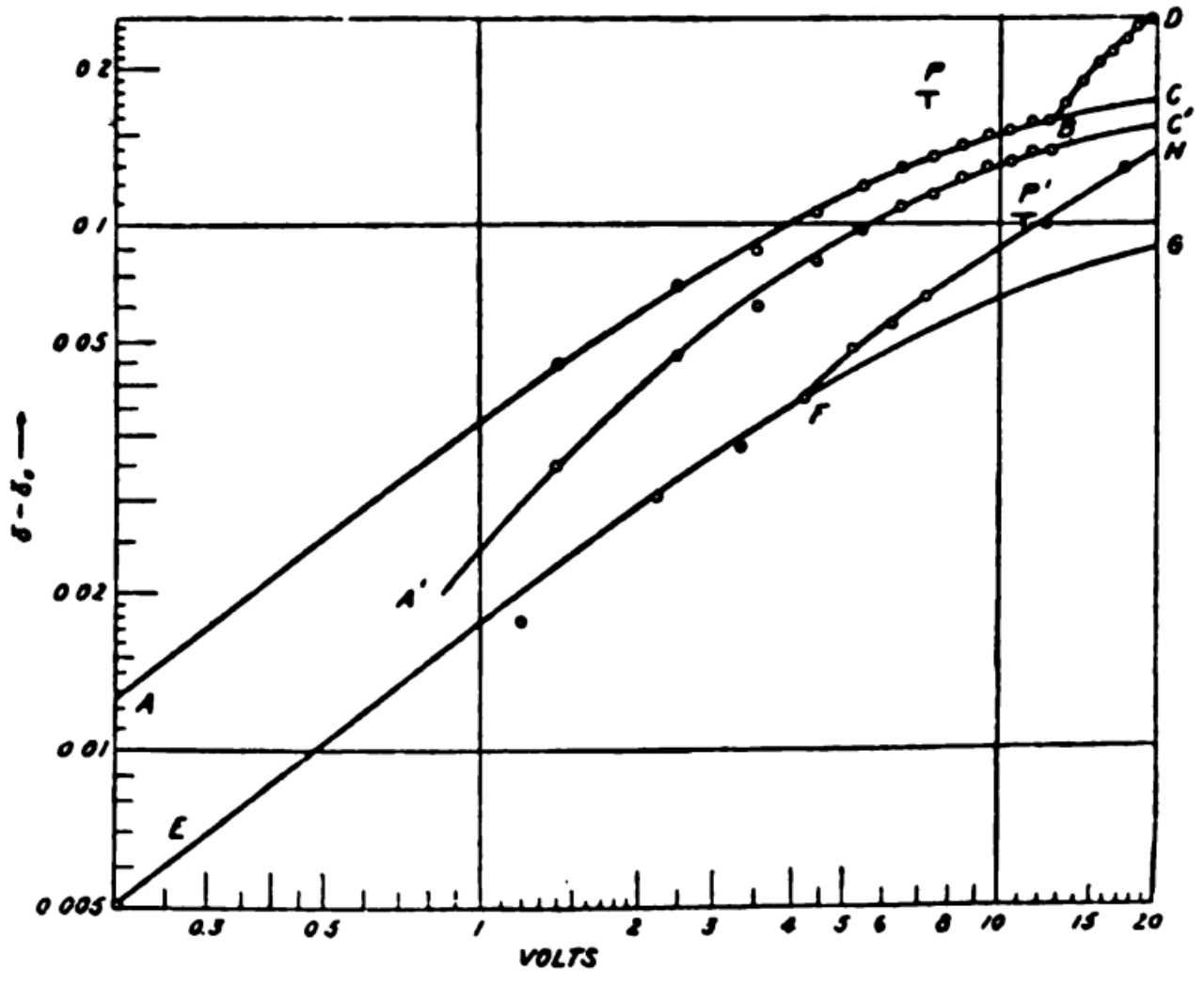

FIG. 9.

γ_0 which must correspond to the steep initial rise $B'C'$ in current near $E = 0$. Suppose we assume $\gamma_0' = 0.33$ for this value. We then plot on double-logarithmic paper (using the same scale as in Fig. 8) the values of $\gamma - \gamma_0'$ as ordinates and the voltages E as abscissas, so that we obtain the points which lie near the curve $A'C'$ in Fig. 9. By superposing[18] Fig. 9 on Fig. 8, keeping the edges of the curve sheets parallel, one figure can be displaced with respect to the other until these points are made to fit as well as possible with one of the family of curves in Fig. 8. We find that the curve $\delta = -0.10$ gives the best agreement. Tracing this curve through on to Fig. 9 we obtain the curve marked $A'C'$.

The curve ST in Fig. 8, for which $\delta = 0$, when transferred through to Fig. 9 gives ordinates in Fig. 9 about 0.02 greater than those of the observed

[18] This is conveniently done by placing one curve sheet on the other, holding both against a glass window by daylight, so that the curves can be seen by transmitted light.

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points. Therefore we see that if we had chosen 0.31 instead of 0.33 for the value of γ_0, the observed points for $\gamma-\gamma_0$ would lie on the theoretical curve ($\delta = 0$). Using this revised value of γ_0 and plotting $\gamma-0.31$ as given in Col. 5 of Table III, we obtain the points indicated in Fig. 9 by circles along the curve *ABD*. This curve can now be superposed upon *ST* in Fig. 8. The curve *ABC* in Fig. 9 is a theoretical curve obtained in this way by tracing through curve *ST* from Fig. 8. The point *P* in Fig. 8 at $\gamma = \gamma_1$, and $x = 1$ when transferred to Fig. 9, gives the point *P* marked by the intersection of the short horizontal and vertical lines. The coordinates of this point give $\gamma_e = 0.180$ and $E_E = 7.2$ volts where E_E is the value of E corresponding to $x = 1$. From Eq. (53) for $x = 1$,

$$E = V_0\theta_0^2 \tag{59}$$

and thus, since $V_0 = 100$ volts, we find $\theta_0 = 0.268$ radians or 15.4° as a measure of the angle through which the electrons are scattered by the elastic collisions. Let us denote this value by θ_E.

The good agreement between the observed points and the theoretical curve in Fig. 9 proves that the electrons are scattered at least approximately according to probability laws. By no choice of γ_0, γ_e or θ_M can the experimental data be made to agree with a curve like that shown by the dotted line in Fig. 8 which corresponds to uniform scattering of the electrons through a cone of semi-angle θ_M, according to Eq. (56).

The values of $\gamma-\eta_0$, as taken from the theoretical curve *ABC* in Fig. 9, are given in Col. 6 of Table III. Up to $E_C = 12$ volts the agreement in good, the errors (Col. 7) being only about 0.2 per cent for the original values of γ. Beyond 12 volts the differences increase rapidly because of electrons which are collected after having made resonance collisions. Let γ' be the fraction of the primary electrons reaching *C* at any given voltage after they have made one resonance collision. As E_C increases, γ' will approach the limit γ_r which we have already defined. Below the resonance voltage V_r, γ' is zero. In the present case, from inspection of the break at *B* in Fig. 9, we will take $V_r = 13.2$ volts. Let us place $E' = E-13.2 = E_C-12.8$ as in Col. 9. The energy of the electrons which reach *C* after single resonance collisions is evidently measured by E' and therefore the reflection coefficient for these electrons will be less than for the electrons that have made no collisions or have made only elastic collisions. An analysis shows that γ' may thus be calculated by multiplying the difference given in Col. 7 by the ratio of the values of $1/(1-\varrho)$ at E' and at E. For example, at $E_C = 18$ we have $\gamma' = 0.065\times(1.147/1.278) = 0.0583$, where 1.147 is the value of $1/(1-\varrho)$ at $E' = 5.2$ volts and 1.278 is the corresponding value at $E = 18$ volts.

The values of γ' have been plotted in Fig. 9 as the small circles which lie along the curve *EFH*. By parallel displacement, the curve *ST* of Fig. 8 has been brought to fit the points in as reasonable a manner as possible, thus giving in Fig. 9 the theoretical curve *EFG*. The coordinates of the point *P'* give γ_r

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$= 0.103$ and $E_R = 11.2$ volts, where E_R is the value of E' corresponding to $x = 1$. The corresponding value of θ_0 which we shall denote by θ_R may be calculated from the equation

$$E_R = (V_0 - V_r)\theta_R^2 \tag{60}$$

which is derived from Eq. (53) by considering that the velocity of the "resonance" electrons corresponds to $V_0 - V_r$ instead of V_0. Thus we find $\theta_R = 0.360$ radians $= 20.6°$ as the root-mean-square angle through which the electrons are scattered by resonance collisions.

The angles θ_E and θ_R which we have thus determined are the mean deflections produced by the combined effect of all the collisions that the electrons may have made in traveling from F to C. We are more interested, however, in knowing the deflections of electrons which have made only one collision.

Assuming that the scattering is of the probability type according to Eq. (47), we may let θ_e be the mean deflection corresponding to a single elastic collision and let θ_r be the mean deflection for a single resonance collision. In successive collisions which an electron may make with gas molecules the transverse components of momentum delivered by the separate collisions, are distributed in random directions in the plane normal to the original path of the electron. Therefore θ^2 for the resultant deflection is found by addition of the values of θ^2 for the separate collisions.

Thus for the N_e elastic collisions which are made on the average by each of the electrons in the group corresponding to γ_e we have

$$\theta_E^2 = N_e\theta_e^2 \tag{61}$$

where N_e is given by Eq. (38).

The resonance electrons, corresponding to γ_r, have each made only one resonance collision and, on the average, N_r elastic collisions. Therefore

$$\theta_R^2 = \theta_r^2 + N_r\theta_e^2 \tag{62}$$

where N_r is given by Eq. (37).

Our analysis of the data of Table III (Figs. 7 and 9) has given:

$\gamma_0 = 0.310$	$V_r = 13.2$ volts
$\gamma_e = 0.180$	$\gamma_r = 0.103$
$E_E = 7.2$ volts	$E_R = 11.2$ volts
$\theta_E = 15.4°$	$\theta_R = 20.6°$.

From these values by means of Eqs. (27), (33), (25), (35), (37), (38), (61) and (62) we thus derive, placing $R = 2.60$ cm,

$P = 0.450$ cm^{-1}	$N_r = 0.458$
$P_e = 0.176$ cm^{-1}	$N_0 = 1.25$
$P_k = 0.274$ cm^{-1}	$\theta_e = 13.8°$
$P_r = 0.081$ cm^{-1}	$\theta_r = 18.4°$.

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These results have been obtained from an experiment with nitrogen at 650°K and a pressure of 50 baryes. The angles θ_e and θ_r are presumably determined by the nature of single collisions so they should not depend on the pressure of the gas. But the probabilities, P, etc., should be proportional to the number of molecules of gas per unit volume, so that we may recalculate them for the standard temperature and pressure of 20°C and 1000 baryes by multiplying the foregoing values by $(1000/50)\times(650/293) = 44.4$, obtaining

$$P = 20.0 \text{ cm}^{-1} \qquad P_k = 12.2 \text{ cm}^{-1}$$
$$P_e = 7.8 \text{ cm}^{-1} \qquad P_r = 3.6 \text{ cm}^{-1}.$$

The ionization probability P_1 has been determined for a number of gases by Compton and van Voorhis.[19] For nitrogen with 100 volt electrons they find for 20°C and 1000 baryes $P_i = 7.7$ cm^{-1}. The inelastic collisions must include those that give ionization and those that produce resonance at 13.2 volts. Thus the remainder P_x given by

$$P_x = P_k - P_r - P_i. \tag{63}$$

measures the probability of all other types of inelastic collisions such as those that excite the molecules to higher energy levels than 13.2 volts. For the case under consideration we find

$$P_x = 0.9 \text{ cm}^{-1}.$$

The curves *ABC* and *EFG* in Fig. 9 are the theoretical curves corresponding to the foregoing values of P, γ_0, γ_e, θ_e etc. By taking points from these curves we can calculate values of Δ and compare them with the original observations. Such reconstructed curves are shown in Fig. 7, the whole curve *ABCDH* being a theoretical curve calculated in this way by our equations. Thus *AB* is the theoretical line corresponding to a positive ion current of −1.12. The curve *ABCF* is the theoretical curve corresponding to the positive ion current together with the current of primary electrons ($\gamma_0 = 0.31$) which have made *no* collisions. The negative slope of the line *CF* shows the effect of the reflection of the electrons from the collector C.

The curve *ABCDG* includes the current due to the electrons ($\gamma_e = 0.18$) which have made elastic collisions and have thus been scattered through angles which average 12.2° (i.e., $\frac{1}{2}\pi^{1/2}\times 13.8°$). Finally the curve *ABCDH* includes the electrons that are collected after resonance collisions.

The theoretical curves can thus be made to fit the experimental data within the probable error of the measurements except in the regions where E is close to zero and where E is above the ionizing potential (in this case 17 volts).

When an electron ionizes a gas molecule it must lose an energy corresponding to the ionizing potential, but then in general it must divide its remaining energy with the new electron produced by the ionization. Thus the

[19] K. T. Compton and C. C. Van Voorhis, *Phys. Rev.* **27**, 724 (1926).

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probability is small that an electron that ionizes a molecule shall lose merely the energy which corresponds to the ionizing potential. Therefore at collector voltages above the ionizing potential the electrons that are collected can no longer be resolved into groups having homogeneous velocities. We see in fact from Fig. 7 that the observed points (along DE) begin to depart from the theoretical line DH at the ionizing potential but this departure begins gradually and not abruptly as in the case of the resonance potential at D.

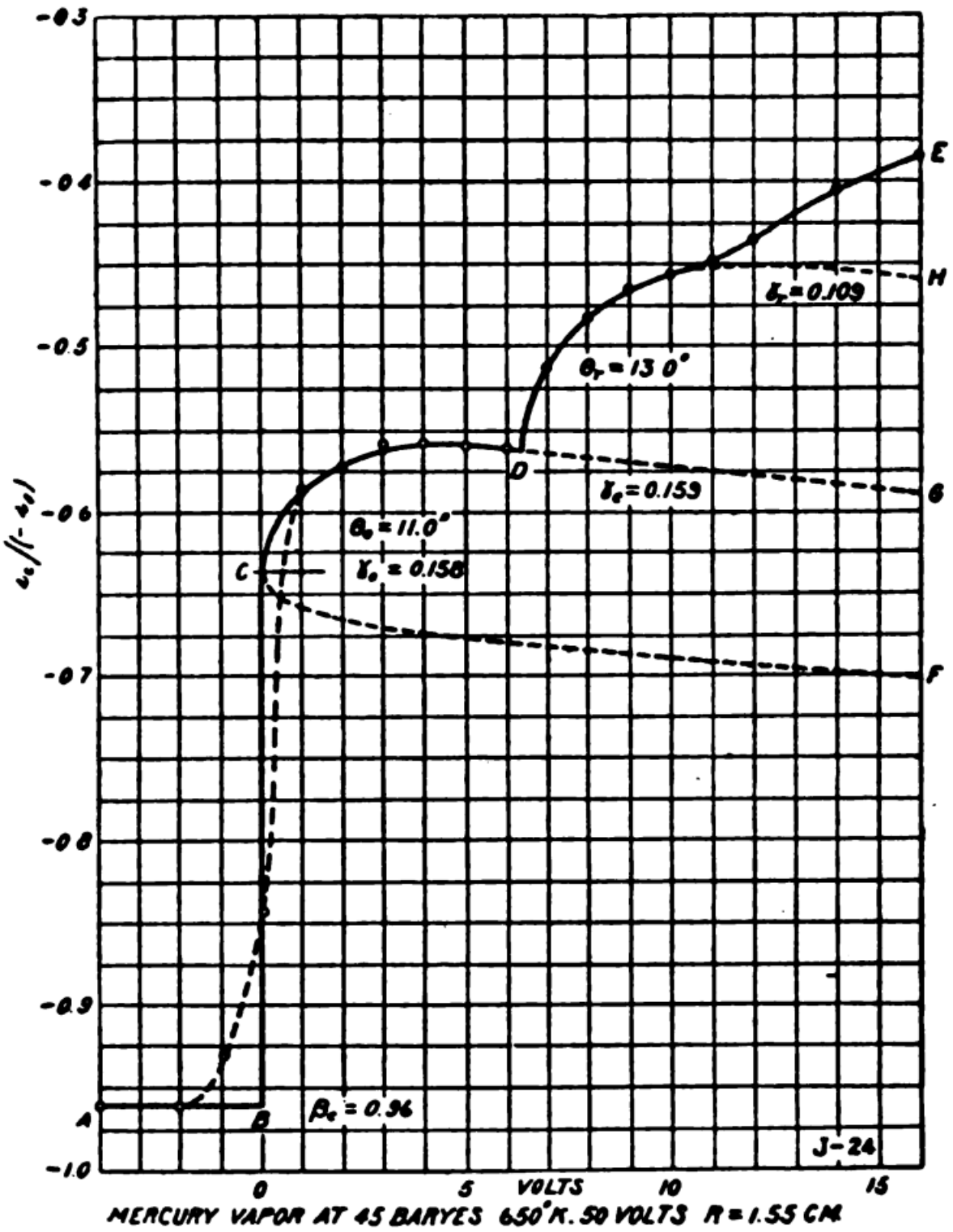

FIG. 10.

The experimental data indicated by circles in Fig. 10, were obtained with mercury vapor at 44.5 baryes and 650°K in a tube having a cylinder C of 1.55 cm radius, the anode potential being 50 volts. By an analysis involving a double logarithmic plot it was found that

$$\gamma_0 = 0.158 \qquad V_r = 6.3 \text{ volts}$$
$$\gamma_e = 0.159 \qquad \gamma_r = 0.109$$
$$E_E = 2.2 \text{ volts} \qquad E_R = 2.8 \text{ volts}$$
$$\theta_E = 12.0° \qquad \theta_P = 14.6°$$

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and from these were calculated

$$P = 1.90 \text{ cm}^{-1} \qquad N_r = 0.364$$
$$P_e = 0.443 \text{ cm}^{-1} \qquad N_e = 1.20$$
$$P_k = 0.743 \text{ cm}^{-1} \qquad \theta_e = 11.0°$$
$$P_r = 0.222 \text{ cm}^{-1} \qquad \theta_r = 13.0°.$$

Under standard conditions (20°C and 1000 baryes) these probabilities become

$$P = 59.0 \text{ cm}^{-1} \qquad P_r = 11.0 \text{ cm}^{-1}$$
$$P_e = 22.1 \text{ cm}^{-1} \qquad P_i = 14.9 \text{ cm}^{-1}$$
$$P_k = 36.9 \text{ cm}^{-1} \qquad P_x = 11.0 \text{ cm}^{-1}.$$

The curve *ABCDH* in Fig. 10 is the theoretical curve corresponding to the foregoing values of the probabilities and the scattering angles. Although the resonance voltage (6.3) and the ionizing voltage (10.0), corresponding to the resonance potential (6.7) and the ionization potential (10.4) in mercury due to a shift of 0.4 volts at zero, are very different from those of nitrogen, the general agreement with the theory and the character of the results are much the same.

With mercury vapor also the experimental points begin to depart from the theoretical curve *DH* as soon as the voltage rises above the ionizing potential.

More than two hundred sets of volt-ampere characteristic curves for various gases, pressures and voltages have been analyzed and in all cases it has been found that by suitable choice of P, P_e, P_r, θ_e and θ_r the curve up to the ionizing voltage can be satisfactorily represented by the equations we have developed. The theory is particularly confirmed by the fact that the experimental values of all these constants are practically independent of the gas pressure. A summary of these results is given in Table VIII.

Discussion of the Accuracy of the Results

In fitting the theoretical curves in Fig. 7 to the experimental data we have 6 adjustable constants at our disposal, and it may therefore seem that the accuracy in the determination of all the constants cannot be high. By trial we find that the permissible limits in the choice of the values of γ_0 are rather narrow, so we estimate the probable error of γ_0 to be less than 0.01.

We find, however, that the curves such as *AC* in Fig. 9 can be displaced a considerable distance in the general direction of their length without greatly altering the agreement between the theoretical curve and the experimental points. Especially is this true with the resonance curve *EFG* which is more nearly straight and for which agreement with experiment is expected only for a rather limited range of voltage.

Trial shows for the data of Fig. 7 that the agreement becomes poor if E_E is altered by more than about 10 per cent which corresponds to a varia-

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tion of 5 per cent in θ_E or θ_e. A 10 per cent change in E_E involves about a 4 per cent change in γ_e, but for any given value of E_E the probable error in γ_e would be not over 1 or 2 per cent.

In the case of the resonance electrons the possible individual errors in γ_r or E_R become relatively large. In Table IV are given a series of pairs of values of γ_r and E_R corresponding to displacements of the curve *EFG* approximately parallel to the nearly straight line *EF*.

TABLE IV

Effect of Displacement of Curve EFG in Fig. 7

Displacement	γ_r	E_R	P_r	P_x	θ_R	θ_r	E_F
D_1	0.182	25.0	6.3	−1.1	30.7°	29.4°	23.0
D_2	0.152	19.8	5.3	0.0	26.8	25.7	18.0
D_3	0.103	11.2	3.6	+1.7	20.6	18.4	17.0
D_4	0.070	7.0	2.4	+3.8	16.3	13.3	14.7
D_5	0.033	2.3	1.2	+4.2	9.3	0.0	13.9

From the values of γ_r and E_R we can calculate P_r, P_x, etc., as given in the table. The displacement D_1 makes P_x negative and therefore must be excluded. The displacement D_2 has been chosen to make $P_x = 0$. Thus the maximum possible values of P_r and θ_r are 5.3 and 25.7° respectively. Displacement D_5 corresponds to the lowest possible value of P_r and the greatest possible value of P_x since any further displacement would make θ_r become imaginary.

For each different displacement, the point corresponding to *F* in Fig. 9 at which the observed points begin to depart from the theoretical curve, occurs at a different voltage E'. Adding 13.2 volts to these values we obtain the voltages E_F given in the last column of Table IV, and this voltage should presumably be approximately equal to the ionizing potential of the gas (17.0 volts for nitrogen). Thus we are led to choose D_3 as the most probable displacement.

In the analysis of the experimental data on the resonance electrons the displacement of the curve, which determines the value of E_r has usually been chosen so as to make E_F come at the ionizing potential. In other cases, especially when the voltage drop along the filament was not eliminated by using the commutator, the point E_F could not be determined with sufficient accuracy. In order to get approximate values of γ_r and P_r in these cases it was assumed that θ_r could be put equal to θ_e. From this, θ_R, and E_R were then calculated and a displacement was then chosen to give this value of E_R.

The values of P_r and θ_r found by such methods are of course only rough, but it is felt that our main conclusions regarding the rather small angles through which the electrons are scattered by the resonance collisions are not vitally affected thereby.

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Estimation of temperature and pressure.—To calculate the *P's* at standard pressure and temperature from the experimental data we must know the pressure and temperature of the gas in the cylinder *C*. The pressure outside the cylinder was measured by a McLeod gauge but because of the effect of thermal effusion the pressure inside the cylinder was usually considerably higher than that outside.

The radiation from the filament (ranging from 11 to 20 watts in the various experiments) heated the molybdenum cylinder to rather high temperatures. If the cylinder and end-plates had completely surrounded the filament so that the whole energy would be re-radiated from the outside surface, the temperature of the electrodes in the absence of gas, would have been 720° to 790°K. But from the small annular spaces about 2 mm wide between the end-plates and cylinder the radiation was nearly that from a black body. Thus the radiative equilibria gave temperatures from 670 to 720°K. The effect of the heat conduction of various gases was calculated, taking into account the Smoluchowski temperature drop at the surfaces.[20] It was found that the cylinder temperature gradually decreased as the pressure rose. For example, argon at 100 baryes would lower the cylinder temperature about 70°. But the heat conduction from the filament tends to raise the gas temperature above that of the cylinder, by amounts that increase as the pressure rises. This effect nearly compensates for the cooling of the cylinder. Thus, the average temperature of the gas along the radius was found to be approximately independent of the pressure and to lie between 650° and 700°K in various typical experiments. At pressures so low that the free-paths of the molecules are large compared to the openings into the cylinder thermal effusion makes the pressure p_c in the cylinder greater than that outside in the ratio of the square roots of the temperatures. The following semi-empirical equation was used in all the experiments to calculate the pressure in the cylinder

$$\frac{p_c}{p_0} = \frac{(T_c/T_0)^{1/2} + bp_0/\lambda_1}{1 + bp_0/\lambda_1} \tag{64}$$

where p_0 and T_0 are the pressure and temperature outside the cylinder, while p_C and T_C are those inside; b is the width of the space between the cylinder and end-plates, while λ_1 is the mean free-path of the molecules of gas at 1 barye pressure at the temperature of the cylinder.[21]

It will be appreciated that the accuracy in this estimation of the temperature of the gas and the pressure increases by effusion cannot be high. Thus in reducing the observed values P_1, P_e, P_k and P_r to standard conditions errors of perhaps 10 per cent have been made. However, it should be noted

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[20] See I. Langmuir, *J. Am. Chem. Soc.* **37**, 421 (1915).

[21] The free-paths λ_1 at 650° were taken to be for Hg 13.1 cm; Ne 51; A 28; He 73; N_2 25 and H_2 47 cm. The slit width b was 2 mm in some cases and 2.5 in others.

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that the effusion decreases the effect of an error in the estimation of the temperature since the pressure increases as the temperature rises and thus causes a slower variation in the density.

To avoid errors due to uncertainty in the temperature of the cylinder a large number of experiments were made with argon, and helium in tubes in which the "cylinder C" consisted in a vaporized magnesium deposit on the inside of the glass wall of the tube. By a blast of air the tube wall was kept at about 30°C as was measured by the resistance of a fine tungsten wire in close contact with the glass. The average temperature of the gas along the radius was usually about 50° above that of the tube wall, because of heat conducted from the filament.

The values of the probabilities P and P_K found with these tubes were practically the same as those found with the molybdenum cylinders, as will be seen in the summary of results (Table VIII). Thus we are justified in believing that the calculated temperatures of 650° to 700° K in the latter case are not seriously in error.

Small angle deflections.—At voltages in the neighborhood of $E_c = 0$ the observed currents, as indicated by the dotted line $B'C'$ (Fig. 7) do not rise as abruptly as they should according to the theoretical curve BC, but increase gradually over a range of about 2 or 3 volts, showing that the electrons which have not made collisions with molecules are not moving in strictly radial directions when they strike the cylinder. Possible causes of these small angular deflections from the radial direction are: (1) Initial velocities of electrons due to thermal agitation; (2) Non-axial position of the filament; (3) Transverse magnetic field (earth's field); (4) Scattering by the electric field of ions; (5) Scattering by the fields of molecules; (6) Scattering by energy transferred from other primary electrons. Let us consider briefly the magnitude of these effects.

1. *Initial velocities.*—The average kinetic energy of the electrons which leave the filament is $2kT_F$, but the the longitudinal component $\frac{1}{2}kT_F$ is not effective in causing them to move against a retarding field at the cylinder. The effective energy $(3/2)\,kT_F$, for a filament temperature of 2300°K corresponds to 0.30 volts, and can thus explain only about one tenth of the observed spread in voltage over which the primary electrons are collected.

2. *Displacement of filament.*—If the filament is displaced a distance δ along a radius, the electrons which move along another radius at right angles to the first reach the cylinder with a transverse energy V_T given by

$$V_T = V(\delta/R)^2 \qquad (65)$$

where V is the original radial energy. Since in the experiments the filaments were displaced from center by not more than 1/20th of the radius, V_T for 100 volt primary electrons, cannot have been greater than 0.25 volts.

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3. *Effect of earth's field.*—The radius of curvature r of a V-volt electron in a transverse magnetic field of H gauss is

$$r = 3.36V^{1/2}/H \text{ cm}. \tag{66}$$

If the electron travels a short distance x (small compared to r) perpendicular to the field, the transverse energy V_T (in volts) given to the electron by the magnetic field is

$$V_T = 0.089\, H^2x^2 \tag{67}$$

this energy being independent of the velocity of the electrons. Thus in a tube of 2.6 cm radius (as for the data of Table III and Fig. 7) a transverse field of 0.5 gauss will give a transverse energy of 0.15 volts. Since the electrons which move parallel to the field are not deflected, the primary electrons are thus spread out over a small range. A longitudinal field, however, would give the same transverse energy to all the electrons so would not produce a spread but only a slight displacement of the current-voltage curve.

In our experiments a transverse field of 0.5 gauss may well have been present but this field can have caused only part of the spread in voltage indicated by the lines $B'C'$ in Figs. 7 and 10.

4. *Scattering by ions.*—H. A. Wilson[22] in considering the scattering of β-rays by matter has derived equations which are useful in calculating the scattering of electrons by the ions in a gas.

Consider a narrow beam of electrons of energy corresponding to the potential V, passing through a layer of ionized gas of thickness t which contains n univalent charges per unit volume (ions plus electrons). The mean square angular deflection produced by all charges which lie within a distance r of the electron beam is

$$\theta_0^2 = 8\pi nt\,(r^2/a^2)(1-\pi^2/8+\log a) \tag{68}$$

where a is a parameter having the value

$$a = 2rV/e. \tag{69}$$

In the derivation of Eq. (68) it was assumed that a is large compared to unity and the scattering is thus nearly all due to numerous small deflections, the close encounters which give large deflections being so rare that they contribute little to θ_0. Expressing V in volts, r and t in cm and n in ions per cm^3 we find

$$a = 1.39\times 10^7 rV$$

and from Eq. (68)

$$\theta_0 = 1.47\times 10^{-6}((nt)^{1/2}/V)(1+0.142\log_{10} rV)^{1/2}. \tag{70}$$

We see that θ_0 becomes infinite if r is infinite so that according to this equation the forces due to the ions at large distance cannot be neglected.

[22] H. A. Wilson, *Proc. Roy. Soc.* **A102,** 9 (1923).

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It was assumed in this derivation that the ions were distributed throughout the gas at random. We know, however, from the work of Debye and Hückel[23] that the positive ions tend to accumulate around the negative and vice versa so that the field of a given ion falls to $1/\epsilon$ of the value it should have by Coulomb's law, in a distance λ, which for the case of univalent ions having a temperature T is

$$\lambda = 6.92(T/n)^{1/2} \text{ cm}. \tag{71}$$

Thus the value of r in Eq. (70) should be placed equal to λ, since the field due to ions at greater distances cannot be important.

For the example we have considered in Table III and Fig. 7 the positive ion current density I_p corresponding to β_C is 5.3×10^{-5} amps per cm^3. From this by Eq. (1) placing $T_p = 3000°$ we find $n_p = 0.88\times10^{10}$ ions per cm^3. Since there must be an equal number of negative charges we may take $n = 1.76\times10^{10}$. Assuming the temperature T in Eq. (71) (average for electrons and ions) to be 5000°, we find $\lambda = 3.7\times10^{-3}$ cm. Taking this for the value of r in Eq. (70) and letting $t = 2.6$ cm, the radius of the cylinder, we find $\theta_0 = 0.003$ radius or 0.17° as the mean angular deflection of the electrons due to the electric field of the ions.

The mean transverse energy corresponding to this deflection is $V(\theta_0)^2$ or only 0.0009 volts which is entirely negligible as a cause of the spread of the voltage at which the primary electrons are collected.

5. *Scattering by the field of molecules.*—In an electric field of intensity X a molecule becomes a dipole of moment αx where α can be calculated from the dielectric constant D of the gas by the equation

$$\alpha = (D-1)/4\pi n \tag{72}$$

where n is the number of gas molecules per unit volume.

Because of this polarization, an electron and a gas molecule at a distance r attract one another with the force

$$F = 2\alpha e^2/r^5. \tag{73}$$

An electron which passes a gas molecule with a velocity corresponding to the potential V is thus deflected through an angle θ given by[24]

$$\theta = (3\pi/8)\alpha e/Vr_0^4 \tag{74}$$

where r_0 is the distance of nearest approach. This equation is derived on the assumption that θ is a small angle. The transverse energy $V\theta_0^2$ acquired by the electron is thus

$$V_T = (3\pi/8)^2\alpha^2e^2/Vr_0^8 \tag{75}$$

expressing V in volts and α and r_0 in terms of cm we find

$$V_T = 2.84\times10^{-14}\alpha^2/Vr_0^8. \tag{76}$$

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[23] P. Debye and E. Hückel, *Physik Zeits.* **24**, 185, 305 (1923).

[24] F. Zwicky, *Physik Zeits.* **24**, 171–183 (1923).

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The dielectric constant of nitrogen at atmospheric pressure and 0°C is 1.000581, so by Eq. (72) we find for this gas $\alpha = 1.71 \times 10^{-24}$. We have found the probability of collision of electrons with nitrogen molecules at 1000 baryes and 20°C to be 20.0 cm^{-1}. The number of molecules per cm^3 is then 2.48×10^{16} so the target area per molecule is 8.05×10^{-16} cm^2 corresponding to a target radius of 1.60×10^{-8} cm. All 100 volt electrons which pass within this distance of the center of the molecule presumably make elastic or inelastic collisions and are thus acted on by forces which are not of the type we are now considering. We may assume, however, that beyond this distance the force F given by Eq. (73) is effective. Thus placing $r_0 = 1.60 \times 10^{-8}$ cm in Eq. (76) and inserting the value of α we find that the maximum transverse energy that a 100 volt electron can acquire from this force is 0.192 volt.

The transverse energy falls off with a very high power of the distance r_0 so that the effect of multiple scattering is negligible compared with single scattering.

We thus conclude that the scattering due to the force acting between electrons and molecules resulting from the polarization of the latter, is very small.

6. *Scattering by energy transferred between primary electrons.*—It has been shown[25] that with high primary electron current density and low gas pressures the longitudinal energy of some of the primary electrons actually increases as the electrons traverse the space between the cathode sheath and the collector sheath. These electrons may thus reach the collector even when it is many volts negative with respect to the cathode.

In some of the present experiments with hydrogen and argon this effect was studied by varying the electron emission of the cathode from 1 to 90 milliamperes.

Fig. 11 gives some typical data obtained with a tube having a molybdenum cylinder 5.0 cm in diameter with end-plates 5.1 cm apart. The filament diameter was 0.18 mm. The full line curves were obtained with argon at a pressure of 13.4 baryes as indicated on the McLeod gauge, corresponding to 19.1 baryes in the cylinder at 650°K. The anode voltage was 60. Cathode currents, $-i_F$, of 3, 9, 29 and 90 milliamperes were used. To prevent overlapping the ordinates of 3 of the curves were increased by the amounts Δ indicated on the curves.

With a current of only 3 milliamperes the electrons begin to be collected in appreciable numbers when the collector is about 3 volts below the cathode, but as the current is increased to 9, 29 and 90, the voltages at which electrons are first collected become respectively —7, —19 and —32 volts. A more rational way of measuring the scattering is to determine[25] the slope of the curve obtained by plotting the electron current against the voltage on probability paper. The square of the reciprocal of this slope divided by 2900 V_0 (V_0 being

[25] I. Langmuir, *Phys. Rev.* **26,** 585 (1925).

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the energy of the primary electrons in volts) gives the temperature corresponding to the random motion of the electrons which is assumed to be superposed on the translational motion of the beam. The temperatures calculated in this way are indicated on the curves of Fig. 11. The probability plots in these cases were straight only for negative voltages (below about —2 volts) so

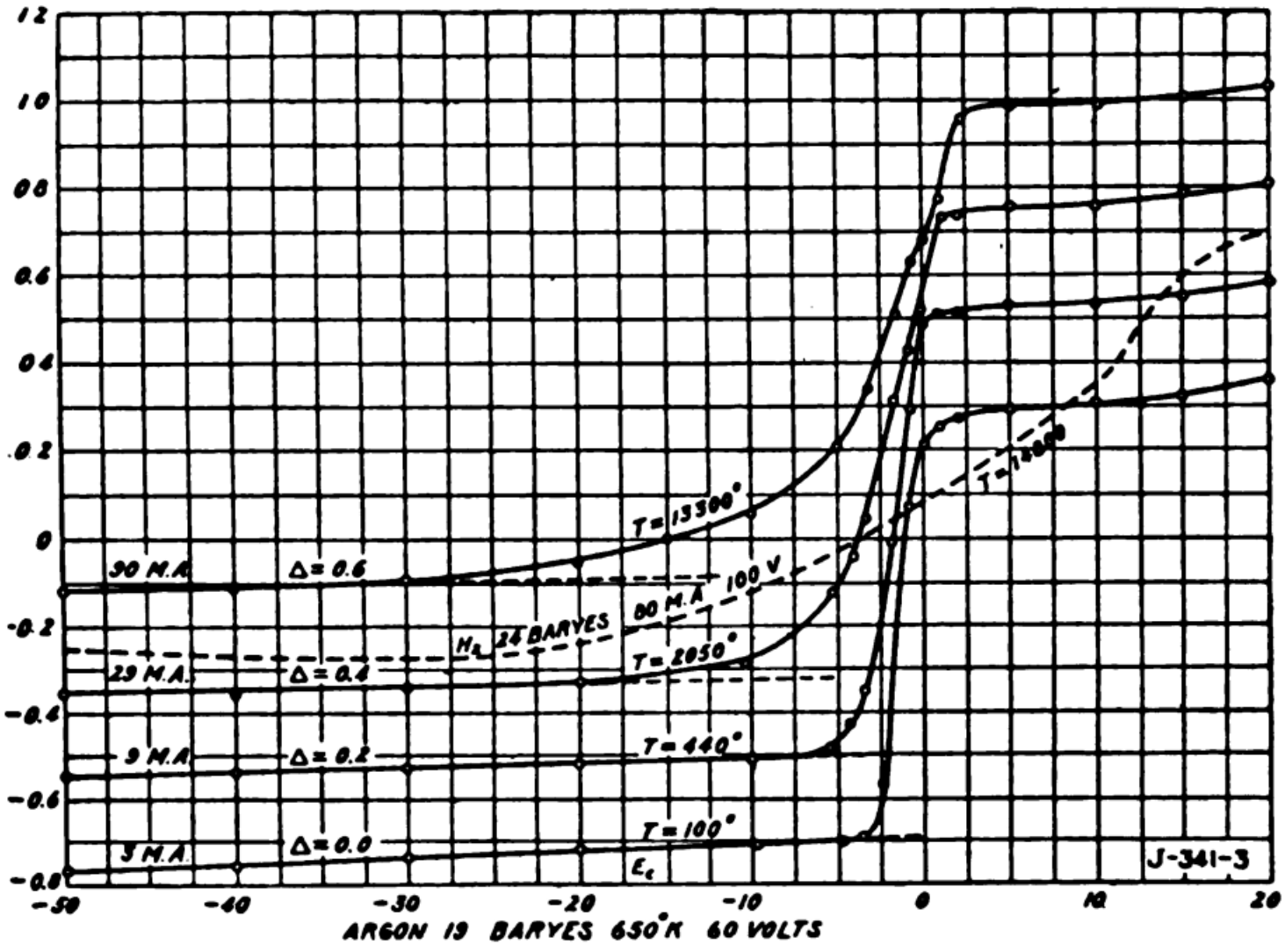

FIG. 11.

the slopes were taken from these straight portions.[26] With hydrogen similar effects were obtained as the current density was increased. The dotted curve in Fig. 11 was obtained with the same tube containing hydrogen at 16.4 baryes (24 baryes in the cylinder at 650°K) with a current of 80 milliamperes and an anode voltage of 100.

This effect was studied at several pressures of argon and hydrogen with various anode voltages. In each case the electron temperatures were calculated from probability plots. A summary of the results is given in Table V. The data given for argon at 19.1 and for hydrogen at 24 baryes are those illustrated in Fig. 11.

The data for argon show that the energy scattering decreases markedly as the pressure is raised, as was previously found for mercury vapor at pressures above 2 baryes.[26]

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[26] Experiments have indicated that the deviations from the straight lines at the positive voltages are largely dependent on the fact that a collector is used which nearly completely surrounds the ionized gas. A small disk-shaped collector in a discharge in a large spherical bulb gives a much more symmetrical curve which gives a nearly straight probability plot.

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TABLE V

Energy Scattering at High Currents

Gas	Pressure (baryes at 650°K)	Anode potential	Current $-i_P$	Electron temperatures
Argon	3.9	50 volts	3 ma.	500°K
			9	2000
			30	16,000
			90	44,000
	19.1	60	3	100
			9	440
			28	2050
			85	13,300
	18.6	100	9	300
			30	2100
			90	12,000
	29.0	100	90	6700
Hydrogen	24.0	100	90	14,800
	53.0	100	9	1200
	53.0	100	90	70,000

On the other hand with hydrogen at a pressure of 53 baryes, a much greater scattering was found than at 24 baryes. With argon, as with mercury vapor, the scattering decreases slightly as the anode voltage is raised.

With 5 ma. emission from the cathode, which was the current usually employed in our studies of electron collisions, the energy scattering effect was always practically negligible.

Effects of current through cathode.—In most of the experimental determinations of the current-voltage characteristics of the collector c which have been described, a commutator was used so that there was no current through the filament when the collector current was measured. To avoid the inconvenience of the commutator and the slight irregular variations in current that were sometimes caused by brush trouble, many characteristic curves were made without using the commutator. Two effects are thus introduced, the voltage drop along the filament (about 3–5 volts) caused the breaks in the curves, such as those at B and D in Fig. 7, to be less abrupt, and the magnetic field of the filament current deflects the primary electrons which leave the cathode with energy V_r in a radial direction, so that they acquire axial energy component V_a in the same direction as the motion of the electrons which constitute the heating current in the filament. From the general equations given by Hull[27] for the motions of electrons near a conductor, we readily find, for the case that V_a is small compared to V_r that

$$V_a = 0.0188 A_F^2 [\log_{10} r/r_F)]^2 \qquad (77)$$

[27] A. W. Hull, *Phys. Rev.* **25,** 652 (1925).

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where V_a is the axial energy in volts at a distance r from the axis of the filament, r_F is the radius of the filament and A_F is the filament heating current in amperes. The axial energy is thus independent of the anode voltage which determines V_r. The effect of the magnetic field is thus to lower the radial component of the energy by the amount V_a so that the break in the curve (B in Fig. 7) is displaced to the right by the voltage V_a. The experiments show this effect distinctly and in complete agreement with the theory.

The effect of the voltage drop along the filament can be calculated if we know the shape of the volt-ampere characteristic in the absence of this voltage drop. Let i be the current to the cylindrical collector when this is at a voltage E above the cathode and there is no voltage drop along the filament. Let i_M be the measured current when the collector is at a voltage E above the mid-point of the cathode, the voltage drop along the cathode being V_F. Then, if the collector is at a potential above that of the positive end of the filament, we find

$$i_M = (1/V_F) \int_{E-1/2V_F}^{E+1/2V_F} i\, dE$$

Expansion by Taylor's theorem and integration gives

$$i_M = i+(1/24)V_F^2\, d^2i/dE^2+(1/1920)V_F^4\, d^4i/dE^4+\cdots. \qquad (78)$$

If the theoretical relation between i and E is not known, but experiment has given i_M as a function of E, we may use the inverted series,

$$i = i_M-(1/24)V_F^2\, d^2i_M/dE^2+(7/5760)V_F^4\, d^4i_M/dE^4+\cdots$$

and thus obtain the relation between the current and voltage that would exist if there were no voltage drop along the filament.

For the electrons that make elastic collisions, the relation between i and E is given by Eqs. (54) and (53). Eq. (78) can be written

$$\gamma_M = \gamma+(1/24)V_F^2\, d^2\gamma/dE^2$$

and by inserting the value of $d^2\gamma/dE^2$ from Eqs. (54) and (53) we find

$$(\gamma = \gamma_M+(1/48)V_F^2\gamma_e/(V_0^2\theta_0^4)\epsilon^{-x}(1+1/x) \qquad (79)$$

where x is defined by Eq. (53). Since approximate values of θ_0 and γ_e can be obtained from the experimental data by neglecting V_F (or by experiments with a commutator) Eq. (79) can be used to correct the values of γ obtained without the use of the commutator. In many cases these corrections have been applied, but usually the corrections are so small as to be unimportant.

Analysis of Current Voltage Curves at Higher Voltages

We have seen from the data of Fig. 3 that in mercury vapor the end-plates do not remove any appreciable number of electrons having normal components of energy as large as 80 volts when the primaries have 100 volts

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energy. We have concluded from this that very few, if any, of the primary electrons are turned through angles as great as 90° by elastic or resonance collisions.

Let us now consider the electrons which begin to be collected when the end-plate potential is raised to +20 volts or more. We may make a rough analysis of these groups of electrons by assuming that they consist largely of electrons having a Maxwellian distribution of velocities.

Mutual effect of electron collectors.—Let a, b, c and f be the currents from electrons A, B, C, and F into space, i.e.,

$$a+b+c+f=0. \tag{80}$$

Let a_e and a_p be the electron and positive ion parts of the current form a, so that $a=a_e+a_p$, $b=b_e+b_p$, etc. Let A_a and B_a be the areas of A and B. Then we may write

$$a = A_a I_e \epsilon^{-e(E_s-E_a)/kT}+a_p \tag{81}$$

$$b = B_a I_e \epsilon^{-e(E_s-E_b)/kT}+b_p. \tag{82}$$

Here E_s is the potential of the space which varies with E_a and E_b and is unknown. Eliminating E_s and I_e between Eqs. (81) and (82) we obtain

$$log(b_e/a_e)+log(A_a/B_a) = e(E_B-E_A)/kT \tag{83}$$

We thus plot the electron current to an electrode divided by the electron current to the anode on semi-logarithmic paper against the differences between the collector and the anode voltages. The slope of the straight line through the points gives the temperature or average energy of the electrons, and the ordinate at the space potential divided by the collector area gives the current per unit area of these electrons in space.

We have analyzed the curve marked B in Fig. 3 for the groups of electrons having a Maxwellian distribution of velocities. The curve B represents the current to the end-plates A and B when they are in parallel and the potential is varied. The cylinder C was anode in this case.

The electron current to the end-plates divided by the electron current to the anode was plotted on semi-logarithmic paper against the differences between the collector and anode voltages. The points lie on a straight line between 30 and 88 volts. The slope of this straight line gives a temperature of 358,000 degrees for this group of electrons which we designate the first Maxwellian group.

The experimental points begin to deviate upward from the straight line above 88 volts. By plotting the current differences between these points and the extrapolated straight line, against the differences between the collector and anode voltages on a semi-logarithmic scale we find that the points lie on a straight line between 90 and 96 volts. This straight line has a slope corresponding to a temperature of 30,500 degrees for a second Maxwellian group. By again plotting the differences in current between the experimental points

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and the extrapolated straight line of the second Maxwellian group on a semi-logarithmic scale, we obtain points which lie on a straight line between 96 and 100 volts. The slope of this line corresponds to a temperature of 7150 degrees for this low velocity group of electrons. We designate this group of electrons as the "Ultimate Electrons" since they are always present in any type of electron discharge in gases, carry practically the entire current and neutralize the space charge created by the relatively low velocity positive ions produced as a result of ionization of the gas molecules.

The curve marked B in Fig. 2 has also been analyzed into three Maxwellian groups of electrons. The first Maxwellian group is measurable when the potential of the cylinder C is raised above $+20$ volts. The temperature of this group is 350,000 degrees. The second appears above 88 volts on the collector and has a temperature of 31,000 degrees. The third Maxwellian group or the ultimate electrons are collected at potentials above 96 volts on the collector and have a temperature of 6900 degrees.

The area of the end-plates A and B is 7.0 cm² each and the area of the cylinder C is 37.0 cm² in the experiments shown in curve B, Figs. 2 and 3.

The positive ion current collected on $A+B = 4.46\times10^{-4}$ amps or 3.18×10^{-5} amps cm^{-2} at $+100$ volts potential. By extrapolation we find that in curve B, Fig. 3, 0.4×10^{-5} and 0.44×10^{-5} amps cm^{-2} of Maxwellian groups 1 and 2 are collected by $A+B$ at $+100$ volts. Similarly the total electron current collected by $A+B$ at 100 volts is 8.18×10^{-5} amps cm^{-2}. By difference there is 7.34×10^{-5} amps cm^{-2} of ultimate electrons collected at 100 volts. Hence there are 2.31 electrons for every positive ion collected at 100 volts. At the potential of the space according to Eq. (1), there should be $(T_e m_p/(T_p m))^{1/2}$ or 855 electrons collected to 1 positive ion if $T_p = (1/2)T_e$.

From the Boltzmann Eq. (3), $\log_e n_1/n_0 = Ve/kT$ we have $\log_e(855/2.31) = Ve/kT$, and thus since $T = 7150°$, $V = 3.65$ volts.

Therefore there is a positive ion sheath with a potential of 3.65 volts when the plates $A+B$ are at the anode potential, i.e., 100 volts positive with respect to the cathode. Thus there is a negative anode drop of 3.65 volts. Each collector in the discharge is then receiving all positive ions headed toward it and repelling all electrons which are not able to penetrate through the 3.65 volt sheath.

From Eq. (1) we have $n_p = (4.03\times10^{13}\times3.18\times10^{-5}\times606)/(3570)^{1/2} = = 1.30\times10^{10}$ positive ions per cc per ma. of emission current.

Similarly the density of electrons in the various groups in space is $n_{e(1)} = 2.9\times10^{5}$/cc ma.; $n_{e(2)} = 4.1\times10^{6}$/cc ma.; $n_{e(ult)} = 1.30\times10^{10}$/cc ma. Thus we observe that the ultimate electrons neutralize the positive ion space charge and practically the entire current in the discharge is carried by them.

The positive ion current to the collector C is equal to 5.13×10^{-5} amps cm^{-2} at 100 volts on C in curve B, Fig. 2. The total electron current to C at 100 volts is 2.90×10^{-3} amps. Of this current 5.4×10^{-5} and 4.5×10^{-5} amps belong to Maxwellian groups 1 and 2 respectively, leaving 2.8×10^{-3} amps

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of ultimate electrons, which is equivalent to 7.57×10^{-5} amps cm^{-2}. Hence there are 1.48 electrons to 1 positive ion collected at 100 volts.

Therefore assuming $T_p = (1/2)T_e$, we have $\epsilon^{Ve/kT} = 574$, or $V = 3.78$ as the retarding potential for electrons in the sheath when the collector is 100 volts positive with respect to the cathode.

From Eq. (1) we have $n_p = 2.10\times10^{10}$/cc ma. emission. Similarly the density of electrons in the various groups in space is $n_{e(1)} = 1.8\times10^{5}$/cc ma. emission; $n_{e(2)} = 9.1\times10^{6}$/cc ma. emission; $n_{e(ult)} = 1.75\times10^{10}$/cc ma. emission.

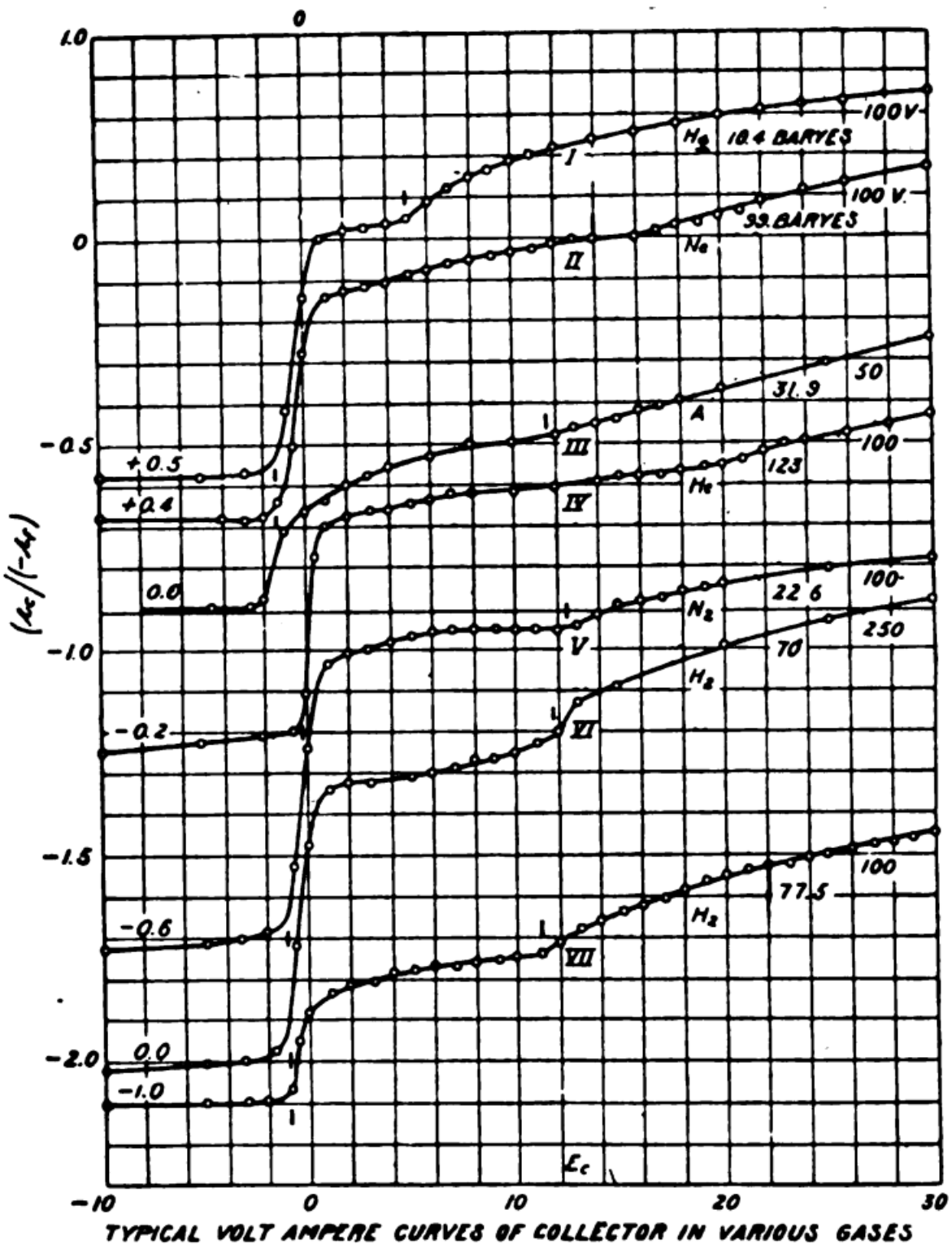

FIG. 12.

We have repeatedly checked this method of calculating the negative anode drop in our experiments by means of the probe wire method of Langmuir and Mott-Smith.[5]

For example in a typical experiment in mercury vapor at 17.3 baryes, the end-plate A and the cylinder C were placed at 100 volts positive with respect to the cathode. A plot of the logarithm of the electron current against the vol-

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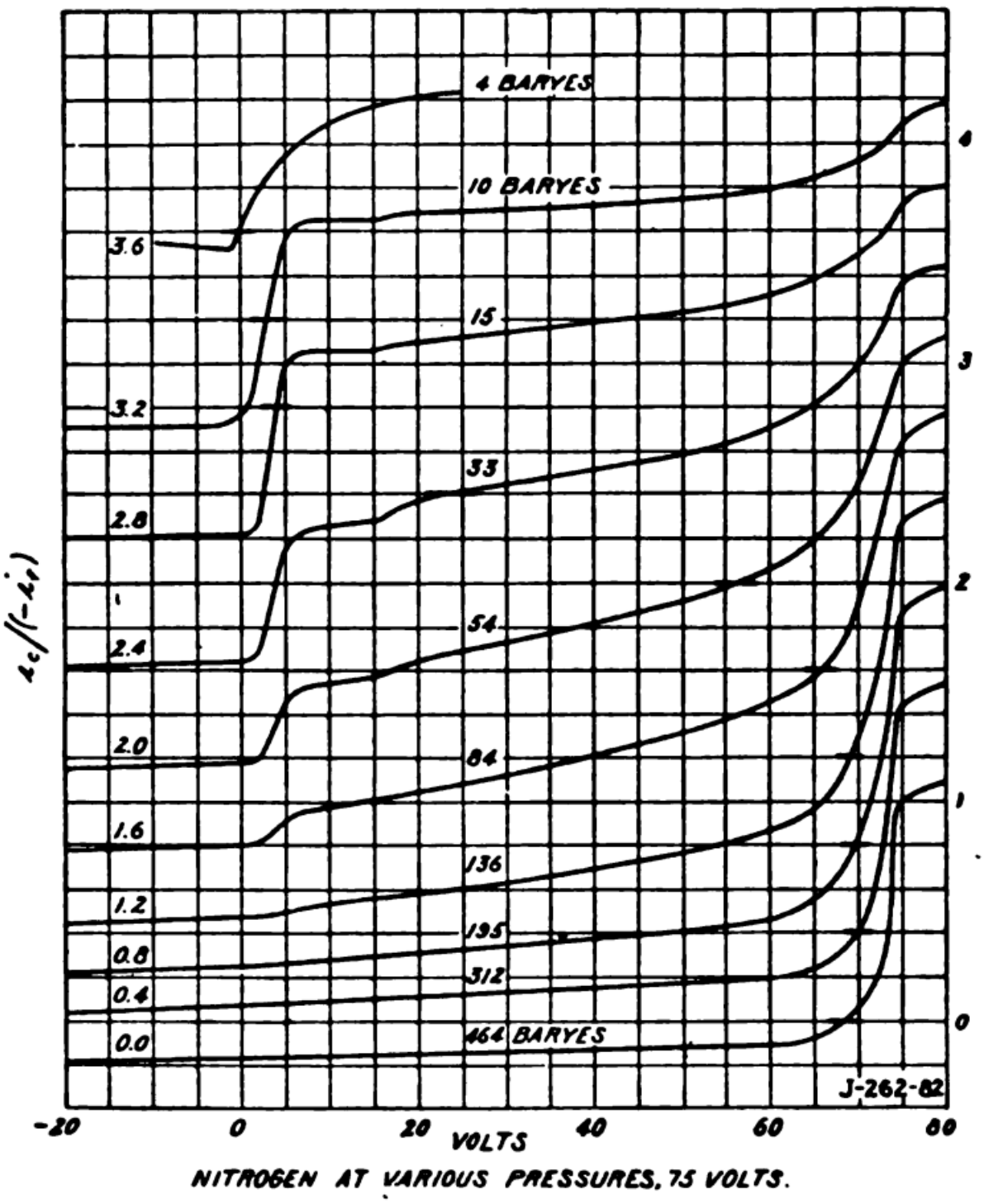
tage of the probe wire which was at a distance of one cm from the cathode gives us the average temperature of the ultimate electrons. Thus $T_e = 3900°K$ in this case and E_s the potential of the space about the probe is 1.25 volts positive with respect to the anode. We find also that $I_e = 0.51$ ma. of ultimate electrons per cm² per ma. emission, and $n_{e(ult)}/ = 3.3 \times 10^8$ electrons per cc at the probe wire per ma. emission from the cathode.



NITROGEN AT VARIOUS PRESSURES, 75 VOLTS.

FIG. 13.

The average current density of ultimate electrons reaching the anode is 0.013 ma. cm^{-2} per ma. emission from the cathode. Hence from Eq. (3) the anode is 1.07 volts negative with respect to the space near the collector. This agrees well with the probe wire measurement and also with the results calculated from the volt-ampere curves of the end-plates and cylinder.

From the results obtained by analysis of the volt-ampere characteristic of both end-plates and cylinder we may draw certain general conclusions. For example, when an electron having 100 volts energy strikes a mercury atom in such a manner that the atom gives up an electron and becomes a positive ion, there is, of course, a 10.4 volt loss by the incident electron in producing an

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ion. The remaining 90 volts energy is distributed between the incident electron and the electron given up by the mercury atom. The incident electrons and the electrons given up by the atoms give rise to the high velocity Maxwellian group of electrons which become random in direction and in the cases which we have considered have an average energy of about 45 volts.

Additional evidence of the formation of this high velocity Maxwellian group is obtained from the detailed analyses of the curves shown in Figs. 7 and 10. From these curves we see that the experimental curve deviates gradually from the theoretical curve starting at the ionizing potential in each case.

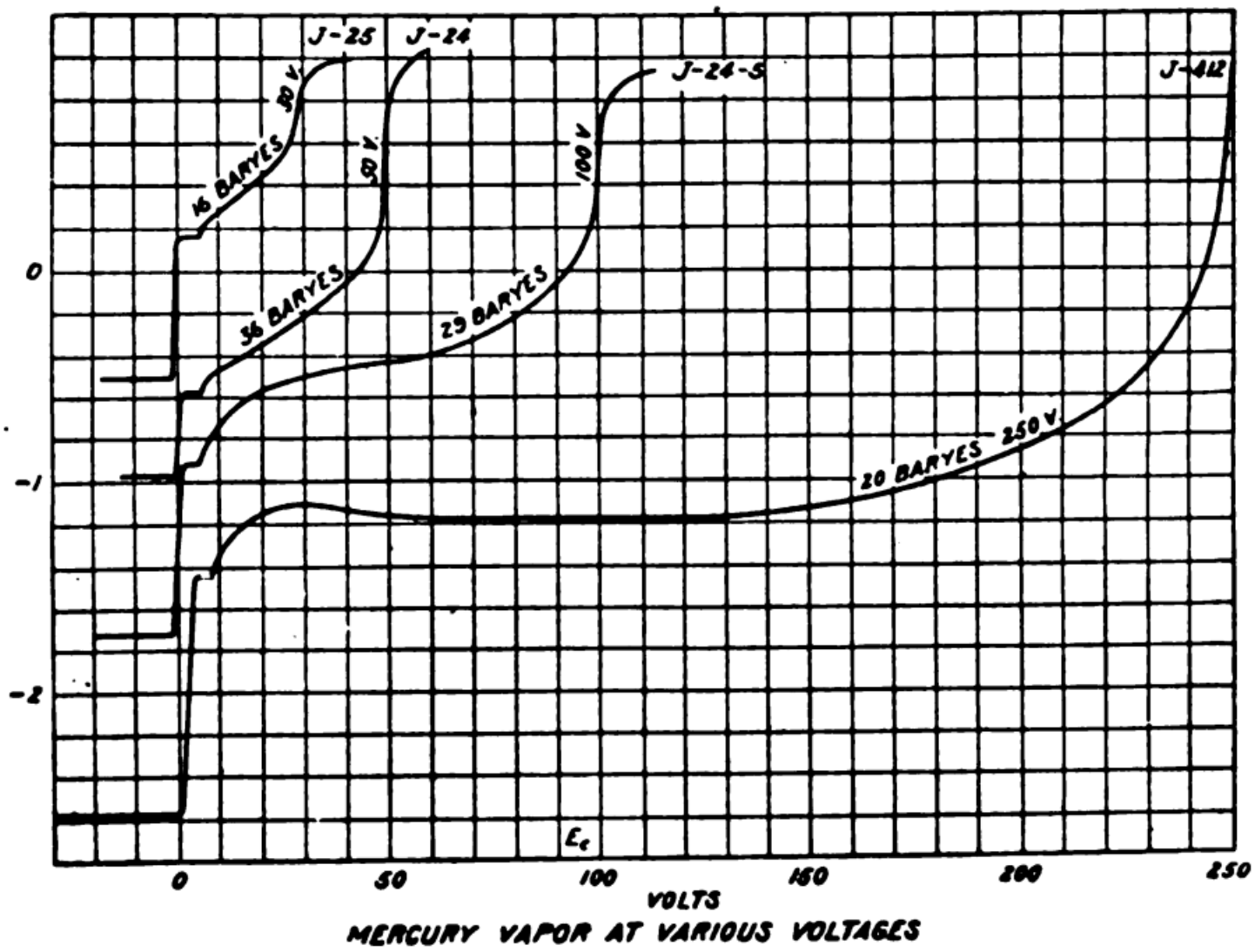

FIG. 14.

Fig. 12 shows typical experimental curves in each of the different gases studied. The collector potential, E_c, is plotted against $i_c/-i_f$ in each case. The curves have been shifted vertically by the amount indicated at the left end of each curve to prevent overlapping. For example, curve 1 gives the results obtained in mercury vapor at 10.4 baryes pressure with the anode A at $+100$ volts potential, the curve having been shifted up 0.5 along the ordinate axis.

The resonance potentials at which the kinks appear on the curves for the various gases have been determined from these curves and also from the large scale plots used in the detailed analysis of the groups suffering elastic and inelastic collisions. These potentials are given in Table VI and are probably correct to ± 0.5 volt.

Fig. 13 shows the experimental curves obtained with nitrogen at a series of different pressures. The anode (A) voltage was $+75$ volts in each case. The

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curves have been raised along the ordinate by the amounts indicated on the left side of the curves, the ordinate scale being at the right. The curves at the higher pressures show that no primary electrons reach the cylinder C without having suffered energy losses. As the pressure is decreased, the primary elec-

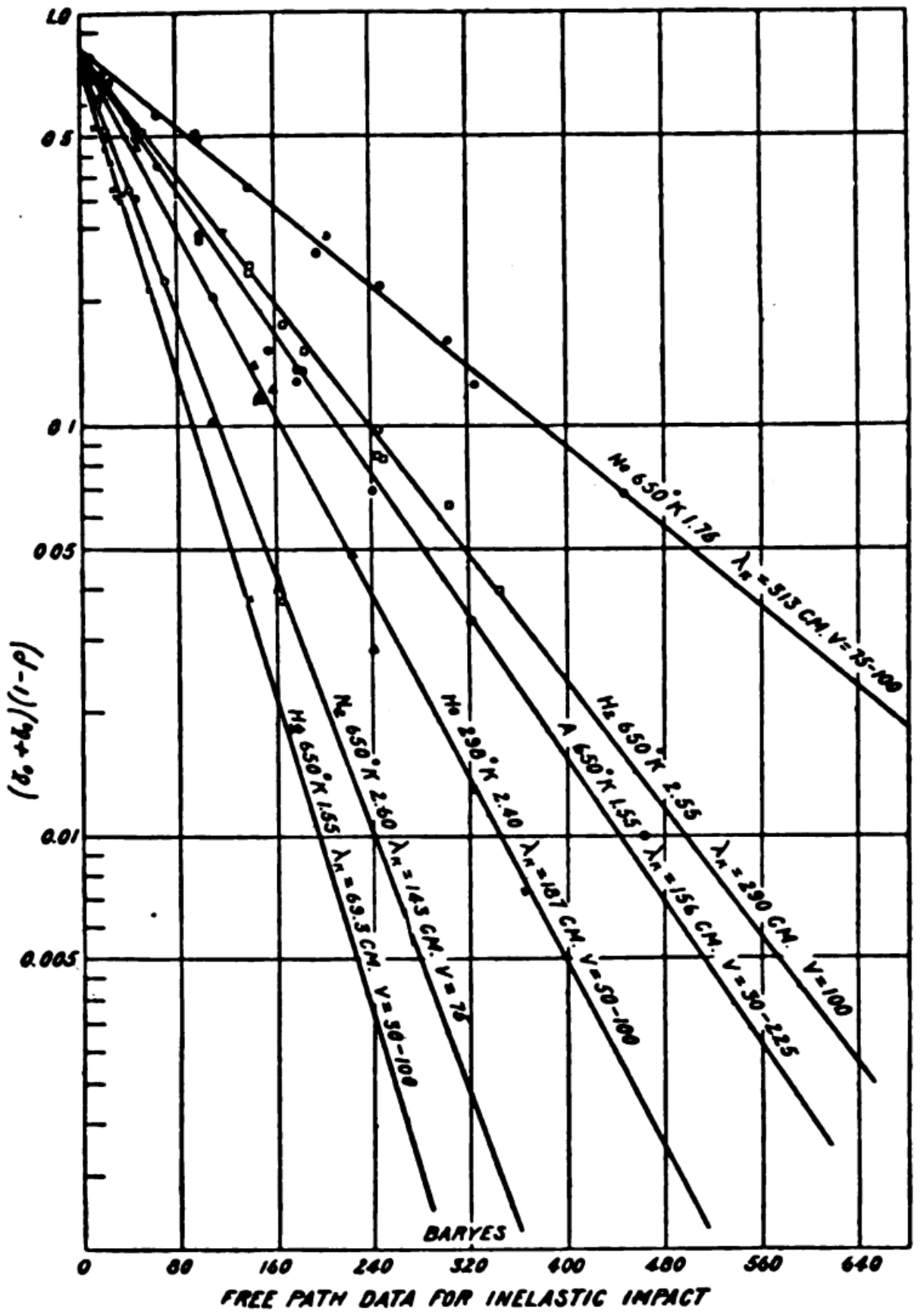

FIG. 15.

trons begin to reach C without loss of energy until finally at 4 baryes pressure the intensity of ionization falls off rapidly at positive voltages on the collector because of the removal of nearly all the primary electrons before they form ions and thus the positive ion sheaths around the cathode and cylinder overlap, the current from the cathode drops suddenly to the "space charge" value and our theory no longer applies.

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TABLE VI

Ionization and Resonance Potentials of Various Gases Used

Gas	Obs. Res. Pot.	Ion. Pot.
Hg	6.7 volts	10.4 volts
Ne	18.5	21.5
A	13.0	15.3
He	21.1	24.5
N_2	13.0	16.8
H_2	12.8	16.1

Fig. 14 shows the curves obtained in mercury vapor at a series of different anode voltages. The decrease in current values between +30 and +70 volts on the 250 volt, 20 barye curve is due to the fact that the reflection coefficient of the collector for electrons increases with the electron velocity. The constancy of the positive ion currents collected at negative voltages on the collector, the sharpness of the break at the resonance potential (6.7 volts) and the relatively large number of electrons which suffer a 6.7 volt energy loss show up particularly well in these curves.

Electron Free Paths

The experimental data on the mean free path of high velocity electrons (above the ionizing potential) in the various gases give results in good agreement with the kinetic theory value calculated from viscosity measurements (i.e., $4 \cdot (2)^{1/2}$ times the free paths of the gas molecules.)

We have plotted $(\gamma_0+\gamma_e)(1-\varrho)$ against the gas pressure in baryes on a semi-logarithmic scale in Fig. 15. This quantity is the fraction of the primary electrons which are collectable by C without having made *inelastic* collisions with molecules and is thus given directly by the experiments when collector voltages just below the resonance potential are used.

Since the free path is inversely proportional to the pressure, we see readily from Eq. (32) that the slope $d \log_e y/dx$ of the lines of the semi-logarithmic plot of Fig. 15 should be R/λ_1, where R is the effective radius of the collector C and λ_1 is the free path of the electrons for inelastic collisions at unit pressure. The intercept of these lines on the vertical axis which corresponds to $1-\varrho$ ranges from 0.77 to 0.83. Hence, ϱ, the reflection coefficient of the collector for electrons striking with a velocity corresponding to ten to twenty volts varies from 0.17 to 0.23 in good agreement with the values derived from Eq. (58).

The values of λ_1 in cm calculated from the slopes of the lines are given in the 5th column of Table VII. These correspond to a pressure of 1 barye and the temperatures given in the second column. The electron free paths at the same temperature and pressure calculated from the viscosities are given in the 6th column. It is seen that the two sets of values agree rather closely.

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In the last column of Table VII are the values of P_k, the probability of inelastic collision per cm at 20°C and 1000 baryes pressure, calculated from λ_1 by assuming that the probability is proportional to the density.

TABLE VII

Free Paths for Inelastic Collisions

Gas	T °K	R	Anode	Voltage	λ_1	Kinetic theory	P_k
Ne	650	1.75 cm	75	and 100	313	288	7.1
H_2	650	2.55	100		290	266	7.6
A	650	1.55	30	50 100	156	160	14.2
He	298	2.40	50	100	187	166	5.4
N_2	650	2.60	75		143	146	15.5
Hg	650	1.55	30	50 100	69.3	72.9	32

We have summarized the results of the detailed analyses of more than two hundred sets of volt-ampere characteristic curves for the various gases studied. The final results, expressed in each case at 1000 baryes pressure and 293°K are given in Table VIII. The various quantities in this table have all been defined with the exception of β (max) The values of θ_r, the mean deflection for a single resonance collision, which appears in brackets, have been assumed to be the same as the mean deflection for a single elastic collision in order to obtain a reasonable value for P_r.

The value β (max) gives the maximum number of ions which can be produced by an electron having a velocity corresponding to the potential given in the second column. The quantity β is independent of the pressure over a very wide range of pressures. At very low pressures, however, β decreases due to the fact that the sheaths around the cathode and the collector become so thick due to the low intensity of ionization that the electrons travel appreciable distances (compared to the tube radius) in the sheaths, (both in being decelerated and in being accelerated) in which the electrons will not have sufficient energy to produce an ion. If an inelastic collision occurs under these conditions the electron will suffer a resonance energy loss instead of producing an ion. Then also at low pressures a larger proportion of the electrons will reach the anode while they still retain enough energy to cause ionization.

At pressures above the range in which β remains constant, β again decreases in value. From the theory of the diffusion of ions in ionized gases due to Schottky and Issendorff[8] we find that, where the free paths of the electrons become small compared to the radius of the tube, the cathode may receive as many, or more ions than the collector. The current (i_F) must then be corrected for the large number of ions flowing to the cathode to obtain the true electron emission. When this correction is made and the pressure is not such that the electrons suffer an appreciable number of collisions in the sheaths we find that β agrees approximately with β(max).

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In some experiments to test the constancy of β, the electron emission from the cathode ($-i_p$) was varied over a range of five hundred fold without changing the value of β. β being independent of the current density of primary electrons indicates that ionization is a single stage process in our experiments and that no two stage processes are effective in producing any measurable number of ions. It also indicates that recombination of electrons and ions is negligible under the conditions of these experiments up to 1000 baryes pressure.

TABLE VIII

Summary of Data on Electron Impacts in Gases

Gas	Volts	P	P_k	P_e	P_r	P_i	P_x	θ_e	θ_r	E_e	β_{max}
He	50	7.4	5.8	1.6	2.1	0.7	3.0	25	[25]	9.5	1.2
	100	5.6	4.5	1.1	0.5	1.22	2.78	19	16	11.0	2.9
Ne	75	7.9	7.0	0.9	1.1	1.38	4.52	21	21]	9.8	1.25
	100	7.6	6.9	0.7	1.0	1.80	4.1	19	[19]	11.0	2.0
A	30	32.4	14.3	18.1	4.4	3.5	6.4	24	[24]	5.3	0.45
	50	29.4	13.8	15.6	1.4	7.1	5.3	18	[18]	5.2	0.9
	100	24.9	14.0	10.9	1.3	8.55	4.15	12	[12]	4.7	1.65
	150	23.4	13.8	9.6	2.0	8.55	3.25	10	[10]	4.5	2.2
Hg	30	61.0	36.0	25.0	13.0	10.2	12.8	17	[17]	2.5	1.1
	50	59.0	36.9	22.1	11.0	14.9	11.0	11	13	3.1	1.4
	100	57.2	37.8	19.4	12.5	16.3	9.0	10	12	3.3	2.7
	250	40.0	24.0	16.0	5.0	15.3	3.7	6	[6]	3.2	5.3
H_2	100	10.9	6.9	4.0	2.6	2.9	1.4	5	16	7.3	1.45
	250	10.1	5.4	4.7	1.6	2.7	1.1	9	5	6.7	2.8
N_2	75	26.4	15.2	11.2	4.0	7.0	4.2	16	[16]	5.9	1.32
	100	20.0	12.2	7.8	3.6	7.7	0.9	14	18	5.8	1.65

The authors wish to take this opportunity to correct an erroneous statement made in a preliminary article in Science.[3] In this article it was stated that 60 or even 70 per cent of the primary electrons lose nearly all of their energy on their first collisions with atoms producing a highly excited atom or ion which subsequently caused the ionization of several other atoms. This incorrect statement resulted from our having neglected to consider the change in the reflection coefficient of electrons from the collector with voltage. As a result of our detailed analyses of the experimental curves with the reflection coefficient for each group of electrons treated separately we may safely say that if any electrons lose all of their energy in the first collision with an atom the number suffering such energy loss is small.

The reflection of electrons from the collectors in our experiments is important because the ion sheaths which cover all electrodes (even the anode) act as *accelerating* grids to return the electrons to the field-free space where after reflection and *acceleration* through the positive ion sheath, it may have suffi-

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cient energy to cause ionization, resonance, etc. This is not true, however, under negative space charge conditions where the field is positive right up to the collector and any reflected electrons are trapped and collected. Thus, under negative space charge conditions in good vacuum, the three halves power law holds over a wide range of voltages.

The authors take pleasure in extending their thanks to Mr. H. M. Mott-Smith, Jr., for valuable assistance with the mathematical derivations, and to Mr. W. G. Baker for many helpful suggestions rendered during the course of the experimental work.

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OSCILLATIONS IN IONIZED GASES

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IN STRONGLY ionized gases at low pressures, for example in the mercury arc, the free electrons have a Maxwellian velocity distribution corresponding to temperatures that may range from 5000° to 60,000°, although the mean free path of the electrons may be so great that ordinary collisions cannot bring about such a velocity distribution. Electrons accelerated from a hot cathode (primary electrons), which originally form a beam of cathode rays with uniform translational motion, rapidly acquire a random or temperature motion which must result from impulses delivered to the electrons in random directions.

In this laboratory we have been studying these phenomena[1] in detail during the last 4–5 years, but the mechanism underlying the Maxwellian distribution and its extremely short time of relaxation have not been understood. At an early date it occurred to me that electric oscillations of very high frequency and of short wave-length in the space within the tube might produce a scattering of the kind observed, but calculation showed that average field strengths of several hundred volts per centimeter would be necessary and this seemed an unreasonable assumption. Experiments capable of detecting oscillations of the electrodes with amplitudes greater than 0.2 volt failed to show such oscillations.

Ditmer[2] although unable to detect oscillations, concluded that oscillations of frequencies higher than 10^8 probably caused the scattering but was not able to suggest why there should be such oscillations.

Penning[3] detected oscillations of frequencies from 3×10^8 to 6×10^8 per second and found that the electron scattering and the oscillations nearly always occurred together. No cause was assigned for the oscillations.

Dr. Tonks and I have repeated and have confirmed Penning's observations. The amplitude of the oscillations is small, less than 0.2 volt, and frequencies up to 1.2×10^9 have been observed. These waves, which are of well-defined frequency, appear to depend on the presence of the ultimate electrons (low velocity electrons) and can be observed in any part of the bulb. Other oscillations of much lower frequency (2×10^7 to 20×10^7 per second) can be observed in regions transversed by the primary electrons, which are injected

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[EDITOR'S NOTE: More detailed paper published in *Phys. Rev.* **33**, 195 (1929).]

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into the tube with velocities corresponding to 25 to 70 volts. These oscillations and the scattering of the primary electrons are observable only when the current of primaries is raised to 10 milliamps or more.

It seemed that these oscillations must be regarded as compressional electric waves somewhat analogous to sound waves. Except near the electrodes, where there are *sheaths* containing very few electrons, the ionized gas contains ions and electrons in about equal numbers so that the resultant space charge is very small. We shall use the name *plasma* to describe this region containing balanced charges of ions and electrons.

For purposes of calculation we may consider the plasma to consist of a continuum of positive electricity having a charge density ϱ with free electrons distributed within it, the average electron space charge being $-\varrho$. If the electron temperature is zero, the electrons will probably arrange themselves in a face-centered cubic space lattice (close packed arrangement).

Any electron after displacement from its equilibrium position oscillates about that point with a frequency ν which is given approximately by

$$\nu^2 = \frac{e}{m} \cdot \frac{\varrho}{3\pi} = \frac{e^2 n}{3\pi m} \tag{1}$$

where n is the number of electrons per cubic cm, e is the charge and m the mass of the electron. At low temperatures or high electron densities (actually if $T < 0.001\, n^{1/3}$) the plasma is thus a kind of crystalline gas.[4] However the electron temperature will ordinarily be far too high to permit this kind of structure even if it were consistent with the actual discontinuous distribution of positive charge. We must therefore consider that the electrons move rapidly throughout the plasma.

By means of the Boltzmann and the Poisson equations, Debye and Hückel[5] have shown that under equilibrium conditions in a plasma, the average potential near a charged plane varies according to the law

$$V = V_0 \epsilon^{-x/\lambda_D}$$

where x is the distance from the plane, $\epsilon = 2.718$ and λ_D which we may call the Debye distance, is given by

$$\lambda_D = \sqrt{\frac{kT}{4\pi n e^2}} = 6.92 \sqrt{\frac{T}{n}} \text{ cm.} \tag{2}$$

where T is the temperature of the electrons.

Thus at any distance from the plane which is large compared to λ_D, the average potential and the average electric field become zero. However, when the potential of the plane is changed it requires a certain time for the redistribution of charges to occur and these transient fields are not given by the Debye-Hückel theory.

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Plasma Oscillations. — If throughout a volume of dimensions large compared to λ_D we change the concentration of electrons by some transient external means, the resulting electric fields act in such a direction as to equalize the concentration, but the potential energy of these fields is converted into kinetic energy of the electrons so that oscillations occur, and electric waves may result.

Mr. Harold Mott-Smith has worked out the theory for infinite trains of harmonic compressional waves of length λ traveling through a uniform plasma containing electrons of negligible velocity. He finds that the velocity of propagation of the waves v is given by

$$v = \lambda \sqrt{\frac{e}{m}\frac{\varrho}{\pi}} \tag{3}$$

Thus no matter what the wave-length, the frequency (v/λ) in sec^{-1} is

$$\nu = \sqrt{\frac{e}{m}\cdot\frac{\varrho}{\pi}} = 8980\sqrt{n} \tag{4}$$

which is $\sqrt{3}$ times that given by equation (1) for the oscillations of single electrons.

Since the velocity is proportional to the wave-length, the waves show high dispersion and the group velocity of the waves turns out to be zero. That is, although these waves can propagate through space with the velocity v they transmit no energy.

Dr. L. Tonks has proved that if the electron density in any region within the plasma is slightly altered so that the potential distribution is $f(x, y, z)$, then after the time t it is $f(x, y, z)\cos 2\pi\nu t$, where ν has the value given by equation (4). This is a stationary wave which remains of constant wave shape and does not spread.

In these derivations it was assumed that the wave-length λ (or the grain size) of the disturbance $f(x, y, z)$ is so large that the plasma electrons do not move distances comparable to λ during one period $(1/\nu)$ of the oscillations. This condition is satisfied if λ is large compared to the Debye distance λ_D.

Beam Oscillations. — A beam of electrons moving with a uniform translational velocity v and containing n_1 electrons per cm^3 may contain similar oscillations of a frequency ν_1 which may be calculated from equation (4) by placing $n = n_1$. In order that these beam oscillations may be unaffected by the plasma electrons it is necessary that their wave-length λ_1 shall be small compared to v/ν, where ν is the frequency of the plasma oscillations. With electron beams of 50 volts energy v/ν is usually about $50\lambda_D$ and thus beam oscillations can exist having a wide range of wave-lengths whose frequency can be calculated by equation (4). Since n_1 is far smaller than n, the beam freqency ν_1 will be less than ν, but because of the Doppler effect the waves will appear to have different frequencies according to the direction from which they are observed. Transversely, the frequency may be ν_1 but longitudinally frequencies as high as v/λ may occur.

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The oscillations that we have observed seem to be in general accord with this theory. Measurements of n, the concentration of ions, by means of the positive ion currents and calculation of the frequency ν from this by equation (4) give values that agree reasonably well with the definite frequencies of about 10^9 that are found by using a Lecher system. The broad band of lower freqencies from 10^7 to 10^8 sec^{-1} which are observed only in the regions traversed by the primary electrons appear to correspond to beam oscillations which are spread over a range in frequencies by the Doppler effect and the non-uniform concentration of the primary electrons.

The smallness of the observed amplitude of the oscillations in the experiments is readily explained by the fact that the plasma oscillations do not transmit energy, or rather, that energy can be transmitted only because of some second order effects neglected in the elementary theory. Thus it is probable that the plasma oscillations may well be of sufficient amplitude to cause the electron scattering that brings about a Maxwellian distribution.

Ionic Plasma Oscillations. — In a recent paper J. S. Webb and L. A. Pardue[6] have described experiments which showed oscillations in low pressure discharges in air with frequencies that ranged from 1 to 240 kilocycles. The frequency increased with the current and with the voltage and decreased in general as the pressure increased. It seems possible that such oscillations might be ionic oscillations in agreement with equation (4) where m is now the mass of the ion. If the electron temperature is very high compared to the ion temperature, as should be expected at high pressure, the diffusion of the electrons will set up fields which tend to make the concentration of ions uniform so that equation (4) should apply.

A more complete analysis by Dr. Tonks, assuming a Maxwellian distribution of electrons but negligible ion temperature, shows that when the wave-length λ of the ionic oscillations becomes small compared to $2\pi\lambda_D$ (calculated by equation (2) from the electron temperature) the frequency approaches the limiting maximum value given by equation (4), and the group velocity approaches zero. On the other hand, as λ becomes large compared to $2\pi\lambda_D$ the wave velocity and the group velocity both approach the limiting maximum value

$$v = \sqrt{kT_e m_p} = 3.9\times 10^{7}\sqrt{T_e m_e/m_p}\ \text{cm per sec.}$$

The frequencies v/λ thus decrease as the wave-length increases just as in the case of sound waves. These waves are in fact electric sound waves and their frequencies lie in the range of Webb and Pardue's experiments.

Thermal Equilibrium of Plasma Oscillations. — Any arbitrarily selected volume (of dimensions large compared to λ_D) which we may choose within a plasma may be regarded as a harmonic oscillator, whose damping coefficient will be of the order λ_D/λ. Since electrons, with a velocity distribution corresponding to T_e are continually entering and leaving this selected volume or cell, the oscillator should acquire on the average $\frac{1}{2}kT_e$ of kinetic energy and $\frac{1}{2}kT_e$ of

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potential energy (electrostatic field) for each degree of freedom. The kinetic energy of the oscillator is included in that of the electrons which coöperate to produce the oscillation but the potential energy is a new factor, not included in the ordinary theory of electron motions.

If we have N independent cells per unit volume the total energy density of the electric field of the oscillations is $^3/_2 NkT_e$; equating this to $X^2/8\pi$, where X is the field intensity, gives

$$X^2 = 12\pi NkT_e$$
$$\text{or } X = 2.16\times 10^{-5}\sqrt{NT_e} \text{ volts per cm.} \tag{5}$$

If there were no lower limit for the wave-length of harmonic plasma oscillations, we could put $N = n$ the concentration of electrons, as in the Debye theory of the specific heats of solids. We have seen that the potential distribution $f(x, y, z)$ in any plasma oscillator is arbitrary. Thus to find N we might expand $f(x, y, z)$ into a Fouriers' series, breaking off the series when we have reached the permissible grain size: the number of independent parameters thus fixes N. Perhaps a better way of attacking the problem is to assume that there are n independent oscillators per unit volume but those of shorter wave-length are non-harmonic and are highly damped so that for these the time average of the potential energy may be far less than the average kinetic energy.

For the present we can obtain an approximate value of N by assuming that harmonic oscillators are possible down to the wave-length $\lambda = a\lambda_D$, where a is a small numerical factor that probably lies between 2 and 10.

Thus $N = 1/(a\lambda_D)^3$ and from equations (5) and (2) the field strength X is

$$X^2 = 96\pi^{5/2}a^{-3}(kT)^{-1/2}n^{3/2}e^3$$

or

$$X = 1.17\times 10^{-6}a^{-3/2}n^{3/4}T^{-1/4} \text{ volts per cm.}$$

For a typical low pressure gas discharge in which $n = 10^{11}$ and $T = 10^4$ this gives $X = 4$ volts per cm if we assume $a = 3$. This is the mean field intensity of plasma oscillations in thermal equilibrium with the ultimate electrons. According to equation (4) the frequency would be 2.8×10^9 per second, corresponding to that of a radio wave of wave-length 10 cm.

Electric Fields Due to a Random Distribution of Electrons. — Let us compare these fields produced by the plasma oscillations with those to be expected from a random distribution of electrons. We assume provisionally that the positive charge in the plasma is continuous, and that the probability per unit volume for the occurrence of electrons is constant throughout the plasma. The radius of a sphere which contains on the average one electron is

$$r_1 = (3/4\pi n)^{1/3} = 0.62n^{-1/3} \tag{7}$$

The average field X_1 at the center of the sphere averaged for all positions is

$$X_1 = 3e/r_1^2 = 1.12\times 10^{-6}n^{2/3} \text{ volts cm}^{-1} \tag{8}$$

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The effect of all the other electrons (out to $r = \infty$) increases the average field only about 10.3 per cent. Thus the average field X_a at any point in space is

$$X_a = 1.23 \times 10^{-6} n^{2/3} \text{ volts cm}^{-1} \tag{9}$$

This field is always stronger than that of the plasma oscillations, as given by equation (6). In our example in which $n = 10^{11}$, we find $X_a = 26.5$ volts per cm as compared with our estimate of 4 volts per cm for the plasma oscillations.

Most of the effects produced by the local fields depend upon the square of the field strength so that we are usually interested in the root-mean square field which we shall denote by X_m. If the electrons are regarded as point charges the value of X_m at any fixed point is infinite. In the neighborhood of any given electron, however, according to the Boltzmann equation the average concentration of electrons is

$$n\epsilon^{-a/r}$$

where

$$a = e^2/kT = 1.66 \times 10^{-3}/T \text{ cm}. \tag{10}$$

We thus find that the mean field X_m which acts on any electron because of the presence of all the others is given by

$$\left.\begin{aligned} X_m^2 &= 4\pi nkT \\ \text{or} \quad X_m &= 1.25 \times 10^{-5} \sqrt{nT} \text{ volts cm}^{-1} \end{aligned}\right\} \tag{11}$$

from which $X_m = 396$ volts per cm if $n = 10^{11}$, $T = 10^4$.

The mean field X_r due to all electrons that lie at a distance greater than r (when $r \geqslant a$) is found to be

$$\left.\begin{aligned} X_r^2 &= 4\pi ne^2/r \\ \text{or} \quad X_r &= 5.08 \times 10^{-7} \sqrt{n/r} \text{ volts cm}^{-1} \end{aligned}\right\} \tag{12}$$

If we put $r = \lambda_D$ as given by equation (2)

$$X_D = (4\pi ne^2)^{3/4}(kT)^{-1/4} = 1.93 \times 10^{-7} n^{3/4} T^{-1/4} \text{ volts cm}^{-1}. \tag{13}$$

Comparing this with equation (6) which gives the field due to plasma oscillations we see that the two equations are identical if $a = 3.31$ which is a reasonable value. We may conclude that the fields due to plasma oscillations in thermal equilibrium with the electrons are of about the same magnitude as the fields acting at any point originating from all electrons at distances greater than λ_D. We have assumed, however, that the electrons are distributed at random, that is, as if they were uncharged. According to Debye-Hückel, the charges cause the distribution to be more uniform so that the field due to electrons at distances greater than λ_D should disappear. The field of the plasma oscillations thus takes the place of the field that is forbidden by the Debye-Hückel theory. The definite frequency characteristic of these oscillations makes them, however, essentially different from the irregularly fluctuating fields that they replace.

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Energy Delivered to Electrons by the Field. — If a steady electric field of intensity X acts for a time τ on an electron which is initially at rest, the electron acquires the energy

$$E = X^2e^2\tau^2/2m$$

The fields acting on a given electron resulting from a random distribution of neighboring electrons fluctuate rapidly in direction and magnitude. If τ is the time of relaxation of these fields we may consider roughly that during each interval τ the field is steady, and of magnitude X, but in each interval the direction is independent of that in the preceding or following intervals. In a time t which is large compared to τ the total energy acquired by an electron is then

$$E = X^2e^2\tau t/2m \tag{14}$$

The field produced by electrons at a distance r will fluctuate at a rate such that, approximately

$$\tau = r/v$$

where v, the velocity of the electrons, is

$$v = \sqrt{8kT/\pi m} = 6.23\times 10^5\sqrt{T}\ \text{cm sec}^{-1}.$$

By integrating (14) over all values of r from 0 to r we find that the rate W_r at which energy is acquired by a given electron is

$$W_r = \pi^{3/2}e^4n\,(2mkT)^{-1/2}\log_e\left(\frac{rkT}{1.78e^2}\right)\text{erg per second.} \tag{15}$$

By dividing this into $3/2kT$ we find the time of relaxation of the electrons, that is, the time required for an electron to gain or lose the average energy. Multiplying this time by the velocity v we find an approximate value for λ_e, the free path of an electron. This free path, which may be defined as the average distance an electron travels before losing most of its energy of motion in that direction, is thus

$$\left.\begin{aligned}\lambda_e &= \frac{6}{\pi^2}\frac{(kT)^2}{e^4n}\frac{1}{\log_e\left(\frac{rkT}{1.78e^2}\right)}\\ \text{or}\qquad\\ \lambda_e &= \frac{9.5\times 10^4T^2}{n\log_{10}(r/1.78a)}\end{aligned}\right\} \tag{16}$$

where a is given by equation (10). We have seen from equation (12) that the field produced by the more distant electrons is small compared to that due to the nearer electrons. However, because of the fact that the field of the distant electrons fluctuates more slowly, this field becomes important in determining the energy transfer between electrons. Thus, according to equation (15) the rate of energy transfer increases indefinitely as r approaches

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infinity. It is clear that equations (12) and (15) are applicable only for values of r that are not much greater than λ_D since the field of electrons at a greater distance fluctuates periodically at a frequency which no longer decreases as r increases (plasma oscillations).

In our example $n = 10^{11}$, $T = 10^4$ and thus by equation (2) $\lambda_D = 2.2 \times 10^{-3}$ cm. Putting these in equations (10) and (16) we find $a = 1.66 \times 10^{-7}$ and the free λ_e is 24.6 cm. The experimental data obtained with mercury arcs shows effective free paths very much shorter than this. With a 0.1 ampere arc in a 3-cm tube containing mercury vapor saturated at 15°C we found[7] $T = 36{,}000°$ and $n = 3.4 \times 10^9$ and yet the Maxwellian distribution, which was destroyed at the tube walls by the removal of the fastest electrons, re-established itself within 1 cm so that λ_e must have been less than 1 cm. But under these conditions equation (2) gives $\lambda_D = 2.3 \times 10^{-2}$ cm and by equations (10) and (16) $a = 4.6 \times 10^{-8}$ cm and $\lambda_e = 6600$ cm. Thus the fields due to a random distribution of electrons are not capable of accounting for the observed short free path.

The fields due to the plasma oscillations also deliver energy to the electrons. The oscillations of large wave-length, being harmonic and only slightly damped, deliver very little energy, but the plasma oscillations of the shortest wave-length, $\alpha\lambda_D$, are highly damped and the electrons which are being acted on are moving distances comparable to $\alpha\lambda_D$ during a single period of the oscillations. Thus as a rough approximation we may consider the time of relaxation τ of the field X to be equal to $1/2\nu$, a half period. The rate at which energy is delivered is then by equations (14) and (4)

$$W_0 = X^2e^2\tau/2m = 1/4 X^2 e \sqrt{\pi/mn} \tag{17}$$

Assuming that the oscillations are in thermal equilibrium with the electrons, X is given by equation (6) so that equation (17) becomes

$$W_0 = \frac{24\pi^3\alpha^{-3}e^4n}{(mkT)^{1/2}} \tag{18}$$

Comparing this with the rate at which energy is delivered by the field of electrons distributed at random, equation (15), we find

$$W_0/W_r = \frac{82.1}{\alpha^3 \log_{10}(r/1.78a)} \tag{19}$$

As before we may identify r with λ_D; then if $n = 10^{11}$ and $T = 10^4$ we find that $W_0 = W_r$ if $\alpha = 2.8$. Thus plasma oscillations in thermal equilibrium, although their fields are much weaker than those produced by the nearer electrons, contribute about as much as the latter to the energy interchanges between electrons. Any complete theory of the kinetics of a plasma cannot ignore the plasma oscillations.

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Plasma Oscillations of Greater Amplitude.—To account for the experimentally observed electron-scattering the amplitude of the plasma oscillations must be far greater than that which corresponds to thermal equilibrium with the electrons. Then, too, with thermal equilibrium, the maximum energy of the electrical oscillations in a coupled Lechter system would be kT whereas actually the oscillations that Penning and we have observed are far more intense than this.

In a mercury arc or a discharge with a hot cathode, ions are continually being generated in a random manner throughout the plasma, and are diffusing to the walls of the tube and to the anode where they recombine with electrons. The electrons set free by the ionization process (secondary electrons) usually have far higher energies than the ultimate electrons in the plasma so that the secondaries are rapidly separated from the ions which were simultaneously produced. Thus the ionization tends to create a random distribution, whereas the electric field in accord with the Debye-Hückel theory, tends to make the field more uniform. In this way energy is delivered to the plasma oscillations, particularly to those of large wavelength, so that the amplitudes become larger than those corresponding to thermal equilibrium.

In a plasma containing n electrons per unit volume, let us consider a sphere of volume V and radius r. With a uniform distribution of electrons there would be nV, which we may call q_0, electrons in the sphere. But with a random distribution the actual number q will differ from q_0 by an amount whose average value is $\sqrt{q}$. Thus the average charge of the sphere is $\pm e\sqrt{q}$. Assuming this charge to be uniformly distributed within the sphere, the total electrostatic energy of the field (of which 5/6 lies outside the sphere) is

$$E = (3/5)qe^2/r = (4\pi/5)e^2r^2n \tag{20}$$

and the field at the surface of the sphere is

$$\text{or} \qquad \begin{aligned} X_r^2 &= (4\pi/3)ne^2/r \\ X_r &= 2.9\times 10^{-7}\sqrt{n/r} \text{ volts per cm.} \end{aligned} \tag{21}$$

If the sphere is of large size compared to λ_D the electrostatic energy given by equation (20) becomes the potential energy of a spherical plasma oscillation which is only slowly converted into thermal energy. Let us compare the energy E of equation (20) with the energy $\frac{1}{2}kT$ which is the average potential energy of the plasma oscillation in thermal equilibrium. Expressing kT by equation (2) in terms of λ_D we find

$$\frac{E}{\frac{1}{2}kT} = \frac{2}{5}\left(\frac{r}{\lambda_D}\right)^2.$$

We see that the energy associated with a sphere of radius r is equal to $\frac{1}{2}kT$, the value required for thermal equilibrium, if $r = 1.58\lambda_D$, but this energy increases in proportion to r^2. Thus, if $n = 10^{11}$, $T = 10^4$, $\lambda_D =$

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2.2×10^{-3} and we had initially a random distribution of electrons, the energy of the plasma oscillations associated with a sphere of 3 cm radius would be 2×10^{6} times greater than if there were thermal equilibrium.

This may possibly account for the amplitude of the observed oscillations detected by the aid of a Lecher system, but it hardly seems able to explain the observed scattering which must depend on the plasma oscillations of shorter wave-length, for these are not so far from the conditions of thermal equilibrium.

We are now investigating other possible causes that may produce plasma oscillations of large amplitude.

References

[1] Langmuir, *Phys. Rev.* **26,** 585 (1925); *Zeit. Physik* **46,** 271 (1927).

[2] Ditmer, *Phys. Rev.* **28,** 507 (1926).

[3] Penning, *Nature* Aug. **28,** (1926), and *Physica* **6,** 241 (1926).

[4] The possibility of gaseous crystals in stellar atmospheres has been recently considered by Eddington, *Monthly Notices, Roy. Astron. Soc.* **88,** 369 (1928).

[5] Debye and Hückel, *Phys. Zeits.* **24,** 185, 305 (1923).

[6] *Bull. Amer. Phys. Soc.* **3,** 19 (1928).

[7] Langmuir and Mott-Smith, Jr., *Gen. Elect. Rev.* **27,** 766 (1924). Table XIV.

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OSCILLATIONS IN AN IONIZED GAS

With LEWI TONKS as co-author

Science
68, 598 (1928).

WHEN, from any cause, a deficiency of electrons occurs in some region of a uniformly ionized gas at low pressure, neighboring electrons flow in to excess and set up a local non-spreading oscillation. Positive ions behave similarly for deficiencies extending over a small region but oscillate with a frequency lower in the ratio of the square roots of the particle masses. These oscillations, however, decrease in frequency and change over to sound-like waves when the original deficiency extends over large regions. Extensive experiments with a hot-cathode mercury arc yielded oscillations quite definitely attributable to the plasma electrons. These oscillations were in the expected range of frequencies, namely, about 10^9 cycles per second. They were found with external electrodes on the tube wall and with internal electrodes both in and out of the path of the primary electrons, and their amplitude was independent of the electrode voltage. Their frequency varied with electron density roughly in agreement with theory. Another oscillation of lower frequency than these may arise from primary electron oscillations stepped up by the Doppler Effect. Another oscillation detected with an external electrode seems to be intimately related to the presence of a negatively charged wire parallel and close to the emitting filament. For instance, the oscillation showed a rapid change of phase as the shadow of the wire on the tube wall was crossed and the amplitude pattern was symmetrical about the shadow. A low frequency and very critical oscillation at 1380 kc may lie at the high frequency limit of ion oscillations calculated to lie at 1500 kc.

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OSCILLATIONS IN IONIZED GASES

With LEWI TONKS as co-author

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ABSTRACT

A simple theory of electronic and ionic oscillations in an ionized gas has been developed. *The electronic oscillations* are so rapid (ca. 10^9 cycles) that the heavier positive ions are unaffected. They have a natural frequency $\nu_e = (ne^2/\pi m)^{1/2}$ and, except for secondary factors, do not transmit energy. *The ionic oscillations* are so slow that the electron density has its equilibrium value at all times. They vary in type according to their wave-length. The oscillations of shorter wave-length are similar to the electron vibrations, approaching the natural frequency $\nu_p = \nu_e(m_e/m_p)^{1/2}$ as upper limit. The oscillations of longer wave-length are similar to sound waves, the velocity approaching the value $v = (kT_e/m_p)^{1/2}$. The transition occurs roughly (i.e. to 5% of limiting values) within a 10-fold wave-length range centering around $2(2)^{1/2}\pi\lambda_D$, λ_D being the "Debye distance." While the theory offers no explanation of the cause of the observed oscillations. the frequency range of the most rapid oscillations, namely from 300 to 1000 megacycles agrees with that predicted for the oscillations of the ultimate electrons. Another observed frequency of 50 to 60 megacycles may correspond to oscillations of the beam electrons. Frequencies from 1.5 megacycles down can be attributed to positive ion oscillations. The correlation between theory and observed oscillations is to be considered tentative until simpler experimental conditions can be attained.

IT HAS been known for some time that if a low pressure mercury arc is maintained using a hot filament as an electron source there is a wide range of conditions under which a large number of the primary electrons rapidly acquire velocities whose voltage equivalent is greater than the total drop across the tube.[1] There is a larger number of electrons which have less than the expected energy, so that as a group the primary electrons have not gained energy. But there is some mechanism which either effects a rapid interchange of energy among the primary electrons or else effects a rapid and random interchange between each primary electrons and some other energy store.

Various hypotheses to explain this phenomenon have been advanced,[1] but none has appeared satisfactory. A natural supposition is that electrical oscillations in the arc cause the scattering both by subjecting the electrons to rapidly changing electronic fields and by causing potential changes on the electrodes.

Dittmer[2] obtained evidence pointing in this direction but was unable to detect the oscillations themselves. To explain the scattering it was thought

[1] Langmuir, *Phys. Rev.* **26,** 585 (1925).
[2] Dittmer, *Phys. Rev.* **28,** 507 (1926).

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that such oscillations must be of about the same amplitude as the observed scattering, namely 20 volts or more, and these should be easily observable.

Penning[3] has observed oscillations of radio frequencies in low pressure discharges in mercury vapor under the same conditions as those that lead to electron scattering. We have undertaken a more systematic investigation of these oscillations in order to determine their nature and cause, for although their voltage-amplitude appears to be very small we believe that they play a fundamental role in high current gaseous discharges. We have found that the oscillatory behavior of the arc is extremely complicated in the types of discharge tube which we have been using. Nevertheless certain features stand out as tending to confirm the theory outlined recently by Langmuir.[4] This theory will be treated in greater detail here and some of the more definite experimental results will be given.

I. Theory

Two types of oscillation seem to be theoretically possible, first oscillations of electrons which are too rapid for the ions to follow, and second, oscillations of the ions which are so slow that the electrons continually satisfy the Boltzmann Law. The mathematical difficulties when any dimension of the disturbed region is of the same order of magnitude as the mean distance between electrons or ions are such as to force us to consider only cases where such dimensions are large compared to the inter-ionic distances. Hence "electric field," "density," etc., refer to the average value of these quantities taken throughout a volume which contains many ions and electrons but whose dimensions are considerably smaller than those of the oscillating region.

A rough idea of the magnitudes involved can be obtained from the density of ionization. Taking 10^{10} as a typical low value of electron density, the inter-electron distance is 4.6×10^{-4} cm. If all the electrons throughout a certain region are displaced this distance, the resulting field strength will be 0.028 e.s.u. or 8.3 volts per cm, and the energy density in the field will be 3.0×10^{-5} ergs cm^{-3}.

A. Plasma-electron[5] *oscillations.*—When the electrons oscillate, the positive ions behave like a rigid jelly with uniform density of positive charge ne. Imbedded in this jelly and free to move there is an initially uniform electron distribution of charge density $-ne$. Choosing an orthogonal system of coordinates, consider the portion of the plasma included between two planes

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[3] Penning, *Nature*, Aug. 28 (1926) and *Physica* **6**, 241 (1926).

[4] Langmuir, *Proc. Natl. Acad. Sci.* **14**, 627 (1926).

[5] The word "plasma" will be used to designate that portion of an arc-type discharge in which the densities of ions and electrons are high but substantially equal. It embraces the whole space not occupied by "sheaths." See Footnote 4.

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each perpendicular to the x-axis. Suppose each electron between these planes to be displaced in the x-direction by a distance ξ which is independent of the y and z coordinates and is zero at each bounding plane. If the displacement ξ is a continuous function of x and $\partial\xi/\partial x$ is small compared to unity, the change in density caused by the electron displacement is

$$\delta n = n\partial\xi/\partial x \tag{1}$$

Originally the net charge was zero, so after the displacement Poisson's equation gives

$$\partial E/\partial x = 4\pi e\delta n \tag{2}$$

E being the electric field strength. Eliminating δn

$$\partial E/\partial x = 4\pi ne\partial\xi/\partial x \tag{3}$$

Integrating, it is found that

$$E = 4\pi ne\xi \tag{4}$$

for the field arising from the electron displacement only, (since the neglected arbitrary constant represents a uniform external field). Eq. (4) shows immediately that the displaced electrons will oscillate about their original positions with simple harmonic motion. The force on each electron is $-Ee = m_e\ddot{\xi}$ giving

$$m_e\ddot{\xi}+4\pi ne^2\xi = 0 \tag{5A}$$

for the equation of motion. The frequency of oscillation is thus seen to be

$$\nu_e = (ne^2/\pi m_e)^{1/2} = 8980\, n^{1/2}. \tag{6}$$

If a spherical or cylindrical instead of a plane disturbance be postulated, the same natural period of oscillation is found, but in those cases some distortion of the oscillation will occur near the center or axis respectively unless ξ/r remains small in those regions. Oscillations of this type will be called plasma-electron oscillations.

The same result in a more general from which shows the oscillations to be independent of any symmetry restriction can be obtained from the fundamental electromagnetic equations applied to an ionized gas. The earlier treatment has been retained, however, because of its simple physical ideas and its application further on to positive ion oscillations. In Gibbs' vector notation the four necessary electromagnetic equations are

$$-(1/c)\dot{\boldsymbol{H}} = \nabla\times\boldsymbol{E} \qquad \boldsymbol{J} = (1/4\pi)\dot{\boldsymbol{E}}-ne\boldsymbol{v}$$

$$(4\pi/c)\boldsymbol{J} = \nabla\times\boldsymbol{H} \qquad -m\dot{\boldsymbol{v}} = e\boldsymbol{E}+(e/c)\boldsymbol{v}\times\boldsymbol{H}$$

For sufficiently small oscillations $\boldsymbol{v}\times\boldsymbol{H}$ is negligible and can be dropped. Eliminating $\boldsymbol{v}$ and $\boldsymbol{J}$ gives

$$\nabla\times\dot{\boldsymbol{H}} = (1/c)\ddot{\boldsymbol{E}}+(4\pi ne^2/mc)\boldsymbol{E}$$

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Taking the curl of the first equation, $\boldsymbol{H}$ can be eliminated, giving

$$\ddot{\boldsymbol{E}}+(4\pi ne^2/m)\boldsymbol{E} = -c^2\nabla\times(\nabla\times\boldsymbol{E})$$

When the electric field arises primarily from electric charges, the portion arising from $\dot{\boldsymbol{H}}$ being negligible, $\nabla\times\boldsymbol{E}$ can be put equal to zero and we have

$$\ddot{\boldsymbol{E}}+(4\pi ne^2/m)\boldsymbol{E} = 0 \qquad (5B)$$

This result has no symmetry restriction and leads to the frequency given by Eq. (6).

When $\nabla\times\boldsymbol{E}$ is not zero the complete equation in $\boldsymbol{E}$ can be multiplied by $\nabla\cdot$ (i.e. operated on with div.),[6] which causes the right member to vanish. Noting that $\nabla\cdot\boldsymbol{E} = 4\pi e\delta n$ we have,

$$\partial^2(\delta n)/\partial t^2+(4\pi ne^2/m)\delta n = 0 \qquad (5C)$$

Hence, under all conditions, the electron density is capable of simple harmonic variation of small amplitude and of frequency $8980\ n^{1/2}$.

If we omit the $\nabla\cdot$ operation but assume that $\nabla\cdot\boldsymbol{E}=0$ we obtain the equation

$$\ddot{\boldsymbol{E}}+(4\pi ne^2/m)\boldsymbol{E} = c^2\nabla^2\boldsymbol{E}$$

which J. J. Thomson has used[7] in the discussion of the electrodeless discharge. The frequency and wave-length of the corresponding plane waves are related according to the equation

$$\nu^2 = ne^2/\pi m+c^2/\lambda^2$$

Thus the lower frequency limit for long waves coincides with the plasma-electron frequency. But there is no natural frequency for these electromagnetic oscillations in an infinite medium. In any actual case, however, reflection at the walls of the discharge tube gives the possibility of the formation of standing waves so that there will be a fundamental resonance frequency and a whole series of overtones. It seems likely, however, that any such oscillation will be very highly damped because of poor reflection at the tube walls.

The density of ions and electrons in the plasma of an arc of the type used experimentally is of the order of 10^{10} cm^{-3}. Using this value for n in Eq. (6) the natural frequency of the plasma-electron oscillation is found to be 9.0×10^8 cycles per sec. The wave-length of this in air is 33 cm, a value to be compared with wave-lengths from 27 cm to 81 cm found experimentally.

To calculate a rough value for the fundamental resonance frequency of the electromagnetic waves we may assume that the diameter of the bulb, 18 cm, is one-half the wave-length. This gives

$$\nu = [(8.9\times10^9)^2+(3\times10^{10}/36)^2]^{1/2} = 12.2\times10^8 \text{ cycles},$$

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[6] We are indebted to Sir J. J. Thomson for this method which he outlined in a letter to one of us.

[7] J. J. Thomson, *Phys. Soc. Proc.* **40,** 82 (1928).

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and the wave-length of this in air will be 24.6 cm, a value so near to those found experimentally that the decision as to whether some, at least, of the observed oscillations may be of this type must be reserved.

The absence of space coordinates in Eqs. (5B) and (5C) shows that there is no tendency for oscillations of this type to propagate through the plasma and their group velocity is zero. As a result one can specify the phases of the electron displacement through a certain region in such a way as to give the semblance of a moving wave, a wave, however, which moves continuously through a fixed region without ever progressing beyond, like the familiar rotating barber poles which appear to move steadily upward without rising.

In spite of the unreality of such a wave motion from the energy transfer point of view, it is necessary to suppose the existence of oscillations in that form if it is shown that these oscillations exhibit the Doppler Effect. An analogy will show, we think, that despite its artificiality, this wave motion is one likely to arise. For this purpose the electrons may be likened to the bobs of a set of regularly spaced identical simple pendulums suspended from a rigid ceiling. If the swing amplitudes are small so that the bobs do not collide, these bobs will exhibit the same behavior in a plane that the electrons do in space. Let these pendulums, originally at rest, be set in motion by passing a horizontal bar along beneath them in such a way as just to make contact with the bobs. The bobs so touched will then oscillate with the particular wave motion under discussion. The simplicity of the exciting means, makes it readily conceivable that the electrons are excited in an analogous way.

There are three factors not yet considered which may result in a transfer of energy by plasma-electron oscillations. First and foremost, of course, the oscillating electrons may themselves be moving in a body through the plasma, an example being the beam of electrons emitted by the hot filament in a mercury arc. Such oscillations will have a lower frequency than the oscillations of the ultimate electrons because of the lower density in the beam. As a typical example we may take the case described later of the 72. 6 v, 25 ma arc in which the current density of primary electrons at the tube wall under the electrode was probably about 3.1×10^{-4} amps cm^{-2}. The electron velocities were 5.1×10^{8} cm sec^{-1} so that the charge density was 6.1×10^{-13} coulombs cm^{-3} corresponding to an electron density of 3.8×10^{6}. By Eq. (6) the natural plasma-electron frequency for this density is 1.8×10^{7} sec^{-1}.

Secondly, the electric field of any unsymmetrical oscillation is unlimited in extent, reaching in varying intensity throughout the discharge tube and extending outside. It may be this factor which is primarily responsible for the detection of the plasma oscillations.

Finally, if a local oscillation be traversed by fast electrons having fairly uniform speeds, the alternate acceleration and retardation or the rhythmical deflection of this beam will excite neighboring portions of the plasma into oscillation.

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The simple mathematical theory outlined leads to a single frequency for the electron oscillations, but there are two factors which may cause departures from this condition. The Doppler effect has already been mentioned. The other factor is that the electrons are not normally at rest but move with their random thermal velocities. Fast-moving electrons completely traverse a local disturbance in a small fraction of a period and do not contribute to the density n which is effective in determining the frequency. Slow-moving electrons, on the other hand, remain in the same neighborhood for several complete oscillations and do contribute to n. The division is not definite but can be fixed roughly by assigning an effective wave-length to the disturbance and treating all those electrons which traverse that wave-length in less than one cycle as fast, the others as slow. The slow-moving electrons are then those which maintain the oscillation. Each fast-moving electron in its "impact" with the disturbance either contributes energy to it or draws energy from it.

It is seen, first, that a "fast" electron for a small (short wave-length) disturbance may be a "slow" electron for a large (long wave-length) disturbance, and second, that a "slow" electron for a high-frequency oscillation may be a "fast" electron for a lower frequency oscillation. Thus the greater the wave-length and the higher the frequency the greater the value of the distinguishing velocity, and the greater the number of slow-moving electrons. In this way the effective electron density n depends on the wave-length of the oscillation and this will give rise to a decrease of frequency with decreasing wave-length in accordance with Eq. (6). Since this decrease in frequency of itself decreases the effective density still further, there is a tendency at the smaller wave-lengths toward a kind of cumulative action which may make oscillations below a certain minimum size impossible. That this may actually occur is indicated by some calculations which we have made, but the present state of the investigation hardly warrants attempting a quantitative treatment here.

Interaction of the fast-moving primary electrons with oscillations of the ultimates has already been mentioned, and of course there is also the interaction of the ultimate electrons with any oscillation of the primary electrons. In either case electrons of one type stream through an oscillation among electrons of the other type and withdraw energy from it. It is immediately evident that the shorter the wave-length the less will the interaction be. There is one very suggestive possibility that should be mentioned. It has already been pointed out that because of their lower density the beam electrons have a longer period than the ultimates. But if the wave-length and wave velocity of the oscillations have suitable values, then the frequency with which the moving beam waves strike the comparatively stationary ultimates may coincide with the higher natural period of the latter and give rise to resonance.

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B. Plasma-ion oscillations.—We are now ready to discuss the slower ionic oscillations. In this case we shall assume the same type of displacement for the ions as we did for the electrons before. We then have

$$\delta n_p = -n\partial\xi/\partial x \tag{7}$$

for the increase in ion density and (by Boltzmann's Law)

$$\delta n_e = n[\exp(eV/kT_e)-1] \tag{8}$$

for the increase in density of electrons. Poisson's equation now gives

$$\partial^2 V/\partial x^2 = -4\pi e(\delta n_p-\delta n_e) \tag{9}$$

and for the relation of motion to field we have

$$e\partial V/\partial x = -m_p\ddot{\xi} \tag{10}$$

Substituting (7) and (8) in (9) and dropping terms in V^2 and beyond under the assumption that $eV/kT_e << 1$ we have

$$\partial^2 V/\partial x^2 = 4\pi en(\partial\xi/\partial x+eV/kT_e) \tag{11}$$

Taking the derivative with respect to x twice and substituting from (10)

$$\frac{\partial^2}{\partial x^2}\left(\ddot{\xi}+\frac{4\pi ne^2}{m_p}\xi\right)-\frac{4\pi ne^2}{kT_e}\ddot{\xi} = 0 \tag{12}$$

A qualitative idea of the frequency behavior of these plasma-ion oscillations can be obtained by solving Eq. (12) for the case of an infinite train of plane parallel waves. Thus if we assume the solution $\xi = \exp[2\pi j(\nu t-x/\lambda)]$ we obtain

$$\nu = \left(\frac{ne^2}{\pi m_p+ne^2m_p\lambda^2/kT_e}\right)^{1/2}$$

for the frequency.

When $ne^2\lambda^2/\pi kT_e$ is small compared to unity the frequency approaches a limiting value analogous to the frequency of the plasma-electron oscillations. The above condition is equivalent to the requirement that λ be small compared to $2(2\pi)^{1/2}\lambda_D$, λ_D being the distance, calculated by Debye and Hückel,[8] the reciprocal of which is like an absorption coefficient of an ionized fluid for electric forces. Thus the potential in the neighborhood of a sphere charged with q units of electricity is $q/(Kr)\exp(r/\lambda_D)$ in an ionized fluid compared to q/Kr in a non-ionized dielectric. Now

$$\lambda_D = (kT_e/8\pi ne^2)^{1/2} = 4.90(T_e/n)^{1/2}\text{ cm}$$

so that in discharges of the intensity used in our experiments where, roughly, $n = 10^{10}$ and $T_e = 10^4$, $\lambda_D = 0.005$ cm. The distance between ions averages $n^{-1/3} = 4.6\times10^{-4}$ cm. This appears to be enough smaller than $2(2)^{1/2}\pi\lambda_D = 4.4\times10^{-2}$ cm so that without approaching the fine structure limit of the

[8] Debye and Hückel, *Phys. Zeits.* **24,** 185, 305 (1923).

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theory too closely there is still room for plasma-ion oscillations near $\nu = (ne^2/\pi m_p)^{1/2}$. For mercury ions this limit is 1/600 that calculated for the electrons. Thus where the frequency of the plasma-electron oscillations is 9.0×10^8 sec^{-1} (as calculated above), that of the limiting plasma-ion oscillation is 1.5×10^6 sec^{-1}.

Most of the considerations applying to plasma-electron oscillations apply also to these short-wave ion oscillations. A factor which may make the latter simpler is the lower "temperature" of the ions, especially as there seem to

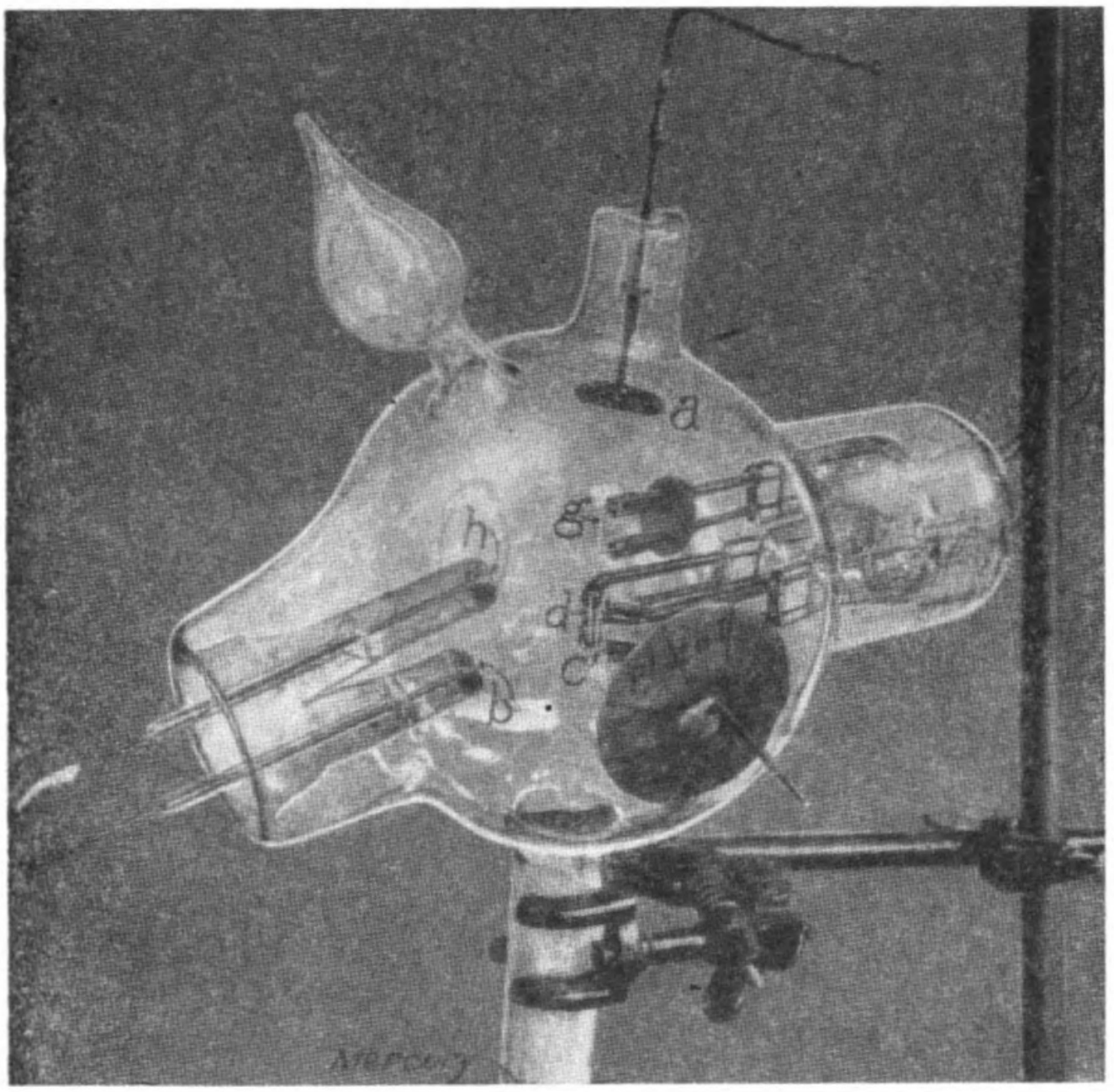

FIG. 1. Experimental tube.

be good reasons for supposing that the ions do not have so much chaotic as general drift motion through the plasma.

If, on the other hand, λ is considerably larger than $2(2)^{1/2}\pi\lambda_D$, the ion oscillations lose their similarity to the electron oscillations and change over to electric sound waves. Their group velocity becomes finite and both it and their wave velocity rapidly approach the limiting value

$$v = (kT_e/m_p)^{1/2} = 3.9\times10^5(T_e m_e/m_p)^{1/2}$$

In our experiments then, putting $T_e = 10^4$ and $(m_e/m_p)^{1/2} = 1/600$ we find $V = 6.5\times10^4$ cm sec^{-1} when λ is greater than about 10^{-1} cm. Thus frequencies originating from these sound waves may extend from 6.5×10^5 per

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sec down, but probably not beyond some point where the wave-length is comparable with the discharge tube dimensions.

The whole transition from the electron type of oscillation to the sound type occurs practically within a 10-fold variation of λ, for when $\lambda = 2(2)^{1/2}\pi\lambda_D/(10)^{1/2}$ the value $(ne^2/\pi m_p)^{1/2}$ for ν is only in error by 5 per cent and when $\lambda = 2(2)^{1/2}\pi\lambda_D(10)^{1/2}$ the value $(kT_e/m_p)^{1/2}$ for v is likewise only 5 per cent out.

J. S. Webb and L. A. Pardue[9] have described experiments in which they found oscillations that ranged from a few hundred cycles to 240 kc. in a low-pressure air discharge. As no data on the intensity of ionization and electron temperatures in their experiments are given, no quantitative comparison between this theory and their results can be made. The frequency range which they observed is roughly the range expected for the sound-like oscillations.

II. Experiments

A. Discharge tubes.—On the experimental side we have worked with two tubes, both containing filamentary cathodes used as electron sources, collectors so placed as to receive a portion of the direct beam of primary electrons from a filament, and an anode off to one side to maintain the discharge. The two tungsten filaments in the first tube used were supported near the middle of the 18 cm spherical bulb by long glass-covered leads. Their exposed portions were about 1.1 cm long, parallel and about 0.5 cm apart. At a distance of 4.2 cm from them was the collector, a circular disk backed by mica and 1.1 cm in diameter. It lay in a plane parallel to that of the two filaments. An appendix containing a little mercury extended from the bottom of the bulb and was immersed in a water bath, the temperature of which controlled the mercury pressure.

The second tube was similar except that it contained three vertical tungsten filaments, *g*, *c*, and *d* in Fig. 1, *g* above, and *c* and *d* about 2.5 cm below it. These two were 0.4 cm apart and all three lay in the same plane. Their diameter was 0.025 cm and their vertical and active portions were 1.1 cm long. Opposite them and about 4 cm away were supported the 1.1 cm disk collectors *h* and *b*. The primary electrons are somewhat deflected by the magnetic field of the heating current, and the collectors are inclined as shown in order to give perpendicular incidence of the primaries on them. The glass tubes surrounding the collector leads prevented collection of ions and electrons by the back of the collector and also reduced the electrostatic capacity between leads and ionized gas. It was thought that the latter precaution might prevent a short circuiting of oscillations picked up by the collector and thus increase materially the observed amplitude of oscillation, but no marked effect of this nature is noticeable in a comparison of the results with the two tubes.

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[9] J. S. Webb and L. A. Pardue, *Phys. Rev.* **31,** 1122 (1928) and **32,** 946 (1928).

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B. Detection of oscillations. The oscillations were detected with a zincite-tellurium detector and galvanometer arranged, for most of the work, in a circuit as shown in Fig. 2. The detector was supported by a spring suspension to shield it from mechanical shocks which were found to destroy its sensitivity. The high frequency potential was applied across the two points X and Y, Y being grounded to one side of a filament and also to the metal-screen cage surrounding the apparatus. The inductance L was often only a 10 to 15 cm length of copper wire. The two condensers, which were of 0.0025 μf each, shunted the galvanometer and 60-cycle crystal-calibrating circuits for the high-frequency oscillations, but at the same time allowed known 60-cycle voltages to be conveniently impressed on the crystal for calibrating purposes at frequent intervals. The impressed 60-cycle voltage in millivolts was plotted against the galvanometer deflection on log-log paper, giving a straight line of slope 0.5 which remained practically unchanged over considerable periods of use. This calibration was then used to interpret the galvanometer deflections arising from the gas oscilla-

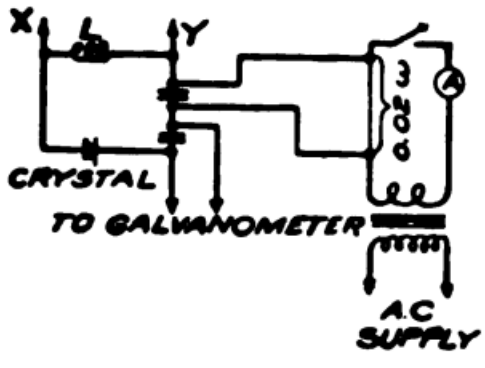

FIG. 2. Crystal detector circuit.

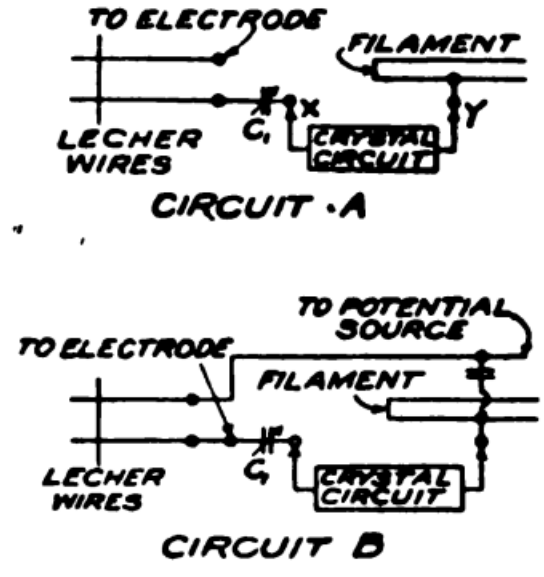

FIG. 3. Oscillation pick-up circuits.

tions as millivolts on the crystal. We found, however, that the same run made at different times always showed the same relative oscillation behavior but in one case the high frequency voltage might appear to be several times that in the other as if a frequency conversion factor for the crystal varied from time to time or with the crystal setting. The comparative amplitudes of a single run thus seem to be reliable, but to make more certain of comparative amplitudes in runs made at various times, high frequency as well as low frequency checking appears to be necessary.

The higher frequencies of the oscillations were determined from their wave-length as measured on a pair of Lecher wires. Circuit *B* of Fig. 3 was used to measure the oscillations of electrodes upon which it was desired to impress a voltage at the same time, while *A* was suited primarily for use with external electrodes.

C. Frequency range of oscillations.—Oscillation frequencies as low as 10^6 and as high as 10^9 cycles per second have been observed. Over a large portion of this range it was not found possible to obtain any resonance phenomena, but

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oscillations were detected throughout so that we lean toward the view that these electric vibrations are irregular, thereby constituting an "electrical noise." Under certain conditions definite frequencies were observed both on the collectors and on external electrodes glued to the tube wall or placed against it.

The frequency range can roughly be divided into three parts, namely from 1 to 100 megacycles, from 100 to 300 megacycles, and from 300 to 1000 megacycles.

D. The range 100 *to* 300 *megacycles.*—A 5 cm circular electrode (Plate I of Fig. 1) was fitted to the spherical tube wall and fastened with sealing wax so that its center was at a level about midway between the two collectors *h* and *b* and it lay opposite the space between collectors and filaments. Point *X* of the detection circuit was connected to this electrode through a variable condenser C_1 (15 $\mu\mu f$ maximum). With an emission from *d* of 25.0 ma, an anode voltage

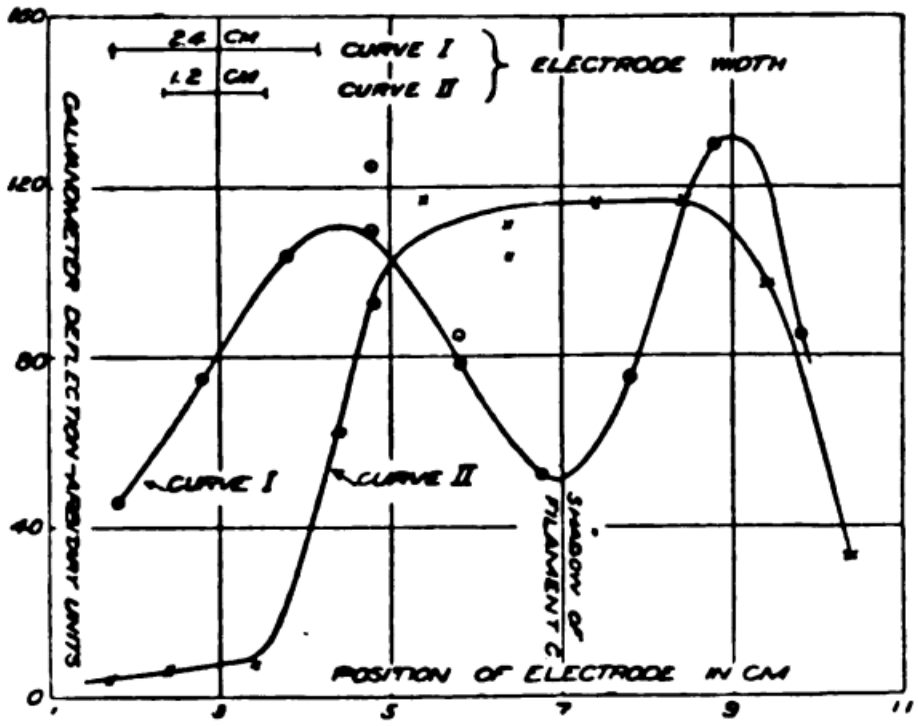

FIG. 4. Oscillations on tube wall.

of 62.7 and the mercury appendix at 20°C a very critical oscillation was found. Practically the whole effect lay within a range of 1.5 ma and 1 v at this temperature. These oscillations did not occur when the appendix was at less than 19° or more than 21°C, the critical voltage decreasing some 5 v in this range. The high frequency voltage at the crystal as measured was about 9 mv. The frequency was 263 megacycles at 19°C, 231 megacycles at 20°C, and 250 megacycles at 21°C, these figures probably being correct to within 2 per cent.

An attempt was made to investigate the effect of additional ionization generated by emission from *g* and also the effect of the magnetic field generated by a current through *c*, but the results are uncertain because of the difficulty of staying on the oscillation peak. It appears that considerable emission from *g* (somewhere above 2 ma) stops the oscillations. Substitution of a smaller pick-up electrode on the tube wall and varying the setting of C_1 failed to change the frequency, but when the Circuit *B* of Fig. 3 was used instead of *A* the 230 megacycle wave disappeared and a 115 to 120 megacycle wave appeared in

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its place. The 230 megacycle wave was not found on the internal electrode using either circuit *A* or *B*. Reversing the filament heating current stopped the oscillations and they were not found in the similar wall position on the opposite side of the tube either when *c* or *d* was used as cathode.

The 115 megacycle wave was first found on *b* in the search for the 230 megacycle wave on an internal electrode. The galvanometer deflection corresponded to 30 to 50 mv. Decreasing the bath temperature from 20°C to 19.5 or less caused the oscillation peak (as the anode voltage was varied) to decrease sharply in magnitude and the frequency to jump to the neighborhood of 900 megacycles. Unlike the 230 megacycle wave, the 115 megacycle persisted over electron emissions from 22 ma to 35, the maximum safe current for the filament. In this

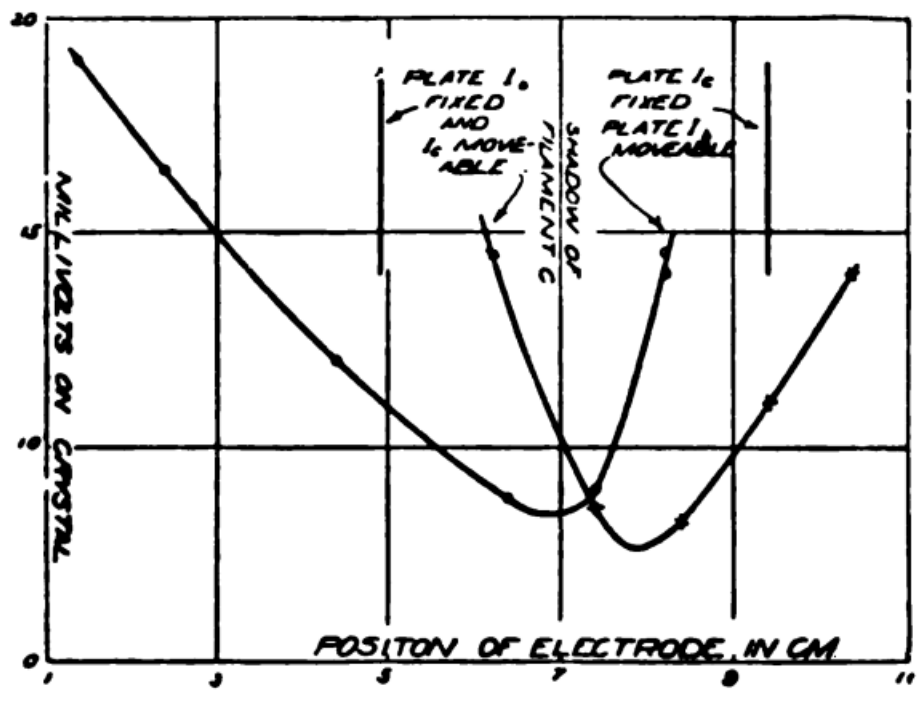
PLATE I_a FIXED AND I_c MOVEABLE
SHADOW OF FILAMENT C
PLATE I_c FIXED PLATE I MOVEABLE
MILLIVOLTS ON CRYSTAL
POSITON OF ELECTRODE, IN CM

FIG. 5. Interference of oscillations on tube wall.

range the wave-length varied from 114 to 125 megacycles. This oscillation was also found on *h* which was out of the direct path of the primaries from the filament, but reversing the heating current in *d* caused the oscillations to disappear.

At this time only the 230 megacycle oscillation had been observed on the tube wall and as it was not found on *b* it was thought that it might arise from some interaction between the electrons from *d* and the sheath about *c*, thus being confined to a comparatively narrow region on the wall. To explore the wall, Plate I was removed and a rectangular electrode 2.4 by 4.1 cm was arranged to slide horizontally along the surface of the tube so that its length was approximately parallel to the filaments.

It was immediately found that the oscillation present was the 115 megacycle one. Curve I of Fig. 4 shows the variation of oscillation amplitude with slider position. Another sliding electrode half as wide (1.2 cm) was then tried. The frequency of oscillation remained unchanged but the distribution on the wall showed a maximum, Curve II of Fig. 4, at the shadow where before it had shown a minimum. The narrower electrode, having the greater resolving power, undoubtedly gives the truer picture of the amplitude distribution. The

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minimum with the larger one must arise from a change of phase centering about the shadow of *c*. As a further test a second narrow electrode was connected in parallel with the first and as symmetrically as possible with respect to the crystal circuit. The curves of Fig. 5 show the effect of moving one while the other was fixed. The minimum followed by a rising branch as the one electrode is moved away from the other indicates that the total phase change is considerably more than 180°.

Apparently the presence of *c* was quite vital to the oscillations so the effect of varying its potential was tried. In the previous work it had been disconnected, its potential then being 2.5 v positive with respect to the cathode. Fig. 6 shows the effect of varying the voltage on *c* with the 1.2 cm external electrode centered on the filament shadow. Nothing critical appears here, but the potential of *c* had a profound effect upon the localization of the oscillation, Fig. 7.

If the oscillation pattern arose from the action of *c* on the passing electrons, a magnetic field which deflected the electrons should affect the oscillation

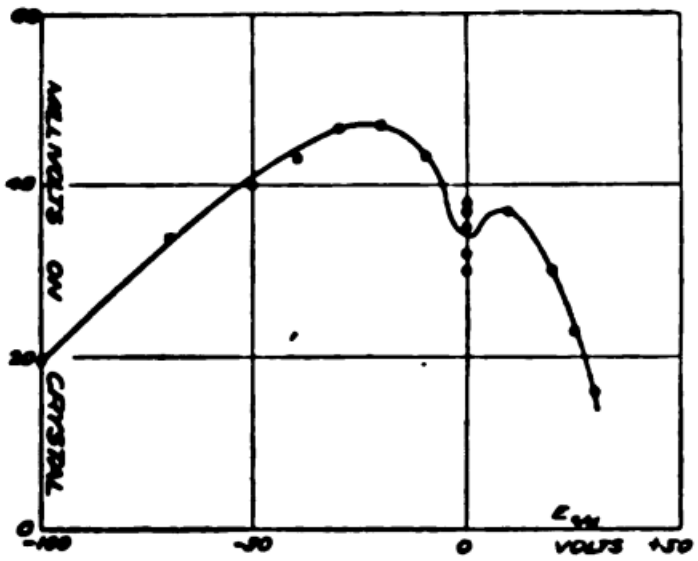

FIG. 6. Effect of potential of *c* on oscillations on tube wall.

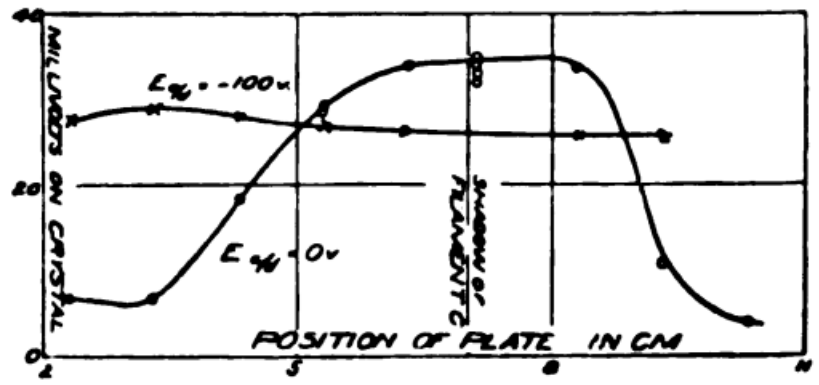

FIG. 7. Effect of potential of *c* on oscillations on tube wall.

pattern equally. When tried, however, it was found that the magnetic field which had been calculated as producing a noticeable deflection caused the 115 megacycle wave to disappear and a 730 megacycle wave to appear in its place.

The frequency of 115 megacycles places that wave among the beam oscillations, but the other evidence is not clean cut. The wall pattern tends to support this point of view and the picking up of the oscillations by a collector even when it was repelling all the primaries is not in opposition to it on account of the low capacity impedance offered by the thin ion sheath to frequencies of this order. But the frequency variation with emission is too slow. A 60 per cent increase in emission from 22 ma should, if the frequency is proportional to the square root of the electron density, cause a 26 per cent increase in frequency from 114 megacycles. Actually a 10 per cent increase was found, the small simultaneous voltage variation being negligible. The 230 megacycle wave may, of course, be related to the 115 megacycle as harmonic to fundamental.

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E. The range above 300 *megacycles.*—These short waves were investigated almost exclusively on the collectors although they were sometimes observed on the external electrode when conditions were not quite right for the longer and more critical oscillations already discussed. The inductance L (Fig. 2) was only that of a short length of wire shunting crystal and condensers and the arrangement of the crystal circuit in conjunction with Lecher wires and electrodes was usually that shown in Circuit *B* of Fig. 3.

One of the most significant features of these oscillations is that the amplitude and frequency was independent of the potential of the receiving collector over the whole range explored, namely from −80 v with respect to the cathode to 5 v above anode potential. Furthermore the frequency was the same whether the primary electrons approached the collector directly, whether they were deflected past by a magnetic field, or whether a collector entirely out of the primary electron stream was used to pick them up. Reversing the

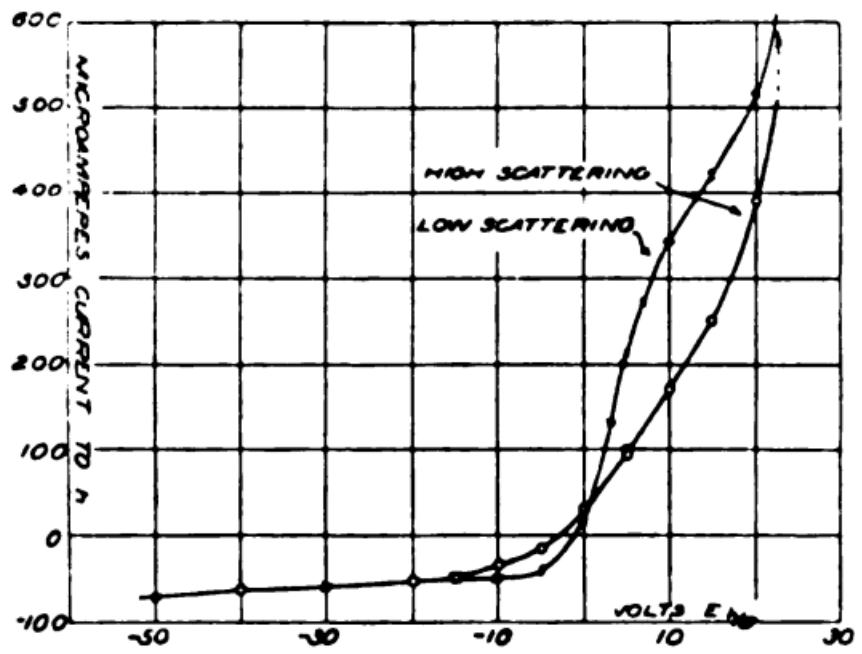

FIG. 8. Volt-ampere characteristics of a collector in the primary electron beam.

heating current in the filament left both frequency and amplitude unchanged. But the situation of the filament used had some effect. With an anode potential of 33 v and an emission of 30 ma a wave-length of 810 megacycles was found when *g* was used as cathode whereas it was 880 megacycles when *d*, one of the two filaments close together, was cathode.

It was natural to seek some correlation between the oscillations and the scattering of primary electrons. Unfortunately the runs meant particularly to elucidate this point are inadequate, first, because compared to the roughness of the present results the range covered was not wide enough; second, because of indeterminateness in the "temperature" of the scattered electrons, and third, because of the absence of high frequency checks on the crystal detector. But some early measurements on the first tube using a 2 mh coil for L in the detector circuit before any frequencies had been determined show that with 10 ma from a 0.0125 cm filament 1 cm long the oscillations disappear

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in the same range of anode potential that the scattering does, namely between 60 and 75 v.

An unexpected behavior of the discharge was discovered in a certain range of current and voltage. At 20 ma emission with 27 v on the anode the arc was stable in either of two conditions which were distinguished, as far as we could ascertain, only by the volt-ampere characteristic of a collector and the frequency of the oscillations. Fig. 8 shows the collector characteristic in each state. The curves are chiefly distinguished by the degrees of scattering shown. In addition the ion currents at large negative voltages are not equal, the ions in the low scattering state preponderating by about 3 per cent in this case. In other cases greater differences in the same direction were found. Transition

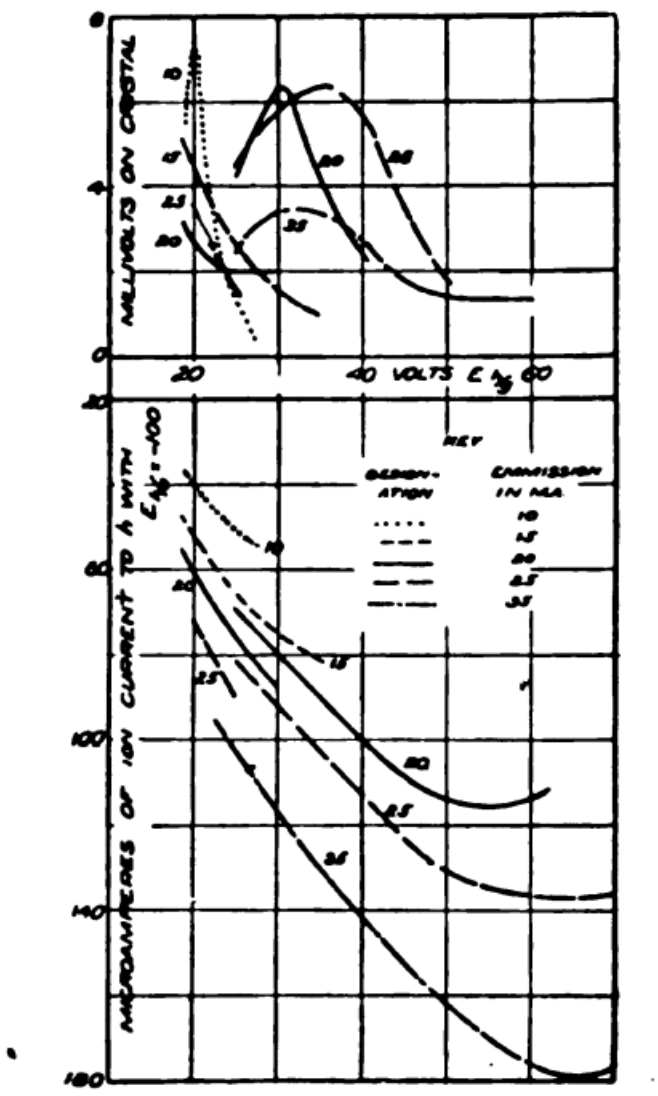

FIG. 9. Oscillation amplitudes and high and low scattering states.

from one state to the otherc ould be accomplished by manipulating a bar magnet. Lowering the collector voltage caused no switch-over but raising it caused a transition from the high to the low scattering state at about 22 volts. Finally, bringing the anode to 27 v from a high value initiated the high scattering, from a low value the low scattering state. In another run, made with 30 v on the anode several transitions occurred back and forth between the two states as the collector voltage was varied from −10 to +20 v. At the same time the oscillation amplitude changed abruptly, but the gaps in both sets of characteristics could readily be bridged so as to bring out the continuity of each state.

The frequency with the low scattering was almost twice that found with the high. Sometimes the arc showed less tendency to stay in one or the other

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state so the low scattering measurement was made with 26 v on the anode, the high scattering with 28 v. The measured frequencies were 1080 and 660 megacycles respectively.

Fig. 9, which shows the variation of oscillation amplitude and positive ion current with anode voltage for different electron emissions, clearly exhibits the transition from the low scattering state at low emissions to the high scattering at high emissions. It is seen that at 10 and 15 ma only low electron scattering was found, that at 20 and 25 ma both low and high scattering appeared, and that 35 ma showed only high scattering.

The frequency of these oscillations places them among the plasma-electron oscillations. Their presence on an internal electrode irrespective of the path of the primary beam and their tendency to appear on the tube wall when other oscillations stop tend to confirm this view. The attempt was made to check them against Eq. (6), but there was no way of finding the absolute value of the

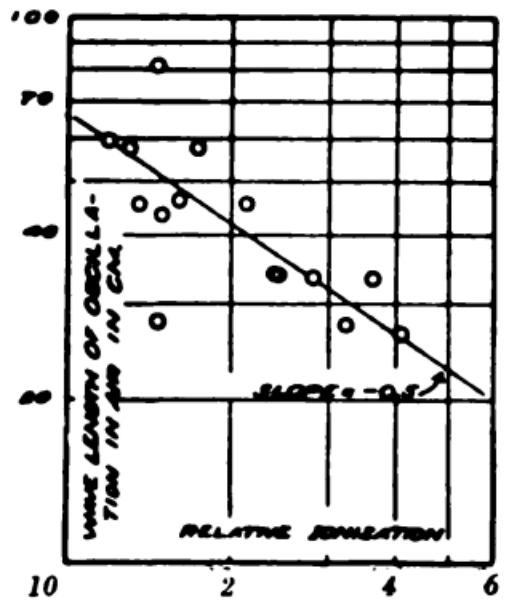

FIG. 10. Oscillation frequency and ionization density.

ion density in the present tube because of large edge effects on the electrodes. The best we could do was to calculate relative values based on an elaborate series of measurements. Partial failure of some internal checks in these calculations show that too much reliance is not to be placed on them. Fig. 10 shows the grouping of the observed oscillations when plotted with wave-length in air against a scale of the relative ionizations. The simple theory requires that they lie in a line of slope $-1/2$. With the exception of two points out of the fifteen their deviations from the line drawn are within the accuracy of the ionization determinations. The deviations of those two may arise from some of the secondary theoretical factors.

F. The range below 100 *megacycles.*—While investigating the effect of Hg pressure on the 230 megacycle wave the appendix temperature was lowered to 15°C. With the emission still at 25 ma the oscillation maximum had apparently moved to 72.6 v and was much less critical. Further tests showed, however, that the frequency was considerably lower than 230 megacycles and using a calibrated series tuned circuit in series with the crystal circuit it was

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located between 50 and 60 megacycles. At first sight it would appear possible to ascribe this frequency to the computed 18 megacycle oscillation for beam electrons together with the Doppler Effect. The frequency, ν_d, arising from the Doppler Effect is

$$\nu_d = (v_w + v_e)/\lambda$$

where v_w is the phase velocity of the waves with respect to the oscillating medium (primary electrons) and ν_e is the velocity of the medium with respect to the observer. Since the beam electrons oscillate only at their natural frequency ν_e, we can replace v_u/λ by this quantity, and solving for λ find

$$\lambda = v_e/(\nu_d - \nu_e)$$

Using the values 55 and 18 megacycles for ν_d and ν_e respectively, and 5.1×10^8 cm sec^{-1} for v_e, we find λ to be 14 cm. This does not seem to be unreasonable, for it might in some way be connected with the 18 cm bulb diameter. Imagining ourselves to be travelling with the primary electrons, however, we see that each plasma electron takes $14/(5.1 \times 10^8) = 2.8 \times 10^{-8}$ sec to traverse a wave-length. The oscillation period is only 5.5×10^{-8} sec, which taken in conjunction with the enormous preponderance of plasma electrons makes it impossible to think that these electrons do not modify the beam oscillations tremendously. In fact, a theory of infinite trains of plane oscillations as worked out by Mr. H. M. Mott-Smith, Jr. indicates that for a stationary observer the plasma-electron frequency ν_e is the minimum to be expected, the lower frequency of the beam oscillations (as seen from the beam) being more than compensated in the Doppler effect by the shortness of wave-length required to avoid interaction with plasma electrons. Thus it seems to be impossible to identify the 55 with the theoretical 18 megacycle oscillation.

Some of the early work with an external electrode on the first tube had, we thought, indicated the presence of 15 or 20 megacycle oscillations. In searching for these Plate I was used as pick-up, C_1 was taken out of the detection circuit, Fig. 3, and a 2 mh coil used for L. An oscillation peak of some 90 mv was located at 25 ma emission and 63 v on the anode — practically the condition for the 230 or 115 megacycle oscillation. Tuned circuits covering successive ranges from 100 megacycles down were tried in various ways in the attempt to locate the frequency. Resonance was finally established at 1.38 megacycles although considerable voltage appeared across the crystal throughout the whole range.

Between 26 and 42 v on the anode the oscillations also reach the neighborhood of 90 mv but the attempts to find the wave-lengths met with no particular success.

We attempted to check the 1.38 megacycle wave by using a tuned radio amplifier. For this purpose a Radiola 17 having the range 550 to 1450 kc was set up in the cage and an antenna wire was brought near Plate I. Unfortunately the 1380 kc wave had become so elusive that even the previous measurements

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of its wave-length could not be repeated so we were unable to check this. At this time the amplifier gave a noise maximum in the middle of its scale. This was not sharp, the noise extending over most of the range. Slight changes in the tube conditions caused large differences in the way the amplifier responded, sometimes several sharp noise peaks appearing, at others a single peak, and at still others only a very dull maximum. This, it seems to us, shows conclusively that there is often "a continuous spectrum" of electric oscillations in this long-wave range.

The strong critical oscillation at 1380 kc lies in the neighborhood of the theoretical short-wave limit at 1500 kc calculated above. Whether this was actually the short-wave limit is not known, for frequencies higher than this were not sought with an amplifier.

G. Future experiments.—The experiments described are in general accord with the outlined. A closer checking and further development of the theory, particularly with respect to the cause and critical nature of some of the oscillations, await experiments under essentially simpler conditions. With this in view we are continuing the experiments with a cylindrical tube containing a cylindrical collector at the tube wall and an axial filament. The ionization can be controlled independently of these electrodes by a hot cathode and an anode at the two ends of the tube.

In addition to Sir J. J. Thomson's letter which has already been mentioned, we gratefully acknowledge the aid and suggestions of Mr. H. M. Mott-Smith, Jr. in the development of the theory and of Mr. S. Sweetser in the experimental work.

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THE INTERACTION OF ELECTRON AND POSITIVE ION SPACE CHARGES IN CATHODE SHEATHS

Physical Review
Vol. XXXIII, No. 6, June (1929).

ABSTRACT

Effect of positive ions generated at a plane anode upon the space charge limitation of electron currents from a parallel cathode. — Mathematical analysis shows that single ions emitted with negligible velocity permit 0.378 $(m_p/m_e)^{1/2}$ additional electrons to pass; but with an unlimited supply of ions the electron current approaches a limiting value 1.860 times that which flows when no ions are present, and the electron current is then $(m_p/m_e)^{1/2}$ times the ion current, both currents thus being limited by space charge and the electric field being symmetrically distributed between the electrodes. Single ions introduced into a pure electron discharge at a point 4/9ths of the distance from cathode to anode produce a maximum effect, 0.582 $(m_p/m_e)^{1/2}$, in increasing the electron current. These conditions apply to a cathode emitting a surplus of electrons surrounded by ionized gas. The cathode sheath is then a double layer with an inner negative space charge and an equal outer positive charge, the field being zero at the cathode and at the sheath edge. The electron current is thus limited to $(m_p/m_e)^{1/2}$ times the rate at which ions reach the sheath edge. If ions are generated without initial velocities uniformly throughout the space between two plane electrodes, a parabolic potential distribution results. If the total ion generation exceeds 2.86 times the ion current that could flow from the more positive to the more negative electrode, a potential maximum develops in the space. Electrons produced by ionization are trapped within this region and their accumulation modifies the potential distribution yielding a region (named plasma) in which only weak fields exist and where the space charge is nearly zero. The *potential distribution in the plasma*, given by the Boltzmann equation from the electron temperature and the electron concentrations, *determines the motions of the ions* and thus fixes the rate at which the ions arrive at the cathode sheath. The *anode sheath* is usually also a positive ion sheath, but with anodes of small size a detached double-sheath may exist at the boundary of the anode glow. In discharges from *hot cathodes in gases* where the current is limited by resistance in series with the anode, the electron current is space-charge-limited, being fixed by the rate of arrival of ions at the cathode sheath. Thus the cathode drop is fixed by the necessity of supplying the requisite number of ions to the cathode. The *effect of the initial velocities of the ions and electrons* that enter a double-sheath from the gas is to decrease the electron current by an amount that varies with the voltage drop in the sheath. A nearly complete theory of this effect is worked out for plane electrodes. A detailed study is made of the *potential distribution in the plasma and near the sheath edge* for a particular case and the conclusion is drawn that the velocities of the ions that enter the sheath can be calculated from the electron temperature if the geometry of the source of ionization is given.

Experiments with double sheaths. — With large cathodes coated with barium oxide in low pressure mercury vapor, simultaneous measurements showed that the electron current density was independent of the cathode temperature and was from 140 to 200 times the ion current density, this ratio being independent of the intensity of ionization and of the gas pressure but varying slowly with the voltage drop in the cathode sheath, in good accord with the theory. The obser-

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ved ratio, however, was about 40 per cent of that calculated, this discrepancy being probably due to non-uniformity in the cathode coating. Similar results were obtained with double sheaths on wire type cathodes, the ratio of the electron current to the ion current through the sheath ranging from 450:1 at high current densities to 2000:1 and more at very low currents, this variation being in agreement with the approximate theory developed for cylindrical sheaths. In these experiments *two cathodes* were used; one at rather large negative voltage to produce any desired intensity of ionization, while from the volt-ampere characteristics of the other cathode the space-charge-limited electron currents were measured. The ion currents were measured either by cooling the test cathode so that it emitted no electrons, or by the use of an auxiliary ion collector.

THE MAXIMUM electron current that can flow from a given hot cathode to an anode in high vacuum is limited by the space charge of the electrons.[1] If even a small amount of gas is present and the applied anode voltage is appreciably higher than the ionizing potential, the positive ions formed tend to neutralize the electron space charge and thus allow the current to increase until, with sufficient gas, the current becomes limited only by the electron emission from the cathode as determined by its temperature.

Since in a given electric field the ions move hundreds of times slower than the electrons, the rate at which the ions need to be produced in order to neutralize the space charge is usually less than one per cent of the rate at which the electrons flow from the cathode. In formulating a quantitative theory for calculating the increase in electron current produced by a given amount of ionization we meet the difficulty that the ions are produced at different points within the gas and therefore are not all moving with the same velocity. Then, too, the probability of ionization as a function of the electron velocity must be known. The problem thus becomes so complicated that it seems hardly worth while to attempt a general solution.

About 13 years ago the writer derived the equations, given in the present paper, for the space charge problem between parallel planes where the cathode emits a surplus of electrons and the anode emits positive ions without initial velocities. The results, although interesting, did not seem to be applicable directly to experimental conditions and therefore the results were not published. Some years later,[2] however, by the discovery that all caesium atoms which strike a tungsten surface at 1300°K are converted into ions, it became practicable to generate positive ions at the anode in any desired number, and thus obtain the conditions which were assumed in the theory.

Still more recently in a study of gaseous discharges at low pressures[3] it was found that the space charge equations could be applied to the positive ion currents flowing to negatively charged collectors.

If we consider a negatively charged hot collector or in fact any hot cathode in a gas we see that there are present in the positive ion sheath electrons as

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[1] Langmuir, *Phys. Rev.* **2,** 450 (1913); *Phys. Zeits.* **15,** 348, 516 (1914).

[2] Langmuir and Kingdon, *Proc. Roy. Soc.* **A107,** 61 (1925).

[3] Langmuir and Mott-Smith, *Gen. Elect. Rev.* **27,** 449, 538, 616, 762, 810 (1924), and *Phys. Rev.* **28,** 727 (1926).

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well as ions. In the present paper it will be shown that the theory which was developed 13 years ago is now applicable to the calculation of the properties of these double sheaths.

Jaffe[4] has attempted to develop a theory of the effect of small amounts of gas ionization on currents limited by space charge in gases at low pressures. However, he based his entire treatment upon the inadmissible assumption that as many ions recombine in each element of volume as are produced by ionization within that volume. We now know that even with the high current densities in a mercury vapor arc carrying amperes, recombination of ions in the gas is negligible, compared to the removal of the ions by diffusion to the walls and to the electrodes. Thus the equations which Jaffe derived are not even approximately correct.

Theory of the Effect of Ions on Space Charge Currents Between Parallel Planes

Consider an infinite plane cathode C at zero potential, and a similar parallel plane anode A at the potential V_A and at a distance a from C. Let the cathode emit a surplus of electrons without appreciable initial velocities. There will thus be an infinite concentration of electrons at the cathode surface and the potential gradient will be zero, but a finite electron current, I_0 per unit area, will flow to the anode, this current limited by space charge being given by the equation[1]

$$I_0 = \frac{(2)^{1/2}}{9\pi}\left(\frac{e}{m_e}\right)^{1/2}\frac{V_A^{3/2}}{a^2} \tag{1}$$

where e is the charge and m_e the mass of the electrons.

Let us consider the effect of introducing positive ions without initial velocities, uniformly distributed over a plane B which is at a distance b from C. Between B and C there is thus an ion current I_p per unit area. Because of the partial neutralization of the electron space charge, the electron current from the cathode will increase to a new value, say I_e per unit area. We assume that the ions and electrons do not collide with gas molecules nor with each other and that no appreciable number of ions or electrons is lost by recombination during the passage between the electrodes. Our problem is to determine how the electron current I_e depends on I_p the positive ion emission and on b the location of the source of ions.

Let v_e be the velocity of the electrons at any point P which is at a distance x from the cathode, ϱ_e the electron space charge density at P. The corresponding quantities for the positive ions are denoted by the subscript p. The signs of all these quantities will be taken as positive.

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[4] George Jaffe, *Ann. d. Physik* **63**, 145–174 (1920).

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Then

$$\varrho_e v_e = I_e \quad \text{and} \quad \varrho_p v_p = I_p \tag{2}$$

and assuming the ions have unit charge

$$\tfrac{1}{2} m_e v_e^2 = V_e' \quad \text{and} \quad \tfrac{1}{2} m_p v_p^2 = (V_B - V)e \tag{3}$$

where V is the potential at the point P and V_B the potential at B.

Poisson's equation gives

$$d^2V/dx^2 = 4\pi(\varrho_e - \varrho_p) \tag{4}$$

We may eliminate ϱ and v by Eqs. (2) and (3) and then after substituting

$$\alpha = (I_p/I_e)(m_p/m_e)^{1/2} \text{ and } \Phi = V/V_A \tag{5}$$

we obtain

$$\frac{d^2\Phi}{dx^2} = 2(2)^{1/2}\pi\left(\frac{m_e}{e}\right)^{1/2} \frac{I_e}{V_A^{3/2}} [\Phi^{-1/2} - \alpha(\Phi_B - \Phi)^{-1/2}] \tag{6}$$

Combining Eqs. (1) and (6) and substituting

$$\lambda = x/a \tag{7}$$

we have

$$\frac{d^2\Phi}{d\lambda^2} = \frac{4}{9} \frac{I_e}{I_0} [\Phi^{-1/2} - \alpha(\Phi_B - \Phi)^{-1/2}] \tag{8}$$

Since the electron current is limited by space charge, we impose the condition $dV/dx = 0$ when $x = 0$, or in other words

$$d\Phi/d\lambda = 0 \quad \text{when} \quad \lambda = 0 \tag{9}$$

Integration of Eq. (8) then gives

$$d\Phi/d\lambda = (4/3)(I_e/I_0)^{1/2}[\Phi^{1/2} + \alpha\{(\Phi_B - \Phi)^{1/2} - \Phi_B^{1/2}\}]^{1/2} \tag{10}$$

Ions Emitted From Anode, $\Phi_B = 1$

According to Eq. (10) the potential gradient at the surface of the anode, ($\Phi = \Phi_B = 1$), is proportional to $(1-\alpha)^{1/2}$ and so becomes imaginary if $\alpha > 1$. When $\alpha = 1$ the potential gradient at the anode is zero and the positive ion current as well as the electron current is thus limited by space charge.

It appears, therefore, that even an unlimited supply of positive ions available at the anode is not capable of neutralizing the electron space charge, for the positive ion current cannot become more than a definite fraction of the electron current, this fraction (according to Eq. 5, when $\alpha = 1$) being equal to the square root of the ratio of the mass of the electron to that of the ion.

Examination of Eqs. (8) and (10) shows that when $\Phi_B = 1$ and $\alpha = 1$ the equations remain unchanged in form if we substitute $1-\Phi$ in place of Φ. Thus the curve representing the potential distribution between the cathode and anode is symmetrical about its central point ($\lambda = \frac{1}{2}, \Phi = \frac{1}{2}$). Between the cathode and this central point there is an excess of negative space charge, while from the central point to the anode there is an excess of positive charge.

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To calculate the potential distribution we integrate Eq. (10) after placing $\Phi_B = 1$

$$\lambda = (3/4)(I_0/I_e)^{1/2}\int_0^{\Phi} [\Phi^{1/2}+\alpha\{(1-\Phi)^{1/2}-1\}]^{-1/2}\, d\Phi. \tag{11}$$

The ratio I_e/I_0 is found by observing that $\Phi = 1$ when $\lambda = 1$, thus

$$(I_e/I_0)^{1/2} = (3/4)\int_0^{1} [\Phi^{1/2}+\alpha\{(1-\Phi)^{1/2}-1\}]^{-1/2}\, d\Phi \tag{12}$$

Table I gives values of λ obtained from Eq. (11) by numerical integration. The values of I_e/I_0 which were used in these calculations as found by Eq. (12) are given for various values of α in the next to last horizontal line of the table.[5]

From the values of I_e/I_0 we see that the electron current increases as more positive ions are emitted from the anode until the positive ion current also becomes limited by space charge. When this occurs the electron current and the positive ion current are each 1.860 times as great as the currents of electrons or ions that could flow (with the same applied potentials) if carriers of the opposite sign were absent.

It is interesting to inquire how large is the effect of single positive ions emitted from the anode, in causing an increased electron flow from the cathode. By differentiating Eq. (12) with respect to α and then placing $\alpha = 0$, and $I_e = I_0$ we find in terms of Gamma functions

$$dI_e/d\alpha = [3-3\Gamma(1.25)\Gamma(1.5)/\Gamma(1.75)]I_0 = 0.378I_0 \tag{13}$$

or by Eq. (5)

$$dI_e/dI_p = 0.378(m_p/m_e)^{1/2} \quad \text{for} \quad \alpha = 0 \tag{14}$$

A similar calculation for the case $\alpha = 1$ involves a numerical evaluation of the resulting integral giving

$$dI_e/dI_p = 3.455(m_p/m_e)^{1/2} \quad \text{for} \quad \alpha = 1$$

A plot of I_e/I_0 as function of α from the data of Table I shows that the slope of the curve increases gradually from 0.378 at $\alpha = 0$ up to 3.455 at $\alpha = 1$. Thus the effectiveness of the ions in raising the electron current increases as the field strength decreases in the region where they originate, but only up to a certain limiting value. Of course when $\alpha = 1$ the further increase in the electron current is stopped by the space charge limitation of the ion current.

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[5] In carrying out these calculations it was found convenient to replace Φ by a new variable σ such that $\Phi = \sigma^4/^3$. By so doing the infinite value of the integrand that occurs when $\Phi = 0$ is avoided. When $\alpha = 1$ and $\Phi = 1$ a similar difficulty still occurs but the value of λ in the range $\Phi = \frac{1}{2}$ to 1 can be obtained from those calculated in the range $\Phi = 0$ to $\frac{1}{2}$ by making use of the fact already noted that in this case λ is symmetrical about the point $\Phi = \frac{1}{2}$.

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The square root of the ratio of the masses of the ions and the electrons is 607 for mercury vapor, 271 for argon, and 60.8 for hydrogen, and therefore each positive ion of these gases liberated at the anode will increase the number of electrons that cross the space by 229, 102 or 23 respectively in the case of a pure electron discharge ($a = 0$).

TABLE I

Potential Distribution between Plane Cathode Emitting Surplus of Electrons and Parallel Plane Anode which Emits Given Numbers of Ions. Table of values of λ, the fraction of the distance to the anode, at which the potential is a given fraction Φ of the anode potential (zero potential at cathode)

$a = (I_p/I_e)(m_p/m_e)^{1/2}$; $\lambda = x/a$							
Φ	$a = 0$	$a = 0.2$	$a = 0.4$	$a = 0.6$	$a = 0.8$	$a = 0.9$	$a = 1.0$
0	0	0	0	0	0	0	0
0.02	0.0532	0.0513	0.0491	0.0467	0.0438	0.0419	0.0396
0.05	0.1057	0.1022	0.0981	0.0934	0.0879	0.0842	0.0798
0.1	0.1778	0.1723	0.1661	0.1588	0.1498	0.1437	0.1367
0.2	0.2991	0.2911	0.2823	0.2714	0.2573	0.2477	0.2363
0.3	0.4054	0.3962	0.3855	0.3721	0.3546	0.3423	0.3274
0.4	0.5030	0.4932	0.4815	0.4667	0.4467	0.4324	0.4146
0.5	0.5946	0.5847	0.5731	0.5580	0.5371	0.5218	0.5000
0.6	0.6817	0.6723	0.6612	0.6461	0.6245	0.6080	0.5854
0.7	0.7653	0.7570	0.7471	0.7332	0.7123	0.6958	0.6726
0.8	0.8459	0.8395	0.8314	0.8198	0.8016	0.7861	0.7637
0.9	0.9240	0.9201	0.9149	0.9074	0.8940	0.8813	0.8633
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
I_e/I_0	1.0000	1.0839	1.1872	1.3237	1.5186	1.6644	1.8605
a/a_0	1.0000	1.0411	1.0896	1.1505	1.2323	1.2901	1.3640

The Source of Ions is at a Plane Between Cathode and Anode $\Phi_B < 1$

Let us consider the effect produced by ions that start from a plane B which lies between C and A. In the region between C and B, Eqs. (6) and (10) are applicable and thus the value of $d\Phi/d\lambda$ at B is found by putting $\Phi = \Phi_B$ in Eq. (10). Equation (6) is applicable also in the region between B and A but there $a = 0$ for there is no ion current. Thus by integration we obtain the two equations

$$4(I_e/I_0)^{1/2}\lambda_B = 3\Phi_B^{3/4}\int_0^1 [\mu^{1/2}+a\{(1-\mu)^{1/2}-1\}]^{-1/2}\,d\mu$$

and (15)

$$4(I_e/I_0)^{1/2}(1-\lambda_B) = 3\int_{\Phi_B}^1 [\Phi^{1/2}-a\Phi_B^{1/2}]^{-1/2}\,d\Phi$$

The constant of the first integration for the second equation was chosen to make $d\Phi/d\lambda$ at $\Phi = \Phi_B$ the same as for the first equation.

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Differentiating Eqs. (15) with respect to α, putting $\alpha = 0$, adding the resulting equations and combining with Eq. (5) gives rigorously[6]

$$\frac{dI_e}{dI_p}\left(\frac{m_e}{m_p}\right)^{1/2} = 3\Phi_B^{1/2} - 2.622\Phi_B^{3/4}. \tag{16}$$

This equation, which reduces to Eq. (14) if $\Phi_B = 1$, allows us to calculate the number of additional electrons that can flow in a pure electron discharge ($\alpha = 0$) if single positive ions are introduced *at any point* in the space between the electrodes. The values given in Column 3 of Table II were calculated by Eq. (16); the second column represents λ_B the fraction of the distance from cathode to anode at which the ions originate. From Eq. (1) we see that $\lambda_B = \Phi_B^{3/4}$.

TABLE II

The Increase in Electron Current Caused by Ions Originating at Various Positions between Cathode and Anode, $\alpha = 0$. Initial velocities neglected

Φ_B	λ_B	$\frac{dI_e}{dI_p} \cdot \left(\frac{m_e}{m_p}\right)^{1/2}$
0.00	0.00	0.0
0.001	0.0056	0.080
0.01	0.0316	0.217
0.1	0.177	0.483
0.2	0.300	0.554
0.338	0.444	0.582 max.
0.5	0.595	0.561
0.6	0.683	0.534
0.7	0.765	0.502
0.8	0.846	0.464
0.9	0.925	0.419
1.0	1.000	0.378

The ions have a maximum effect in increasing the electron current when they are introduced at a point which is 4/9 of the distance from cathode to anode. If a trace of gas is present and the voltage is so high that we may assume the probability of ionization per cm of electron path is uniform, we find readily from Eq. (16) (integrating with respect to λ) that the average value of dI_e/dI_p is 0.489 $(m_p/m_e)^{1/2}$.

If the probability of ionization is greater near the end of the path, as in the case of low anode voltages, the coefficient will lie between 0.489 and 0.378.

Low Pressure Discharges With Hot Cathode

Let us consider for a moment the phenomena that are observed as we pass from a pure electron discharge to a discharge at low gas pressure in which there is abundant gas ionization. When the ions originate at the anode or

[6] The coefficient 2.622 is equal to 3 minus the coefficient 0.378 as given in Eq. (14).

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at a definite plane between the electrodes we have been able to follow through the effects produced by even an unlimited supply of ions. But when the ions are produced throughout all or a large part of the space between the electrodes, we have been able to analyze only the effects produced by a very small total number of ions ($\alpha = 0$). To understand the typical characteristics of gas discharges, however, we must devise methods of treating the problem involving a large intensity of ionization throughout a volume. The nature of the problem will best be realized by considering briefly some experimental observations.

Suppose, for example, we have a hot tungsten cathode (at zero volts) capable of emitting 50 ma in a bulb containing mercury vapor saturated at room temperature. With an anode at 10 volts the current is limited by electron space charge and is practically the same as in the absence of mercury vapor. Beginning at the ionization voltage (10.4 volts) the current increases with the anode voltage more rapidly than in good vacuum until at a voltage of 15 to 25 volts depending on the vapor pressure, and the geometry of the tube, the current rises abruptly to the saturation value corresponding to the cathode temperature.

In this second state of the discharge practically the whole of the voltage difference from cathode to anode is concentrated in a cathode sheath in which there is a positive ion space charge; the rest of the volume is nearly field-free, the space charge of the positive ions being neutralized by low-velocity or "ultimate electrons" which accumulate in this space until their concentration is hundreds or thousands of times greater than that of the primary electrons from the cathode.

By means of a sufficiently large resistance in series with the anode it is possible to observe points on the negative resistance part of the current voltage curve that lies between the parts corresponding to the two regions we have just considered. In this transition region the current is limited by the electron and ion space charges in a double layer or double sheath on the cathode.

In order to understand the formation of this double sheath, the final neutralization of space charge and the accumulation of the ultimate electrons, we will first consider the following problem.

Potential Distribution and Current Flow Resulting from the Production of Ions Uniformly throughout the Volume Between Two Planes

Consider two parallel plane electrodes A and C separated by the distance a and let V_c be the potential of C taking that of A to be zero. We assume that S ions of charge e are generated per unit time in each unit volume, and for the present will assume that no electrons are generated by this ionization.

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We will first consider the case in which there is no maximum potential in the space between A and C. We will take V_c negative so that the ions move towards C. Then the ion current density I at any plane at a distance x from A is

$$I = Sex \tag{17}$$

but this current is composed of ions having widely different velocities depending on the potentials of their points of origin. The space charge ϱ at any point at a distance x from A where the potential is V_1 is thus given by the integral $\int (1/v)dI$ where v, the velocity of an ion which originated at a point of potential V, is found from

$$\tfrac{1}{2}m_p v^2 = (V - V_1)e \tag{18}$$

Thus

$$\varrho = S(em_p/2)^{1/2} \int_0^{x_1} (V - V_1)^{-1/2}\, dx. \tag{19}$$

The problem is now to find V as a function of x which satisfies this integral equation simultaneously with Poisson's equation. By trial the following is found to be a particular solution

$$V = V_c x^2/a^2 \tag{20}$$

which we may now prove as follows.

Substituting this value of V and the corresponding value of V_1 in Eq. (19) we obtain

$$\varrho = \tfrac{1}{2}\pi I_a [m_p/(-2eV_c)]^{1/2} \tag{21}$$

where

$$I_a = S_1 ea \tag{22}$$

is the ion current which reaches C, and S_1 is the particular value of S which corresponds to the potential distribution assumed in Eq. (20).
Differentiation of Eq. (20) gives

$$dV/dx = 2V_c x/a^2 \tag{23}$$

and a second differentiation and combination with Poisson's equation gives

$$d^2V/dx^2 = 2V_c/a^2 = -4\pi\varrho \tag{24}$$

whence

$$\varrho = -V_c/2\pi a^2 \tag{25}$$

We see that both Eqs. (21) and (25) give ϱ independent of x or a uniform space charge between the planes. Equating the two expressions for ϱ and solving the resulting equation for I_a gives

$$I_a = (2e/m_p)^{1/2}(-V)^{3/2}/\pi^2 a^2. \tag{26}$$

If, as before, I_0 is the current calculated by the ordinary space charge equation, Eq. (1), we see that

$$I_a = (9/\pi)I_0 = 2.865 I_0 \tag{27}$$

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We now recognize that the parabolic potential distribution assumed in Eq. (20) is a solution of our problem only when the rate of ionization S has the particular value S_1 given by Eq. (22) and where I_a is given by Eq. (26). Curve 1 in Fig. 1 illustrates this parabolic distribution. It will be noted by Eq. (23) that the potential gradient is zero at A, i.e. at $x = 0$.

If $S < S_1$ the potential distribution curve will lie between the straight line 0 and Curve 1 in Fig. 1, but will not be parabolic. On the other hand, if $S > S_1$ it is evident that there will be a potential maximum between A and C and that Eq. (26) can then be applied separately to the two branches of

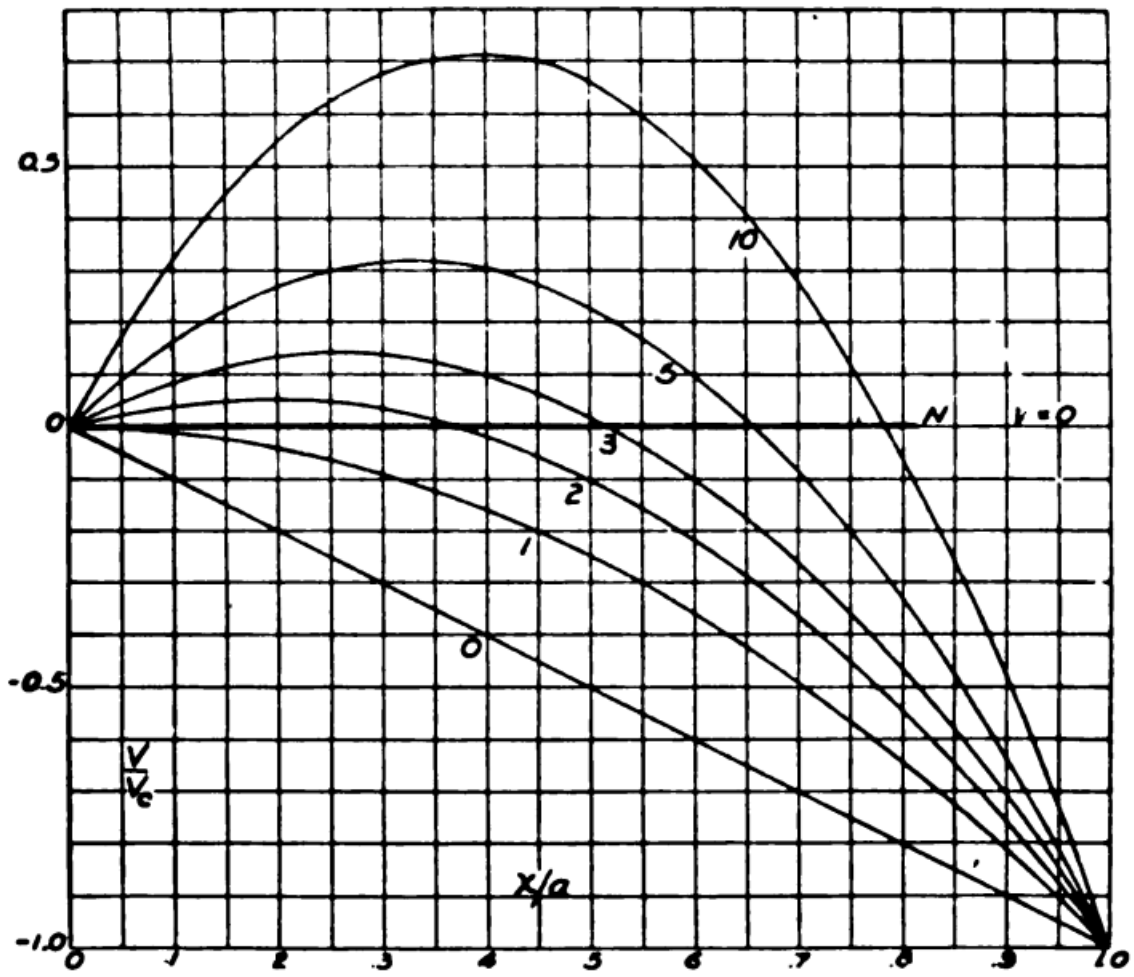

FIG. 1. Potential distributions between plane electrodes when ions are generated uniformly between them.

the curve on the opposite sides of the maximum. The potential distribution curve is thus still a parabola but the origin is no longer at A. The curves marked, 2, 3, 5 and 10 in Fig. 1 have been calculated for values of S equal respectively to 2, 3, 5 and 10 times S_1. The equation of these parabolas is

$$\frac{V}{V_c} = \frac{x}{a}\left[1 - \left(\frac{S}{S_1}\right)^{2/3}\left(1 - \frac{x}{a}\right)\right]. \tag{28}$$

Effect of the Electrons Generated by Ionization

If the positive ions in the foregoing problem are produced by the ionization of a gas an equal number of electrons will be generated simultaneously. If $S < S_1$ there will be no potential maximum between the electrodes so that these electrons will flow to the electrode A without having appreciable effect

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on the space charge, for it would take an electron current hundreds of times greater than I_a to neutralize the positive ion space charge due to I_a.

The situation is very different, however, if $S > S_1$ for there is then a tendency to develop a potential maximum as illustrated in Fig. 1. In any region at a potential higher than that of both electrodes the low velocity electrons produced by ionization will accumulate until they nearly neutralize the positive space charge. The potential of the region which would otherwise be above anode potential (above line ON in Fig. 1) is thus lowered to a value at which the electrons, in virtue of their initial velocities, can just escape to the anode as fast as they are produced. The accumulation of the ultimate or low velocity electrons is greatly favored by the smallness of the field available for drawing away the positive ions. In general there will still be a maximum potential in the space but this usually exceeds the anode potential by not more than a volt or so, and thus the ions flow in nearly equal numbers to anode and cathode, while the electrons go to the anode only. Any calculation of the exact potential distribution must involve some knowledge of the velocity distribution of the electrons and ions.

We see from Fig. 1 that when S/S_1 is as great as 10, the fields at the cathode and anode which are necessary to draw these ion currents, are very large.

These regions of strong field due to space charge which cover the electrodes will be referred to as the *sheaths*. The relatively field-free regions between the sheaths where the positive and negative space charges are nearly balanced will be called the *plasma*. We shall find in general that these two regions are rather distinct and have very different properties. Let us first consider some of the characteristics of the plasma.

Random Currents and Potential Distribution in the Plasma

Experiment has shown[3] that the ultimate electrons in the plasma usually have a velocity distribution corresponding closely with that of Maxwell. Thus we may define the electron velocities in terms of an electron temperature T_e. Through any imaginary plane there is a certain current density I_e of electrons passing from one side to the other and a nearly equal current in the reverse direction, this current being related to n_e the number of electrons per unit volume by the equation

$$n_e = (2\pi m_e/kT_e)^{1/2} I_e/e = 4.03 \times 10^{13} I_e/T_e^{1/2} \tag{29}$$

if I_e is expressed in amperes per cm^2 and n in cm^{-3}, k being the Boltzmann constant 1.372×10^{-16} erg per degree.

The ion velocity distribution is not so easily determined and is far less accurately Maxwellian. Measurements with perforated collectors have shown

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that the normal components of velocity of the ions that reach the edge of a cathode are roughly that of a Maxwellian distribution[7] corresponding to a temperature T_p which is about half that of the electron, i.e., $T_p = \frac{1}{2}T_e$. In these low pressure discharges the ions probably acquire practically all the kinetic energy they possess from the electric fields within the plasma. The momentum of the electrons is so small that ionizing collisions of electrons with gas molecules cannot impart to the ions appreciable kinetic energy.

All the ions which reach the edge of a cathode sheath pass to the electrodes and thus only half of the ions corresponding to a Maxwellian distribution T_p can be present in the plasma near the sheath edge, so that the directions of the ions can be distributed only over a hemisphere. Thus if I_s is the positive ion current density reaching the sheath edge, and therefore the electrode, we have by analogy with Eq. (29)

$$n_p = \tfrac{1}{2}(2\pi m_p/kT_p)^{1/2}I_s/e = 2.02\times 10^{13}I_s(m_p/m_eT_p)^{1/2} \tag{29a}$$

The accumulation of ultimate electrons in the plasma will go on until n_e and n_p are very nearly equal and thus from Eqs. (29) and (29a)

$$I_e/I_s = \tfrac{1}{2}(m_pT_e/m_eT_p)^{1/2} \tag{30}$$

or if

$$T_p = \tfrac{1}{2}T_e \tag{30a}$$

$$I_e/I_s = (m_p/2m_e)^{1/2}$$

For mercury vapor this gives

$$I_e = 429I_s \tag{30b}$$

The electron current density I_e which enters this equation is practically the same as that in the plasma at a small distance from the sheath edge for since no electron current flows to the collector the electrons are in a state of equilibrium and have a full Maxwellian velocity distribution. The values of n_e and n_p at the sheath edge are thus the same as in the plasma.

To measure I_s we need merely to determine the current density of the ions flowing to a negatively charged plane collector. By then making the collector positive it is frequently possible in a similar way to obtain an ***electron current*** independent of applied voltage and thus to measure the rate I_e' at which electrons reach the edge of the electron sheath. Although now equilibrium of the electrons is disturbed, so that only half of the full Maxwellian distribution can exist at the sheath edge, we may assume with reasonable accuracy that the observed electron current density I_e' at the sheath edge is the same as that in neighboring regions of the plasma where the distribution has more nearly spherical symmetry. Thus the values of n_e and n_p at the sheath edge are half of those in the interior of the plasma. This non-uniform distribution n_e and n_p is made possible by the fact that the positive ions contain

[7] Tonks, Mott-Smith and Langmuir, *Phys. Rev.* **28**, 104 (1926). See pp. 120–123.

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no intrinsic energy by which they could force the electron concentration to become uniform. We can therefore conclude that the measured I_e' is the same as the I_e in Eq. (30b).

Direct measurements[8] of I_e' and I_s in mercury arcs have given for the ratio I_e/I_s the value 405 ± 25 for 14 experiments, 423 ± 30 for 8 experiments and 407 ± 15 for 3 other experiments or for the average of all 25 experiments 411 ±17 which is in good agreement with the value 429 from Eq. (30a).[9] Returning to our consideration of a plasma discharge between parallel plane electrodes we see that, since the current of ultimate electrons flowing to the anode cannot exceed the positive ion current which is simultaneously produced by ionization, there must be an anode sheath with a field sufficient to repel all but this small fraction of the electrons that move towards the anode, but the ions are not repelled.

To obtain clearer conceptions of the typical distribution of ions and electrons in the plasma and of the resulting plasma fields, we may consider, as before, that there is a uniform rate of ionization throughout the plasma and we may neglect for the present the currents carried by primary electrons.

If a is the distance between the anode and cathode sheaths then the current density I_s of ions flowing to each sheath is approximately

$$I_s = \tfrac{1}{2} Sea \tag{31}$$

and to the anode the ultimate electron current density is $2\ I_s$ since no electrons pass to the cathode.

At somewhat higher gas pressures than we have been considering the mean free paths λ_p of the ions may be small compared to a and then the ions and electrons that are formed in the interior of the plasma must move to the sheaths by diffusion. Since the electrons tend to diffuse more rapidly they leave the interior of the plasma positively charged and thus there is a field which aids in moving the ions. The general differential equations for this "ambipolar diffusion" of ions and electrons have been given by Schottky and Issendorf.[10] The "ambipolar diffusion coefficient" is

$$D = \frac{M_p M_e k(T_e+T_p)}{(M_p+M_e)e} \tag{32}$$

where M_p and M_e are the mobilities of the ions and electrons (velocity per unit field). The effect of the field produced by the electrons, in accelerating

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[8] These data are the averages of the values given as I_e/I_p in Tables III, XIII and XV on pages 544, 765 and 769 of Vol. 27 *Gen. Elect. Rev.* Reference 3. At that time the necessity for introducing the factor 1/2 in Eq. (29a) was not realized and therefore it was thought that I_e/I_p should be equal to the square root of the ratio of the masses, viz. 607. The difference between this and the observed value of about 410 was attributed to the possible presence of negative ions.

[9] Dr. Tonks is developing the more rigorous theory of the positive ion velocity distribution assuming that the energies are acquired from the plasma fields.

[10] W. Schottky and J. v. Issendorf, *Zeits. f. Physik* **31,** 163 (1925).

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the ions, has been taken into account in this derivation and is represented by the term T_e in the equation.

The current density I of ions (drift current, not random current) at any point in a direction normal to the sheaths is then

$$I = -De(dn/dx) \tag{33}$$

n being the number of ions, or electrons per unit volume. At any point at a distance x from the central plane of the plasma the current will be Sex and therefore from Eq. (33) by integration we find that the ion concentration at x is

$$n = n_M - Sx^2/2D \tag{34}$$

or at the sheath edges where $x = \pm a/2$ we have

$$n_S = n_M - Sa^2/8D. \tag{35}$$

The drift ion current I is zero at center $x = 0$ and increases to I_S at the sheath edge while the random ion current is proportional to n and therefore should decrease from a maximum value I_M at the center to a value $I_M(n_S/n_M)$ at the sheath edge. Although strictly speaking the current at the sheath edge is not a random current for it flows in one direction only we may provisionally[11] identify this calculated value of the random current with the drift current I_S at the sheath edge, so that

$$I_S = I_M(n_S/n_M). \tag{36}$$

The mobility M_p of Eq. (32) is very small compared to M_e and therefore M_e drops out. The ion mobility is

$$M_p = e\lambda_p(2/\pi m_p k T_p)^{1/2} \tag{37}$$

and thus by Eq. (32)

$$D = \lambda_p\left(\frac{2k}{\pi m_p T_p}\right)^{1/2}(T_e+T_p) = 3.15\times 10^5\lambda_p(T_e+T_p)\left(\frac{m_e}{m_p T_p}\right)^{1/2} \tag{38}$$

if we use c.g.s. units.

Substituting this value of D and the value of S from Eq. (31) in Eq. (34) and combining with Eqs. (36) and (29a)

$$\frac{I_M}{I_S} = \frac{n_M}{n_S} = 1+\frac{Sa^2}{8n_S D} = 1+\frac{aT_p}{8\lambda_p(T_e+T_p)} \tag{39}$$

or if we put $T_e = 2T_p$

$$I_M/I_S = 1+a/24\lambda_p. \tag{40}$$

This simple result indicates that the ion concentration will be nearly uniform throughout the plasma if the free path of the ions is large compared to 1/24th

[11] Although this assumption is of doubtful accuracy because of the non-Maxwellian character of the ion velocity distribution near the sheath edge, it is far more nearly in accord with the facts than the assumption made by Schottky and Issendorf and others that $n_S = 0$ at the sheath edge.

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of the distance between the electrodes. With mercury vapor at 1 barye pressure and 20°C, λ_p is of the order of 5 cm and therefore if a, the distance between the electrode sheaths, is 5 cm the pressure may be as great as 24 baryes (saturated vapor at 56°C) before the concentration of ions at the center of the plasma becomes twice as great as at the edges.

The potential distribution within the plasma may now be calculated by means of the Boltzmann equation

$$n/n_M = \exp\,[(V-V_M)e/kT_e] \tag{41}$$

which will be applicable within the plasma since the electron drift currents are negligible compared to the random currents and therefore the conditions will not depart appreciably from those of equilibrium. The difference of potential between the sheath edge and the center of the plasma is thus by Eqs. (41) and (40), since $e/k = 11{,}600$ degrees per volt,

$$V_M - V_S = 1.98\times 10^{-4} T_e \log_{10}(1+a/24\lambda_p) \text{ volts.} \tag{42}$$

These fields are ordinarily very weak, for example if $T_e = 10{,}000°$ in a mercury arc at 24 baryes pressure and with $a = 5$ cm we find $V_M - V_S = 0.6$ volts. At a pressure of 240 baryes this voltage difference would be 2.1 volts if T_e remained the same, but actually T_e decreases as the pressure rises so that the potential difference would probably remain approximately constant at about 0.6 volt. Although this field is extremely small compared to that in the sheaths, it must play an essential role in gaseous discharges in drawing the positive ions towards the sheath edges. In fact the kinetic energy of the positive ions is probably almost entirely derived from the action of this field.

Anode Sheath

The electron current flowing to the anode is usually dependent on the conditions that exist in other parts of the circuit which supplies the energy for the anode current. For example, it may be fixed by the electron emission from the cathode or by the current that can flow through an external resistance placed in series with the anode. If the anode area is comparable with the cross-section of the plasma the current density I_a of the electron current flowing to the anode is usually far smaller than I_e the electron current density in the plasma near the anode. There will then be a retarding potential V_a (negative anode drop) in the anode sheath whose magnitude can be calculated from the Boltzmann equation which takes the form

$$V_a = 1.98\times 10^{-4} T_e \log_{10}(I_e/I_a). \tag{43}$$

In cases of low pressure plasma discharges with hot cathodes when no measurements of I_e are available, methods of the following type will usually give I_e with sufficient accuracy to give a fairly reliable value of the anode drop.

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Each primary electron from the cathode, in falling through the cathode sheath, acquires a high kinetic energy, which it retains until the energy is dissipated in excitation and ionization of the gas molecules. The glass walls of the tube become so strongly negatively charged that they receive no more electrons than positive ions and thus in ordinary cases only a negligible fraction of the primary electrons are lost to the walls. It has been shown[12] under a wide range of conditions of this kind, that the total number of ions formed per primary electron, which we may call β, is independent of current density, of pressure and of the geometry of the tube, but depends only on the nature of the gas and the energy of the primary electrons. Thus 50 volt electrons in mercury vapor give 1.4 ions per primary electron. Neglecting the relatively small ion current to the cathode and anode, the primary electron current from the cathode is equal to i_a the electron current to the anode. Thus the total rate of production of ions within the tube is βi_a.

Since recombination in the gas is negligible at low pressures, all these βi_a ions will flow to the electrodes and to the walls of the tube. With envelopes of roughly spherical shape, the ion current density will be fairly uniform over the whole surface. Thus if B is the area of the plasma envelope (sheaths over all electrodes and glass surfaces) we find that the positive ion current density I_p is given by

$$I_p = \beta i_a / B. \tag{44}$$

Substituting this value of I_p for I_s in Eq. (30) we can thus calculate I_e, the random electron current density and then Eq. (43) gives the negative anode drop

$$V_a = 1.98 \times 10^{-4} T_e \log_{10}\left[\frac{A\beta}{2B}\left(\frac{m_p T_e}{m_e T_p}\right)^{1/2}\right] \tag{45}$$

where A is the anode area.

Experiments with discharges from hot cathodes in mercury vapor in spherical and cylindrical bulbs and with anodes of various sizes have demonstrated the general usefulness and reasonable accuracy of this equation.

With large anode area A the anode sheath is thus a positive ion sheath but as A decreases a point is reached where the sheath disappears and is then replaced by an electron sheath, the anode drop becoming positive. When the positive anode drop approaches the ionizing voltage an anode glow appears and with sufficiently high gas pressure the rate of positive ion production becomes so great as to break down the electron space charge, causing a *second plasma* to develop near the anode. This second plasma usually takes the form of a globular luminous protuberance from the anode. Its interior is a typical plasma with high electrical conductivity; its boundary is a double sheath with an inner

[12] I. Langmuir and H. A. Jones. *Phys. Rev.* **31,** 357–404 (1928). See particularly pp. 402–3.

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positive space charge and an outer negative space charge, the potential drop being of the order of the ionizing potential.

The conditions at this double layer are essentially like those that we postulated in the first section of this paper; the potential gradient vanishes at the inner and outer edges and the potential distribution is determined by the charges of moving ions which acquire their kinetic energy mainly from the field within the double layer.

Cathode Sheaths

An electrode which is at a negative potential with respect to the plasma repels all the ultimate electrons which move towards it, except those that have a sufficient component of energy normal to the surface of the electrode, to enable them to move against the retarding field. The positive ions in the plasma that move towards the electrode are collected by it. The fraction of the ultimate electrons that penetrate through this positive ion sheath is given by the Boltzmann equation or may be calculated from the ratio I_a/I_e in Eq. (43). Thus if the temperature of the ultimate electrons is 10,000° the current density of these electrons which reach the electrode will be only 1/1000th of that in the plasma if the electrode is 5.9 volts negative with respect to the plasma.

If the electrode is 50 volts or more negative with respect to the plasma the sheath will contain no appreciable number of electrons and the positive ions will enter the sheath with energies negligible compared to those they acquire in the sheath itself. Thus the velocity of the ions is given by

$$\tfrac{1}{2} m_p v_p^2 = -Ve$$

and since there is no appreciable field in the plasma itself we will have at the outer edge of the sheath $dV/dx = 0$.

Since these conditions are exactly those that are postulated in the derivation of the ordinary space charge Eq. (1) we may apply this equation to cathode sheaths in gaseous discharges, merely replacing m_e by m_p.

In the usual applications of Eq. (1) the voltage and the distance between the electrodes are known and we wish to calculate the current density that can flow when the current is limited by space charge. In the present case, however, the current density I_p is fixed by conditions within the plasma and since the applied voltage is usually known the equation can be used to calculate only the thickness x of the sheath (a in Eq. (1)). Thus the current density I_p is independent of the applied voltage and x varies in proportion to $V^{3/4}$.

The cathode sheath can usually be seen as a dark space and under favorable conditions its thickness can be measured. In low pressure discharges in mercury vapor the observed sheath thickness agrees well with that calculated by the space charge equations.[13]

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[13] Reference 3, p. 545.

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Of course, modifications or corrections to this theory may be necessary under special conditions, for example, when the gas pressure is so high that the ions collide with gas molecules within the sheath or when the voltage is so low that the initial velocitites of the ions and electrons at the sheath edge cannot be neglected.

Sheath on Hot Cathode. Double Sheath

If the cathode is heated so that it emits electrons these flow out through the positive ion sheath and if this electron current becomes sufficiently great it will neutralize the positive ion space charge in the immediate neighborhood of the cathode. Since the electrons start with negligible velocities from the cathode, the conditions are the same as those postulated when we calculated the effect of ions liberated at the anode upon the electron currents from a hot cathode emitting a surplus of electrons, for example, in our derivation of Eqs. (5), (11) and (12). In the present case, however, it is the sheath edge, functioning as anode, that emits a surplus of ions (since $dV/dx = 0$ at this point) while a limited number of electrons is emitted from the cathode. Thus, in applying our equations to the present problem we need merely interchange the subscripts p and e.

Let us consider the currents of ions and electrons that flow to or from a plane cathode in a plasma having given characteristics, such as I_c, I_p and T_e. Let the cathode, at potential $-V$ with respect to the plasma first be at such low temperature that it emits no electrons. We let a_0 denote the sheath thickness under these conditions, as calculated from the known value of I_p by means of Eq. (1), in which for this purpose we replace I_0 by I_p, V_a by V, m_e by m_p and a by a_0.

Now let the cathode temperature be raised until the current density of the emitted electrons is I_e. To calculate the coefficient α we interchange e and p in Eq. (5) giving

$$\alpha_e = (I_e/I_p)(m_e/m_p)^{1/2} \tag{46}$$

The effect of these electrons in neutralizing the ion space charge cannot cause an increase in I_p for this is fixed by the plasma, but manifests itself by changing the sheath thickness so that this becomes a instead of a_0. In Eq. (12) the ratio I_e/I_0 was calculated on the assumption that the distance between the electrodes was fixed. Evidently for our present problem we must replace I_e in this ratio by I_p and I_0 must be replaced by the positive ion current density that would flow to a cathode of voltage $-V$ if no electrons were emitted and if the sheath thickness were a. This current may be calculated from Eq. (1) by replacing m_e by m_p, and V_a by V. Thus we see that $(I_e/I_0)^{1/2}$ in Eq. (12) must be replaced by a/a_0. The last horizontal line of Table I gives the values of a/a_0 (which are merely the square roots of the numbers in the line above) for various values of α_e. An electron emission for which $\alpha_e = 0.2$ would corre-

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spond to $I_e = 0.2 \times 607 I_p = 121 I_p$ in a discharge in mercury vapor. The electron current would then be 121 times the positive ion current and yet we see from Table I that this would cause the sheath thickness to increase merely 4.1 per cent. The potential distribution functions of Table I are of course applicable to these sheaths on a hot cathode.

As the cathode temperature is raised the electron current density I_e increases and is equal to the electron emission from the cathode until, when a_e becomes equal to unity, the current becomes limited by space charge and a further increase in electron current cannot occur. The cathode is then covered by a double layer or double sheath and the ratio of the electron current to the ion current is equal to $(m_p/m_e)^{1/2}$. Even a change in cathode voltage will not cause a change in electron current if the positive ion current I_p remains constant.

These facts are of vital importance in any understanding of discharges from hot cathodes in low pressures of gas. We see that the use of a hot cathode, no matter how high its temperature, does not destroy the cathode drop.

In discharges with a single cathode the intensity of ionization in the plasma depends on the electron current from the cathode and on the cathode drop which gives to these electrons their velocity. Under these conditions, when the total current is restricted by a resistance in series with the anode, and a surplus of electrons is emitted by the cathode, the cathode drop adjusts itself to such a value that I_p in the plasma bears the proper ratio (corresponding to $\alpha = 1$) to the electron current that is drawn. For example, with mercury vapor, the cathode drop must be such as to cause an ionization in the plasma that will give a positive ion current equal to 1/607th of the electron current that flows from the cathode.

For very low pressure discharges in which the plasma is very uniform we may calculate the magnitude of the cathode drop and thus learn upon what factors it depends. If C is the effective area of the cathode, then the electron current from it will be CI_e and this will be practically equal to the anode current i_a. Substituting CI_e for i_a in Eq. (44) and then putting for I_e/I_p its value $(m_p/m_e)^{1/2}$ we find

$$\beta = (B/C)(m_e/m_p)^{1/2} \tag{47}$$

where as before, B is the surface of the plasma envelope. The cathode drop will become such as to give to β (which is a function of voltage only) this value. The ratio C/B measures the fraction of the positive ions that are generated that get back to the cathode. The greater this ratio the smaller will be the cathode drop.

At higher gas pressures the intensity of ionization in the plasma will not be uniform but there will be a greater intensity near the cathode; this will greatly decrease the cathode drop. Under the most favorable conditions nearly all the ions formed will pass to the cathode and then β (for mercury vapor) will only need to be 0.0016, a value which will be reached at about 0.1 volt above the ionizing voltage.

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Theory of Double Sheath Considering Initial Velocities

The foregoing theory of the cathode double sheath, simplified by neglecting the effects of the initial velocities of the ions and electrons, has given us a useful picture of the essential character of the phenomena at a hot cathode in gas. We have seen, however, that its main field of application will be to cases involving cathode drops of 10 or 20 volts so that we are not wholly justified in ignoring these initial velocities. Fortunately there is no great difficulty in calculating the ratio I_e/I_p even when these velocities are taken into account.

Let us consider a sheath having parallel plane boundaries or "edges." Any group of electrons or ions entering the sheath with a given velocity distribution will contribute to the space charge at any point P within the sheath the amount $f(1/v)dI$, where dI denotes the current density, normal to the plane of the sheath, of an elementary group which has the normal component of velocity v. Since with a given initial distribution, both dI and v are functions solely of the potential at the point P, it follows that the total space charge ϱ at any point depends only on the potential at that point.

Multiplying both members of the Poisson equation (for plane case) by dV and integrating we obtain

$$(dV/dx)^2 = -8\pi \int \varrho dV + \text{const.} \tag{48}$$

In the case of the double sheath where the electron and ion currents are both limited by space charge, $dV/dx = 0$ at both the inner and outer edges so that we have

$$\int_{V_M}^{V_S} \varrho \, dv = 0 \tag{49}$$

where V_M and V_S are the potentials at these two edges, or more specifically at the places where dV/dx is zero, one of these being at the minimum potential region very close to the cathode and the other S, the outer edge of the sheath.

Later we shall find it necessary to analyze more closely the nature of this sheath edge S, but for the present we will define it as a place where $dV/dx = 0$ and $\varrho = 0$ (as in the plasma) so that according to the Poisson equation d^2V/dx^2 is also zero.

Within the sheath there are three groups of carriers to be considered — the positive ions from the plasma, the ultimate electrons from the plasma and the electrons emitted by the hot cathode. For each of these groups we shall express its space charge in terms of the potential.

1. *Plasma ions.*—We have already seen that the velocity distribution at S cannot be Maxwellian and we shall see that it cannot even have hemispherical symmetry. Assuming that the velocities of the ions originate from the weak plasma fields which we now wish to ignore, the velocity must in general be

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nearly normal to the plane of the sheath. Although the velocities of all the ions cannot be equal, we shall make no great error (especially since the ions are accelerated into the sheath) if we replace the actual distribution by a homogeneous swarm of ions moving normal to the plane of the sheath with a kinetic energy corresponding to the potential E_p. If ϱ_{pS} is the space charge of these ions at S then their space charge ϱ_p at a point of potential V is

$$\varrho_p = \varrho_{ps}\{E_p/(V_s - V + E_p)\}^{1/2} \tag{50}$$

We shall wish to calculate the integral of Eq. (49). If we let H_p be the part contributed to this integral by the ions, we have

$$H_p = \int_{V_M}^{V_S} \varrho_p \, dV = 2\varrho_{pS}E_p^{1/2}\{(V_{SM}+E_p)^{1/2} - E_p^{1/2}\} \tag{51}$$

where V_{SM} is the difference of potential between S and M, i.e. $V_S - V_M$.

The space charge ϱ_{pS} at the sheath edge can be determined from the positive ion current density I_p flowing to the cathode from the equation

$$\varrho_{pS} = I_p(m_p/2E_p e)^{1/2}. \tag{52}$$

2. *Plasma electrons.*—We assume that the cathode is sufficiently negative with respect to the plasma so that no appreciable fraction of the ultimate electrons reaches the cathode and thus the velocity distribution of these electrons at S will be a complete Maxwellian distribution corresponding to a temperature T_e and a random electron current density I_{eS}. Then according to the Boltzmann equation the space charge at any point due to these electrons is

$$\varrho_e = \varrho_{eS} \exp\{(V - V_s)/E_e\} \tag{53}$$

where E_e is the potential which corresponds to the energy kT_e that is, $E_e = T_e/11{,}600$ volts. The integral H_e is

$$H_e = \int_{V_M}^{V_S} \varrho_e \, dV = E_e\varrho_{eS} \tag{54}$$

Finally ϱ_{eS} may be expressed in terms of I_e

$$\varrho_{eS} = I_{eS}(2\pi m_e/E_e e)^{1/2} \tag{55}$$

3. *Electrons from cathode.*—If the cathode emits more electrons than can flow across the sheath there will be a potential minimum V_M at a short distance from the cathode surface.

If V_c is the potential of the cathode then V_M may be calculated[14] from the equation

$$I_{eM} = I_c \exp\{(V_M - V_c)/E_c\} \tag{56}$$

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[14] Eqs. (56) and (57) have been derived from equations in a paper by I. Langmuir, *Phys. Rev.* **21**, 419 (1923).

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where I_c is the current density of the saturation emission from the cathode corresponding to its temperature T_c and I_{eM} is that which gets past the potential minimum and is thus electron current density passing through the sheath; E_c is the potential corresponding to kT_c, that is $T_c/11{,}600$ volts. The space charge of the electrons at any point in the sheath, between M and S is thus rigorously

$$\varrho_c = \varrho_{cM} \exp\{(V-V_M)/E_c\} \operatorname{erf}\{(V-V_M)/E_c\}^{1/2} \tag{57}$$

where erf denotes the error function defined by

$$\operatorname{erf}(x) = 2\pi^{-1/2}\int_x^\infty \exp(-y^2)\,dy$$

When x is greater than about 2 a very close approximation is

$$\exp x^2 \operatorname{erf} x = 1/\pi^{1/2}(x+1/2x). \tag{58}$$

The quantity E_c is only 0.1 or 0.2 volts and if $V-V_c$ is more than 4 times as great this approximation is justified and Eq. (57) thus becomes

$$\varrho_c = \varrho_{cM}\{(E_c/\pi(V-V_M+E_c)\}^{1/2} \tag{59}$$

Here ϱ_{cM} is the space charge density of the emitted electrons at the point of minimum potential.

Integration of Eq. (57) gives rigorously

$$H_c = \int_{V_M}^{V_S} \varrho_c\,dV = E_c[\varrho_{cS}-\varrho_{cM}+2\varrho_{cM}(V_{SM}/\pi E_c)^{1/2}]. \tag{60}$$

Putting $V = V_S$ in Eq. (59) we can express ϱ_{cM} in terms of ϱ_{cS}. Thus Eq. (60), expanded as a power series in E_c/V_{SM}, becomes

$$H_c = 2\varrho_{cS}V_{SM}(1-(\pi E_c/4V_{SM})^{1/2}+E_c/V_{SM}\ldots]. \tag{61}$$

The space charge ϱ_{cS} expressed as a function of I_{eM} is

$$\varrho_{cS} = I_{eM}[m_e/2(V_{SM}+E_c)e]^{1/2} \tag{62}$$

When the current I_M is limited by space charge Eq. (49) can now be used to determine the relation between I_M and I_p. The equation takes the form

$$H_p = H_e+H_c. \tag{63}$$

The values of these quantities from Eqs. (51), (54) and (61) substituted in this equation give a relation between the ϱ's and then by means of Eqs. (52), (55) and (62) we find the following relation between the currents

$$I_{eM}[1-(\pi E_c/4V_{SM})^{1/2}+E_c/2V_{SM}] = I_p(m_p/m_e)^{1/2}[1-(E_p/V_{SM})^{1/2}+E_p/2V_{SM}]-I_{eS}(\pi E_e/V_{SM})^{1/2} \tag{64}$$

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We have defined S as a place where dV/dx is zero. At this region we may also put $\varrho = 0$, so that

$$\varrho_{pS} = \varrho_{eS} + \varrho_{eS}. \tag{65}$$

Expressing the ϱ's in terms of the I's by Eqs. (52), (55) and (62) and using this equation to eliminate I_{eS} we obtain

$$I_{eM}/I_p = (m_p/m_e)^{1/2} \frac{1-(E_p/V_{SM})^{1/2}(1+E_e/2E_p)+E_p/2V_{SM}}{1-(\pi E_c/4V_{SM})^{1/2}-(E_e-E_c)/2V_{SM}}. \tag{66}$$

An examination of this result shows that when $(1/2)E_e < E_p < V_{SM}$ an increase in either E_p or E_e causes a decrease in I_{eM}. Thus the effect of the initial velocities of both the ions and the electrons at the sheath edge is to decrease the electron current that can flow from a hot cathode. As a typical example we may take $E_e = 2$ volts (corresponding to $T_e = 23{,}000°$), $E_p = 1$ volt, $E_c = 0.2$ volt. The ratio I_{eM}/I_p instead of being 607 for mercury vapor, will be only 31 per cent of this for $V_{SM} = 5$ volts, 53 per cent for 10 volts and 71 per cent for 25 volts.

The Sheath Edge

If the initial velocities of the plasma electrons and ions could be neglected we would be justified in regarding the edge of the sheath as sharp. In our study of the double sheath we must now recognize that the velocities of the ions which are measured by E_p are derived from the weak fields that extend from the sheath a considerable distance into the plasma.

To develop clear conceptions of the relation between the plasma and the sheath we will analyze in some detail a typical although somewhat simplified example. Let us imagine two parallel plane electrodes with a plane between them which acts as a source of ions and electrons, but which is not to be regarded as an electrode. Thus the potential of the source will be determined by space charges and cannot be arbitrarily varied as that of an electrode could be.

If the source emits only ions having no initial velocities, the potential will rise until the ions can flow to the two electrodes in accord with the space charge equation for ions (similar to Eq. (1)). Thus at the source the potential gradient is zero. Without loss of generality we may take the source as the origin both for the potential V and for the distance x and for simplicity may consider phenomena only on one side of the source, i.e. for positive values of x. Let I_p be the positive ion current density that flows from the source in this positive direction.

Now let us introduce electrons having a Maxwellian velocity distribution corresponding to a temperature T_e. We assume that the electrode is at a negative potential sufficient to prevent any appreciable number of electrons from reaching it, and thus the Maxwellian distribution is not disturbed. We assume also that the electrons pass freely through the source in both directions, which

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is equivalent to assuming that the source is a perfect reflector for electrons. Let I_{e0} be the electron current density passing in one direction through the source (random current). At any point having the potential V the positive ion space charge is

$$\varrho_p = I_p(m_p/-2Ve)^{1/2}$$

and the electron charge density according to Eqs. (29), (41) and (53) is

$$\varrho_e = I_e(2\pi m_e/E_e e)^{1/2} \exp(V/E_e).$$

We now substitute the total space charge $\varrho_p - \varrho_e$ into the Poisson equation and then reduce the result to a dimensionless form by introducing the variables η and x_p defined by

$$\left.\begin{aligned} \eta &= -V/E_e \\ I_p &= (1/9\pi)(2e/m_p)^{1/2} E_e^{3/2} x_p^{-2} \end{aligned}\right\} \tag{67}$$

This last equation is similar in form to the space charge Eq. (1). Thus x_p is the positive ion sheath thickness calculated for a collector voltage E_s ignoring the effects of initial velocities. The Poisson equation thus gives

$$d^2\eta/d\lambda^2 = (4/9)[\eta^{-1/2} - 2\pi^{1/2}\alpha_{e0} \exp(-\eta)] \tag{68}$$

where λ is a dimensionless parameter proportional to length defined by $\lambda = x/x_p$ and α_{e0} in accordance with Eq. (46) is

$$\alpha_{e0} = (I_{e0}/I_p)(m_e/m_p)^{1/2}. \tag{69}$$

Multiplication by $d\eta/d\lambda$ and integration gives

$$d\eta/d\lambda = (4/3)\{\eta^{1/2} - \pi^{1/2}\alpha_{e0}[K - \exp(-\eta)]\}^{1/2} \tag{70}$$

where K is a constant of integration. The potential distribution is given by

$$\lambda = (3/4)\int_0^{\eta} \{\eta^{1/2} - \pi^{1/2}\alpha_{e0}[K - \exp(-\eta)]\}^{-1/2} d\eta. \tag{71}$$

For the case that we are now considering where the ions start from the source without initial velocity, we can determine K by the condition $d\eta/d\lambda = 0$ when $\eta = 0$, and find from Eq. (70) that $K = 1$. Let us now consider the nature of this solution to our problem given by Eqs. (70) and (71).

The family of curves shown in Fig. 2 represents the solutions of Eq. (71) for the case $K = 1$ for a series of different values of α_{e0}. By plotting $-\eta$ as ordinate instead of η the curves give directly the potential distribution V as a function of x in terms of E_e and x_p as units. The data for the curves shown in Fig. 2 have been calculated accurately either by numerical quadrature of Eq. (71) or by using some of the equations given below.

If $\alpha_{e0} = 0$, that is, if no electrons are present, Eq. (71) becomes

$$\lambda = \eta^{3/4}$$

which corresponds to the ordinary space charge equation for ions.

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For values of α_{e0} small compared to unity, expansion of Eq. (71) gives

$$\lambda = \eta^{3/4}[1+0.3\pi^{1/2}\alpha_{e0}\eta^{1/2}+(9/56)\pi\alpha_{e0}^{2}\eta-\cdots]$$

if η is small, while if $\eta > 4$

$$\lambda = \eta^{3/4}+2.66\alpha_{e0}\eta^{1/4}-2.41\alpha_{e0}.$$

Thus the effect of the electrons is to increase λ and decrease the potential gradient.

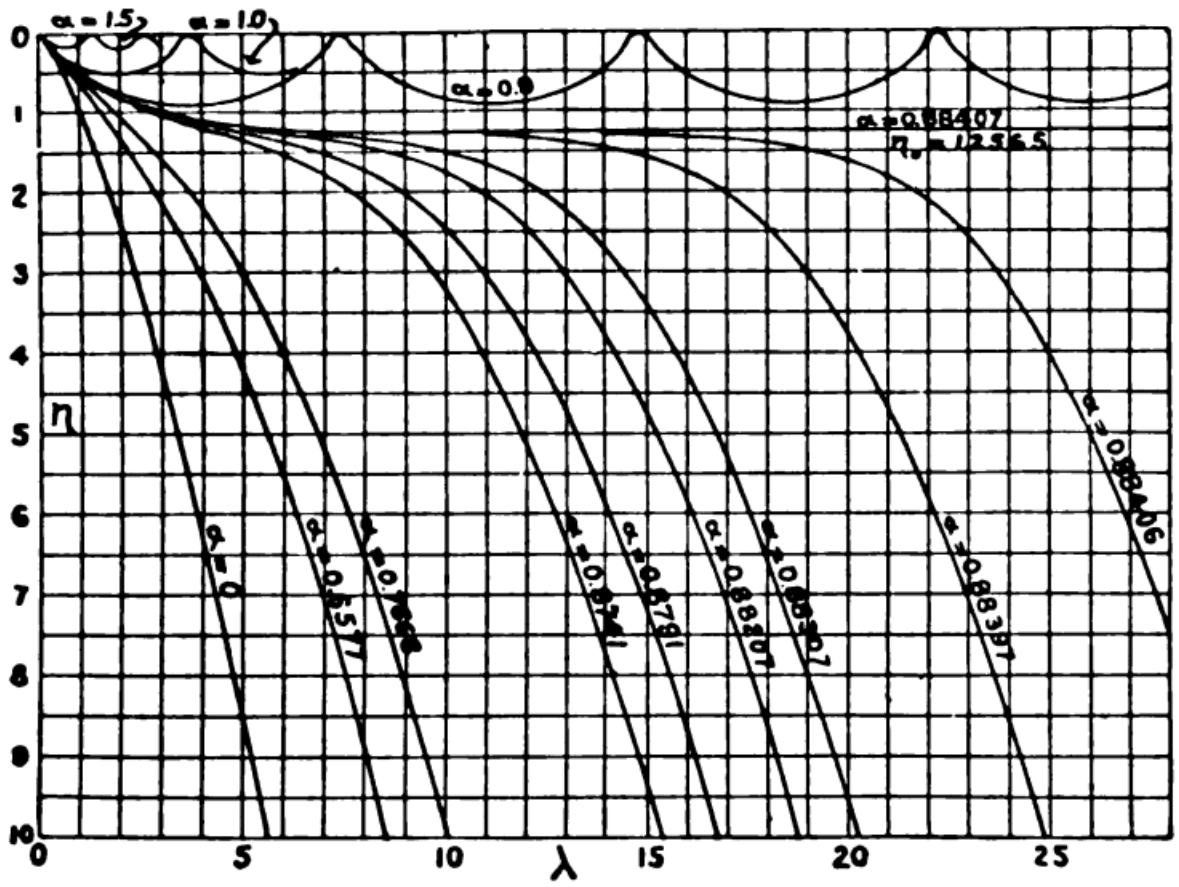

FIG. 2. Effect of electrons of temperature I_e on the potential distribution between planes at one of which ions are generated.

When α_{e0} is zero there is an infinite positive space charge at $\eta = 0$ and elsewhere the charge decreases continuously as η increases. Electrons, when introduced, are confined almost wholly to regions where η is less than 3 or 4. These electrons tend to produce a minimum in the space charge. The condition for the occurrence of such a minimum is that

$$d^3\eta/d\lambda^3 = (2/9)[-\eta^{-3/2}+4\pi^{1/2}\alpha_{e0}\exp(-\eta)]d\eta/d\lambda = 0. \qquad (72)$$

Before a negative space charge can occur as α_{e0} increases, it is necessary that $d^2\eta/d\lambda^2$ shall pass through zero. Equating both the third and the second derivatives to zero we find $\eta = 0.5$ and $\alpha_{e0} = (\epsilon/2\pi)^{1/2} = 0.6577$. The curve obtained with this value is plotted in Fig. 2.

When α_{e0} increases beyond 0.6577 there are two points of inflection between which there is a negative space charge. At the upper one ($\eta < 0.5$) the potential gradient $d\eta/d\lambda$ is a maximum while at the lower one it is a minimum. At a certain value of α_{e0} which we shall call α_{e1} the slope at this second inflection becomes zero. We find this value by placing the second members of Eqs. (68) and (70) each equal to zero, thus obtaining

$$\alpha_{e1} = 0.88407; \quad \eta_1 = 1.2565.$$

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This solution as shown in Fig. 2 is one that gives a region where the space charge and the potential gradient are both zero, conditions which characterize an ideal plasma. We shall return to a consideration of this case later.

When α_{e0} increases beyond α_{e1} the second point of inflection becomes imaginary and the curve no longer dips indefinitely downward but after passing through a minimum rises again to $\eta = 0$. Thus for larger values of α_{e0}, η must remain so small that we can replace $1 - \exp(-\eta)$ by η and then Eq. (71) after integration gives

$$\lambda = (3/2)\pi^{-3/4}\alpha_{e0}^{-3/2}[\sin^{-1}\nu - \nu(1-\nu^2)^{1/2}] \tag{73}$$

where

$$\nu = \pi^{1/4}\alpha_0^{1/2}\eta^{1/4}.$$

Thus the potential is a periodic function of the distance λ, the wave-length corresponds to

$$\lambda_1 = (3/2)\pi^{1/4}\alpha_{e0}^{-3/2}$$

and the minimum potential at $\lambda = 1/2\lambda_1$ corresponds to

$$\eta_{max} = 1/\pi\alpha_{e0}^2.$$

Periodic solutions of this kind are illustrated in Fig. 2 for values of α_{e0} of 0.9, 1.0 and 1.5. These data have been obtained directly from Eq. (71) by quadrature since Eq. (73) is only applicable accurately for still larger values of α_{e0}.

Examination of Fig. 2 shows that when α_{e0} is only slightly less than the critical value $\alpha_{e0} = 0.88407$ the curves show an extended region in which the potential gradient is nearly zero. This clearly corresponds to a plasma. Close to the source of ions at $\lambda = 0$ there is a field which draws the ions outward into the plasma. However, for large enough values of λ, the potential begins to drop again and we thus pass from the plasma into a positive ion sheath. As α_{e0} approaches α_{e1} this descending portion rapidly approaches a limiting form in which the successive curves differ from one another merely in a horizontal displacement. Thus for the limit $\alpha_{e0} = \alpha_{e1}$ the lower part of the curve is like that shown in the curves for $\alpha = 0.88207$, 0.88307, 0.88397 etc. except that it lies at a infinite distance to the right.

To investigate this transition from plasma to sheath we may write Eq. (71)

$$\lambda = (3/4)\int_0^n f^{-1/2}\,d\eta \tag{74}$$

where

$$f = \eta^{1/2} - \pi^{1/2}\alpha_{e0}[1 - \exp(-\eta)]. \tag{75}$$

An expansion in the neighborhood of $\eta = \eta_1$ and $\alpha_{e0} = \alpha_{e1}$ gives to 3 terms

$$f = A\Delta\alpha + B\mu\Delta\alpha + C\mu^2$$

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where

$$\Delta\alpha = \alpha_{e1} - \alpha_{e0}$$

$$\mu = \eta - \eta_1$$

and $A = 1.2679$, $B = -0.5046$ and $C = 0.13428$.

Substitution of this value of f in Eq. (74) and integration gives

$$\lambda = 0.75C^{-1/2} \log_e [f^{1/2} + C^{1/2}\mu + 0.5BC^{-1/2}\Delta\alpha] + \text{const.}$$

When $\Delta\alpha$ is small this reduces to

$$\Delta\lambda = 4.713 \log_{10} [0.424\mu_2(-\mu_3)/\Delta\alpha] \qquad (76)$$

where μ_2 and μ_3 correspond to 2 points η_2 and η_3 that lie on opposite sides of η_1, and $\Delta\lambda$ is the distance between the points η_2 and η_3. It is assumed that μ_2 and $-\mu_3$ are large compared to $\Delta\alpha$.

When η is larger than about 4 the exponential term in Eq. (75) can be neglected in comparison with unity. Integration of Eq. (74) for $\alpha_{e0} = \alpha_{e1}$ then gives

$$\lambda = (\eta^{1/2} + 3.134)(\eta^{1/2} - 1.567)^{1/2} - 4.713 \log_{10} \Delta\alpha - 1.93 \qquad (77)$$

the integration constant having been determined from the data of Curve VII in Fig. 2 together with Eq. (76).

Our equations enable us to study the transition between plasma and sheath. Let us consider a typical numerical example. A one ampere arc in a 3 cm diameter tube containing mercury vapor saturated at 30°C gives for a collector flush with the tube wall $I_p = 2\times10^{-4}$ amperes per cm², $T_e = 21{,}000°$ and thus $E_e = 1.81$ volts. Then from Eq. (67) we find $x_p = 0.00686$ cm as our unit of length for the sheath measurements ($\lambda = 1$). If a collector is at -25 volts with respect to the ionized gas then at the collector surface $\eta = 1.256 + 25/1.81 = 15.1$. The effective distance between the collector and the source of ions may be taken to be 1 cm so that in Eq. (77) we put $\lambda = 1/0.0068 = 146$, and obtain $\Delta\alpha = 10^{-29}$. From Eq. (76) we see that the plasma field near the sheath falls to 1/10th for an increase in distance from the sheath equal to 0.032 cm (i.e. 4.71×0.0068).

According to the data of Fig. 2 the potential distribution within the sheath is given by

x	0	0.049	0.073	0.089	0.114	0.149
V	25	5.0	1.34	0.44	0.080	0.006

where V is the negative voltage at a point whose distance from the collector surface is x cm. If the initial velocities of the ions and electrons could be neglected the total thickness of the 25 volt sheath, calculated by the ordinary space charge equation, like Eq. (67), would be 0.049 cm.

From the foregoing analysis it appears that the motions of the positive ions in the plasma are not to be chosen arbitrarily but are fixed by the electron temperatures and by the geometry of the ion source. With the plane source we

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have assumed above, we found $\eta_1 = 1.2565$ which means according to the nomenclature of Eq. (66) that $E_p = 1.2565E_e$.

This particular assumption, however, is of course not directly applicable to experimental conditions. Dr. Tonks has been able to solve the integral equations involved in the problem of potential distribution resulting from a uniform generation of ions throughout the plasma (plane, cylindrical or spherical) and also for the more usual case where the rate of generation at each point is proportional to n_e the concentration of electrons at that point. It is intended to publish these results soon. The way in which the distribution of ion generation in the plasma affects the conditions in the sheath is by altering the average velocity of the ions which enter the sheath. The equations that we derived for the double sheath, such as Eq. (66), are applicable if we choose the proper value for the ratio E_p/E_e. For the case of ion generation proportional to n_e and an infinite plane collector the effective value of this ratio is 0.751 for very high intensity of ionization, decreasing somewhat as the intensity of ionization decreases but probably never falling as low as 0.5 which according to our discussion of Eq. (72) is the limit at which only periodic plasma solutions are possible.

For most experimental conditions we are thus justified in placing $E_p = 0.7E_e$ in Eq. (66).

Experimental Data on Double Sheaths

In determining the space potential by the hot probe method,[15] the volt-ampere characteristic of a small filament is determined first when the filament is cold and then again when heated to a temperature at which electrons are emitted. Only when the filament is below the space potential do the electrons escape. Thus the voltage at which the two curves separate is the space potential. At lower potentials the difference between corresponding ordinates measures the electron current from the probe.

In using this method in ionized gases it was soon found, when the probe potential was lowered below space potential, that the electron current did not rise abruptly to the saturation emission corresponding to the probe temperature, but often a transition region of several volts was observed. Clearly in this range the current is limited by space charge in spite of the fact that an abundant supply of positive ions is present in the ionized gas. The double sheath theory should apply in such cases.

Experiment 559.—Similar limitation of current by space charges in double sheaths is observed in hot cathode tubes containing gas if the current to the anode is lowered below its saturation value by using a large resistance in series with the anode. For example, a spherical bulb 12.5 cm in diameter contained a tungsten filament 1 cm long and 0.18 mm diameter, a disk-shaped anode 2.2 cm in diameter, and a mica-backed collector 1.1 cm in diameter. The filament

[15] I. Langmuir, *J. Franklin Inst.* **196,** 754 (1923).

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was heated to a temperature at which it gave a saturation current of 22.5 ma. The volt-ampere characteristics of the anode were measured when the bulb contained mercury vapor saturated at 20°C, while simultaneous readings were made of the positive ion current flowing to the disk collector, this being kept at −80 volt with respect to the anode. Until the anode potential was raised to about 12 volts above that of the cathode, negative charges on the walls of the tube prevented any appreciable current from flowing. At 15 volts, however, the current rose to about 1 ma and as the voltage was increased to 25 the current increased only slowly to 1.9 ma. Electrons of 25 volts energy have an ionizing power 3.3 times as great as those of 15 volts. This small increase in current in spite of the considerable rise of voltage is probably due to the fact that the sheaths on the walls are relatively thick. As the ion production increases the sheaths become thinner so that the area of the sheath boundary, (which collects the ions that pass to the walls) increases and prevents any large rise of ion concentration. The small ion currents to the collector, ranging from 5 to 8 microamperes, indicate that the sheaths must have been several cm thick.

At voltages above 27 volts the behavior is quite different; as the current is raised from 2 to 22.5 ma the voltage rises slowly from 27 to a sharp maximum 33 at 15 ma and falls to 28 at 22 ma. As saturation is approached the voltage can be raised to 60 or more without increasing the current above 22.5.

In the range of relatively constant voltage (27–33) the collector current rose from 8 to 80 microamperes while the electron current changed from 2 to 22. With such low intensity of ionization the collector currents do not measure accurately the ion current density since the sheath on the collector is many mm thick.

Similarly the diameter of the double sheath on the cathode must have been 10 to 30 times that of the cathode itself. Rough calculations based on the space charge theory of sheaths indicate that the electron current from the cathode, at the higher currents (20 ma) was about 2000 times as great as the positive ion current that flowed to the cathode and at lower currents the ratio may have increased to 4000:1.

In this range the electron current from the cathode is evidently limited by the rate of arrival of positive ions at the cathode sheath. If the total current is fixed by an external resistance, the voltage drop in the sheath adjusts itself to a value which will give just the required number of ions. Since the number of ions formed in the plasma is proportional to the current and increases rapidly with voltage, we have an explanation of the relatively constant voltage.[16]

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[16] Approximate calculations of the ratio of electron current to ion current for cylindrical sheaths indicate that this ratio should increase from $(m_p/m_e)^{1/2}$ for very thin sheaths up to 2.25 $(m_p/m_e)^{1/2}$ for sheaths of large diameter. However, this theory neglects the fact that for sheaths of large diameter many of the ions may escape from the sheath after describing orbits about the cathode. Thus the ratio i_e/i_p may increase to values much greater than that just given. In the case of mercury vapor, values of this ratio as high as 2000 or 4000 are therefore consistent with the theory.

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Tube with Oxide Coated Cylindrical Cathode

Conditions more nearly approaching those of the theory we have developed are to be found in discharges using relatively large oxide coated cathodes. The following data were obtained by F. B. Vogdes. The cathode was a nickel cylinder of 0.63 cm diameter, 2.5 cm length closed at one end and heated by radiation from an inner tungsten helix. The outer cylindrical surface was coated with barium oxide. This cathode was mounted in a well-exhausted spherical bulb 10 cm in diameter provided with an appendix containing mercury. A collector disk 0.6 cm in diameter was placed 2 cm opposite the central part of the cathode, the axis of the cathode lying in the plane of the disk. Shortly after starting to operate the tube (appendix at 12°C) with an anode current of 4 amperes, the arc drop was 50 volts and the current depended greatly on the cathode heating current. Thus the current was limited by the cathode emission. The positive ion current density given by the collector was 14 ma per cm² and the electron temperature T_e was 140,000°. The ratio I_e/I_p was thus 57.

After operation for some time, the activity of the cathode suddenly increased, the arc drop fell to 17 volts, and T_e became 38,000. On the following day the characteristics were steady and reproducible and the data given in the accompanying table were taken. The arc current was now independent of the cathode heating current and adjusted to 4 amperes by means of a resistance in series with the anode.

Appendix temp.	Pressure baryes	Arc drop	T_e	I_e ma cm^{-2}	I_p ma cm^{-2}	I_e/I_p		Ratio obs./cal.
						obs.	cal.	
19.7°	1.5	16.0	24,000	800	5.6	143	372	0.385
8.8	0.55	20.2	35,000	800	6.0	133	350	0.380

The effect of lowering the vapor pressure of the mercury in a ratio of about 3:1 was to increase the electron temperatures and the arc drop but the ratio of the observed electron to ion currents remained nearly constant at about 140:1. In the last column of the table are given the ratios calculated by the double sheath theory according to Eq. (66) placing $E_p/E_e = 0.7$; $E_e = T_e/11600$, $E_c = 0.1$ and $V_{SM} =$ arc drop $+0.5$.

We see that the small variation of the observed value of I_e/I_p from 143 to 133 is in accord with the effect to be expected from the increased electron temperature. However, the observed ratio is only 38 per cent of that given by our theory. Such a difference may probably be explained by a lack of uniformity in the emission from the oxide coated surface.

Experiment 560.—In this experiment the intensity of ionization and the voltage drop in the sheath could be varied independently of one another. Two cathodes were used, one to produce the ionization while the volt-ampere characteristics of the second were determined. The spherical bulb having a mercury

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appendix was of 12.5 cm diameter and contained a cathode K of tungsten wire 0.25 mm diameter wound as an open helix, and an oxide coated cathode C like that used before: a nickel cylinder 2.5 cm long and 0.63 cm diameter heated by an internal tungsten coil. There was also a collector disk, B, 1.1 cm in diameter, backed by mica and a disk-shaped anode A, 2.2 cm diameter. The collector was so placed in the bulb that primary electrons reflected from the sheath on the bulb were not focussed upon it, for if this precaution is not taken serious errors may sometimes result, especially at low pressures of mercury vapor.

In exhausting the tube all metal electrodes were heated to bright red by high frequency induction and the bulb was baked at about 400°C. During the whole experiment the tube remained connected to a glass condensation pump through a liquid air trap, the readings of a McLeod gauge showing pressures of non-condensible gases less than 10^{-5} barye.

A typical set of data obtained with this tube containing saturated mercury vapor at 20°C is illustrated in Fig. 3. In this case the tungsten cathode K was maintained at —50 volts with respect to the anode (taken as zero potential) and heated to a temperature at which it emitted 60 ma of electrons. The ionization produced by these 50 volt electrons was such as to give 0.127 ma of ion current to the collector B (kept at —75 volts). This corresponds to $I_p = 65 \times 10^{-6}$ amps cm^{-2} after correction for the edge effect.[17]

Curve I represents the observed volt-ampere characteristic of the oxide-cathode C when the heating current in the tungsten coil was 5 amperes, — the cathode then being at a temperature too low to emit an appreciable electron current. The rise in current as the voltage approaches zero is due to electrons collected from the plasma. At positive voltages C becomes anode and the current is then limited by the emission from K. By plotting the ratio[18] of the electron currents to C and to the anode A (the sum being approximately 60 ma) on semi-logarithmic paper against the voltage of C, a straight line was obtained extending over a range of 6 volts, in which the ratio increased 10^4 fold. From the slope of this line, the temperature of the plasma electrons was found to be 8500° which gives $E_e = 0.733$ volt.

When the current through the tungsten coil was raised to 9.5 amperes, the saturation emission from C, calculated by Richardson's equation from measurements of saturation currents at lower temperatures, increased to 2.2 amperes. We see, however, from the observed volt-ampere Curve II, Fig. 3, that the electron currents were less than 80 ma. These currents remained unchanged if the heating current was lowered to 8.0 amperes (calculated emission 180 ma) but the electron current became saturated at —14 ma with a heating current of 7.0 amps. Thus the currents of Curve II are limited by space charge.

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[17] Reference 3, p. 540.

[18] Langmuir and Jones, *Phys. Rev.* **31,** 396 (1928).

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For positive voltages the two curves coincide since no electrons can escape. The difference between corresponding ordinates for negative voltages thus represents the current of thermionic electrons that leaves the cathode, which we shall call i_e. As the voltage is lowered below 0 volts, a few electrons escape because of their initial velocities which correspond to about $E_c' = 0.1$ volt. The point of inflection at about $E_c = -0.5$ volt thus lies close to the space

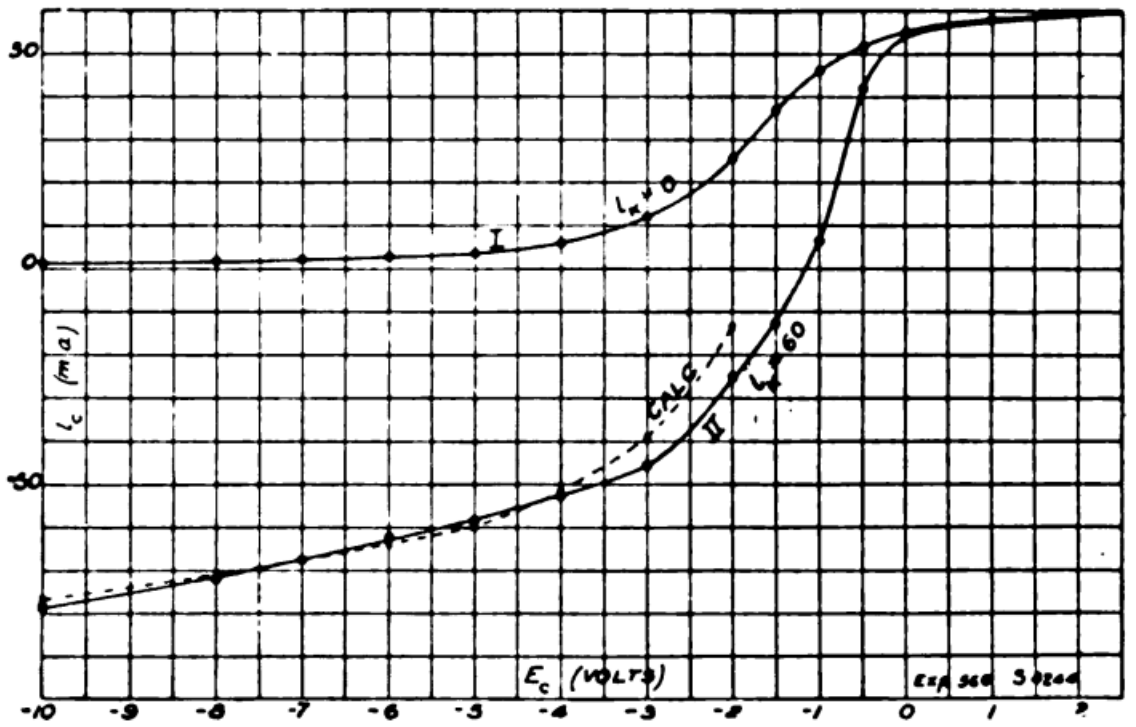

FIG. 3. Volt-ampere characteristics of oxide coated cylindrical cathode; E_c is voltage of the cathode C with respect to the anode A; $E_k = -50$; $i_k = 60$ ma (disregard "$i_k = 0$" on Curve I).

potential. This would seem to indicate a positive anode drop of 0.5 volt. However, because of the contact potential of the oxide on the cathode the effective cathode potential is probably a volt or so more positive than indicated by the value of E_c.

The values of i_e obtained by subtracting the ordinates of Curve II from those of Curve I are tabulated in Column 6 of Table III. A corresponding set of data obtained with an ionizing current $i_k = 20$ ma and a heating current of 9.5 amperes for the oxide cathode C is given in Column 3. We see that although these currents are independent of the temperature of C they are nearly proportional to the current from K showing that the electron current is determined by the supply of ions to the cathode.

Let us first examine these data from the point of view of Eq. (66), that is, let us see whether the variation of i_e with E_c can be explained as the effect of the velocities of the plasma electrons and ions. Placing $E_e = 0.733$ volt from the data of Curve I, $E_p = 0.7\, E_e = 0.513$ volt and $E_c' = 0.1$ volt, we calculate from Eq. (66) the values of the ratio I_e/I_p given in Column 2. Dividing the observed values of i_e by this factor we should obtain the positive ion current to the cathode. However, although I_p in the plasma may be assumed to be independent of E_c (since $-E_c$ is less than the ionizing potential) the positive ion current will vary slightly with E_c because of the changes in the sheath thickness.

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TABLE III

Variation of Electron Current with Cathode Potential

Mercury vapor sat. at 20° $E_k = -60$ Emission of C 2.2 amps.

1	2	3	4	5	6	7	8
E_c	$\frac{I_e}{I_p}$	$i_k = 20$ ma; $I_p = 0.025$ ma cm^{-2}			$i_k = 60$ ma; $I_p = 0.061$ ma cm^{-2}		
		i_e obs.	i_e cal.	x cm	i_e obs.	i_e cal.	x cm
−3	310	22.4	20.7	0.040	57.7	50.9	0.026
−4	347	23.9	23.7	0.048	58.9	57.8	0.031
−5	375	25.5	26.2	0.056	62.2	63.4	0.036
−6	393	27.0	27.9	0.063	65.2	67.1	0.040
−8	420	31.5	30.9	0.076	73.8	73.5	0.048
−10	437	34.0	33.0	0.087	79.9	78.1	0.056

The fifth and eighth columns give the sheath thickness in cm calculated by the space charge equation making allowance for the initial velocities of the ions.[19] The values of I_p used in these calculations according to a method which will be described later, were taken to be 0.025 and 0.061 ma cm^{-2} for $i_k = 20$ and 60 ma respectively.

From the diameter of the sheath $(0.635+2x)$ the collecting area of the cylindrical sheath was found (length 2.54 cm) and thus a positive ion current density I_p' was calculated from each value of i_e. The averages of these, excluding only those at $E_c = -3$, gave 0.0117 and 0.0299 ma cm^{-2} for $i_k = 20$ and 60 ma. These multiplied by the sheath areas and the factor I_e/I_p of Column 2, gave the values of i_e (calc.) in Columns 4 and 7. The dotted curve in Fig. 3 represents the values in Column 7. A comparison of these observed and calculated values (which involve only one empirically determined parameter I_p') shows that the observed variation of i_e with E_c is in satisfactory agreement with Eq. (66) and is thus explained by the velocities of the plasma electrons and ions. Only about 1/4 of the observed variation is due to changes in the sheath thickness.

The slight differences in the shape of the curves obtained from the calculated and the observed values of i_e are probably to be explained; — 1. By uncertainties in the contact potentials which would displace the curves horizontally a volt or so; — 2. By changes in the distribution of ion currents in the plasma, resulting from the space charges of the escaping electrons. If the cathode is made more negative than −10 volts ionization is produced by the emitted electrons which may cause considerable departures from the theoretical curve; for this reason we have confined ourselves in this discussion to the region above −10 volts.

[19] Calculated by Eqs. (10) and (11), Reference 3, p. 452. The positive ion "temperature" which enters only as a small correction, was taken to be 5000° and the voltage in the sheath $-E_c$.

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Since in deriving Eq. (66) we have assumed that plasma electrons in appreciable numbers are not able to reach the cathode we must not expect this equation to apply when, according to Curve I, any large electron current passes to this electrode. Thus we should not expect agreement between the observed and calculated values of i_e at voltages much higher than -4. We see in fact that considerable differences are observed at -3 volts.

TABLE IV

Relation between Electron and Ion Currents

Hg at 20°C $E_k = -50$ $E_B = -85$ $E_c = -8$

Currents in milliamps

Hg temp.	1	2	3	4	5	6	7
	i_k	i_e	i_B	I_p	i_p	x	I_p
20°	10	16.7	0.0376	0.0063	0.0923	0.1316	0.0129
	20	28.6	0.0591	0.0168	0.1580	0.0994	0.0237
	30	38.0	0.0765	0.0269	0.2100	0.0854	0.0327
	40	47.9	0.0940	0.0379	0.2646	0.0755	0.0422
	50	57.7	0.1098	0.0484	0.3188	0.0685	0.0517
	60	66.5	0.1270	0.0604	[0.3674]	0.0636	[0.0604]
Exponent		*0.77*	*0.67*	*1.17*	*0.77*		*0.86*
10°	60	63.9	0.117	0.0534	0.328	0.0676	[0.0534]
Exponent		*0.80*	*0.67*		*0.80*		*0.88*

Studies were also made of the variation of i_e with ionizing current i_k, of the relation of i_e to the ion current density I_p and the effect of varying the mercury vapor pressure. For these purposes E_c was kept constant at -8 volts, and, while i_k was varied from 10 to 60 ma measurements were made of i_e and of i_B, the ion current to the collector B, this being at voltages ranging from -70 to -120. The cathode C was heated to a temperature at which it emitted 2.2 amps. Table IV gives data obtained in this way.

Plotting i_e against i_k on double logarithmic paper gives a straight line of slope 0.77 showing that the electron current increases in proportion to the 0.77 power of the ionizing current i_k. The ion current to the collector i_B, plotted similarly gives approximately a straight line of slope 0.67.

Before we can test the proportionality between i_e and i_p, the ion current to the cathode, which is to be expected by our double sheath theory, corrections must be made for the effects of the thickness of the sheaths on C and B. The theory of these edge corrections[17] indicates that $i_B^{1/2}$ should be a linear function of $v^{1/2}$, the slope S of this line being

$$S = 0.00306g(m_e/m)^{1/4}\text{amp}^{1/2}\text{volt}^{-3/4} \tag{78}$$

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where g is a numerical factor approximately equal to unity and which is dependent solely on the geometry of the collector and where

$$\nu = V^{3/2}\{1+0.0247(T/V)^{1/2}\} \tag{79}$$

is the $V^{3/2}$ of the space charge equation multiplied by a correction factor which allows for the effects of the initial velocities of the ions as they enter the sheath. Numerous experiments in which collectors like B have been used for ions in mercury vapor have given $g = 0.98$ which corresponds by Eq. (78) to $S = 1.21 \times 10^{-4}$. By this theory, to correct i_B for the edge effects, we subtract $S\nu^{1/2}$ from $i_B^{1/2}$ and thus obtain $i_0^{1/2}$ where i_0 is the corrected current; — that is, the current that would flow if no edge correction were needed. Dividing i_0 by the actual area of the collector gives I_p the ion current density. The figures in Column 4 were obtained in this way. Putting $V = 85$ volts, $T = 5000$ for the ion temperature, gives $\nu = 932$ and $S\nu^{1/2} = 3.694 \times 10^{-3}$ amps$^{1/2}$, so that if i_B is expressed in microamps we subtract 3.694 from $i_B^{1/2}$ to obtain $i_0^{1/2}$, i_0 also being in microamps. Dividing i_0 by 0.95 cm^2, the collector surface, gives I_p.

For the lower currents at least, these values are certainly over-corrected. We realize this if we consider that the sheath thickness for $i_k = 10$ ma calculated from the value of I_p in Column 4 is 0.7 cm as compared to a collector diameter of 1.1 cm. We also find by a double logarithmic plot that I_p is proportional to the 1.17 power of i_k, whereas numerous experiments (unpublished) with guard-ring collectors, that need no corrections, have shown that I_p, as observed by a collector mounted not far from the center of a spherical bulb, varies with a power of the ionizing current that ranges from 0.85 to 0.90.[20]

Let us see whether the experimental data of Columns 1, 2 and 3 are consistent with the assumption that the electron current from the cathode is proportional to i_p, the ion current that flows to the cathode. The thickness of the double sheath x according to the last line of Table I, is 1.364 times that calculated by the ordinary space charge equation. The current density used for this calculation should be that at a surface half way between the cathode and the outer edge of the sheath, so that the effective area of the cathode sheath is $\pi(0.635+x)$ 2.54 cm^2. Thus from Eqs. (1) and (79) taking $T = 5000$, $V = 8$ and expressing i_p in ma we get

$$x^2 = 0.00212(0.635+x)/i_p. \tag{80}$$

[20] In mercury arcs, (Reference 3, p. 764) I_p varies in proportion to the 1.25 power of the arc current indicating appreciable ionization of excited atoms. Experiments (Langmuir and Jones, *Phys. Rev.* **31,** 403 (1928)) with ionization by 30—100 volt electrons in which the *total* ion currents were measured using the walls of the cylinder enclosing the discharge as collector have shown that I_p varies with the 1.0 power of the ionizing current even when this varies in a ratio of more than 1:100. The reason for exponents less than unity is undoubtedly to be sought in a variation of the plasma fields which causes the distribution of ions between a central collector and the glass walls to vary with the intensity of the discharge. The fact that the best data (with guard rings) give a straight double logarithmic plot, is entirely inconsistent with recombination of ions and electrons as a cause of the low value of the exponent. Recombination would introduce a term varying with I_p^2 which cannot be reconciled with the observed data.

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The collecting area for ions, however, is $\pi(0.635+2x)$ 2.54 cm² and therefore

$$i_p = 7.98(0.635+2x)I_p. \tag{81}$$

For the highest current, $i_k = 60$, we may assume that the value of I_p in Column 4 is approximately correct. With this value of I_p, Eqs. (80) and (81) solved as simultaneous equations, give $x = 0.0636$ cm and $i_p = 0.3674$ ma. We obtain the other values of i_p in Column 5 by taking them proportional to i_e in Column 2, and from these the sheath thickness x (Column 6) is found by Eq. (80). Equation (81) then enables us to calculate I_p as given in Column 7. These values of I_p give a straight logarithmic plot against i_k of slope 0.86 which agrees with the exponents 0.85 to 0.90 found in the experiments with guard-ring collectors, and therefore our experimental data are consistent with the double sheath theory that requires a proportionality between i_e and i_p.

The ratio i_e/i_p given by these data is 181:1. By Table III at this cathode voltage (−8) we should have the ratio 420:1 so the observed electron current is 0.431 of that calculated, only a little greater than the ratio 0.38 we found from the data supplied by Mr. Vogdes. Undoubtedly part at least of this discrepancy is accounted for by lack of uniformity of the oxide coating and perhaps the lower temperature of the ends of the cathode cylinder.

The last two lines of Table IV give data obtained with mercury vapor saturated at 10° at which the pressure is only 0.40 that at 20°. The exponents were found from logarithmic plots of data with values of i_k ranging from 10 to 60. The ratio i_e/i_p is here 195, somewhat closer to the theoretical value 420.

Filamentary Cathodes

A large number of experiments (Experiment 562-a) have been made with two tungsten cathodes 0.25 mm in diameter, one (K) to produce the ionization of the mercury vapor and the other (C) to give the volt-ampere characteristics. The values of I_p were determined simultaneously using a guardring collector. With C cold the positive ion currents i_p to C could be measured as a function of voltage. Then with C hot enough to emit a surplus of electrons the electron current was measured. The ratio i_e/i_p depended on I_p and on the voltage, E_c, but was independent of the pressure of mercury vapor (10 to 20°C) and of the voltage E_k of the ionizing electrons. For $E_c = -10$ volts the following values were obtained

I_p	i_e/i_p
0.140 ma cm^{-2}	940
0.38	530
0.57	450

A more detailed analysis of the curves obtained in these experiments will be reserved for a subsequent paper.

The writer wishes to express his appreciation of the assistance of Mr. S. P. Sweetser who carried out most of the experiments.

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A GENERAL THEORY OF THE PLASMA OF AN ARC

With LEWI TONKS as co-author

Physical Review
Vol. XXXIV, No. 6, September (1929).

ABSTRACT

The conception of random positive ion velocities corresponding to ion temperatures in a plasma has serious theoretical difficulties and is lacking in direct experimental verification. It is more reasonable to assume that each ion starts from rest and subsequently possesses only the velocity which it acquires by falling through a static electric field which is itself maintained by the balance of electron and ion charges. This new viewpoint thus ascribes motions to the positive ions which, for long free paths, are ordered rather than chaotic, each negative body in contact with the discharge collecting ions from a definite region of the plasma and from it only. The resulting integral equations for the plasma-sheath potential distribution have been set up for plane, cylindrical, and spherical plasmas, for long, short and intermediate length ion free paths, and for both constant rate of ionization throughout the plasma and rate proportional to electron density, and these equations have been solved for the potential distribution in the plasma in all important cases. The case of short ion free paths in a cylinder with ion generation proportional to electron density gives the same potential distribution as found for the positive column by Schottky using his ambipolar diffusion theory, with the advantages that ambipolarity and quasineutrality need not appear as postulates. The calculated potential distribution agrees with that found experimentally. The potential difference between center and edge of plasma approximates $T_e/11{,}600$ volts in all long ion free path cases. The theory yields two equations. One, the *ion current equation*, simply equates the total number of ions reaching the discharge tube wall to the total number of ions generated in the plasma, but it affords a new method of calculating the density of ionization. The second, the *plasma balance equation*, relates rate of ion generation, discharge tube diameter (in the cylindrical case), and electron temperature. It can be used to calculate the rate of ion generation, the resulting values checking (to order of magnitude) those calculated from one-stage ionization probabilities. The potential difference between the center of the plasma and a non-conducting bounding wall as calculated from the ion current equation agrees with that found experimentally.

The solution of the general plasma-sheath equation has been extended into the sheath surrounding the plasma to determine the first order correction which is to be subtracted from the discharge tube radius to obtain the plasma radius. The wall sheath in the positive column is several times the thickness given by the simple space charge equation.

Actually the ions do not start from rest when formed but have small random velocities corresponding to the gas temperature, T_g. In the long ion free path cases this leads to an error of the order of only T_g/T_e in the calculated potential distributions.

In the plasma surrounding a fine negatively charged probe wire the potential difference between plasma potential maximum and sheath edge may be so small that the ions generated within the plasma potential maximum are not trapped but can traverse the maximum by virtue of their finite initial velocities. This justifies the use of a sufficiently fine negatively charged wire in the usual way to measure positive ion concentrations, although certain difficulties appear

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which are thought to be connected with the collector theory rather than the present plasma theory.

A positively charged cylindrical probe collects electrons in the same manner as previously supposed, except that the sheath about it is considerably thickened by the presence of ions generated in the sheath.

The plasma balance equation completes the number of relations needed to determine completely the conditions in a positive column. Taking the arc current as the independent arc variable, the five dependent variables are axial electric field, density of ionization, electron temperature, positive ion current to the wall, and the rate of generation of positive ions. The five relations which determine them are the plasma balance equation, the ion current equation, an ion generation equation, a mobility equation, and an energy balance equation. The essential nature of these relations is recognized even though present knowledge is insufficient to complete all of them.

Stability in the positive column has not been considered. The possibility exists that instability of one type or another may lead to the oscillations which can occur in an arc.

LIST OF SYMBOLS

It has been thought advisable to give below a list of the symbols used in the present paper together with their meanings and a reference to the place of their first appearance in the body of the article. The occasional duplication of symbols will, it is thought, not lead to confusion.

A	Coefficient of $\Delta_s^2\eta$;	Eq. (58)
A	Area of probe electrode;	Eq. (87)
A'	Coefficient of $\Delta_q^2\eta$, (similar to A);	Eq. (79)
a	Discharge tube radius, maximum value of r in plasma.	
a_0, a_1, etc.	Numerical coefficients;	Eq. (17)
B	Numerical coefficient;	Eq. (80)
C	Numerical coefficient;	Eq. (80)
C	As in "Case $CL\lambda$" indicates cylindrical plasma;	Table II
D	Ion-into-gas diffusion constant;	Eq. (27)
e	Electronic charge;	Eq. (1)
f_λ	Defined by	Eq. (93)
H_1, H_2	Integrals;	following Eq. (80)
h_0	Numerical plasma constant;	Eqs. (51) and (53)
I_n	Random electron current density at tube wall;	following Eq. (55)
I_p	Positive ion current density, random;	Eqs. (1), (87), and (88)
	at discharge tube wall;	Eq. (49)
i_B	Arc current;	Eq. (97)
i_e, i_p	Electron, positive ion currents to probe;	Eq. (87)
J	N_z when N_z is constant;	Eq. (24)
J	As in "Case CLJ" denotes that N_z is constant;	Table II
J_0	Bessel Function of zero order;	Table IId
k	Boltzmann constant;	Eq. (1)
L	As in "Case $CL\lambda$" indicates long ion free paths;	Table II
L and M-S	Refers to I. Langmuir and H. M. Mott-Smith, Jr. articles;	Footnote (1)
l_e	Mean free path of electrons;	Eq. (96)
l_g	Mean free path of gas molecules.	
l_p	Mean free path of positive ions.	
M	As in "Case $CM\lambda$" indicates "medium" ion free paths;	Table II
M. D.	Maxwell Distribution.	
m_e	Mass of electron;	Eq. (1)

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m_p	Mass of pasitive ion;	Eq. (1)
N	Number of electrons per cm length of arc;	Eq. (97)
N_z	Number of ions generated $cm^{-3} sec^{-1}$ at z;	Eq. (2)
n_e	Electron density;	Eq. (3)
n_0	Electron density at plasma potential maximum;	Eq. (3)
n_p	Positive ion density;	Eq. (2)
n_z	Electron density at z;	Eq. (8)
P	Number of ions formed per electron per cm of path;	Eq. (91)
P	As in "Case PLJ" indicates plane plasma;	Table II
$p+1$	Term number in a power series;	following Eq. (17)
p_0	Gas pressure in baryes;	Eq. (91)
q	Ion mobility per unit charge;	Eq. (29)
q	Dimensionless length parameter;	Eq. (79)
q'	Ion mobility factor;	Eq. (35)
r	Distance in plasma;	Eq. (2)
S	Value of s at plasma potential maximum;	Eq. (82)
S	As in "Case $SL\lambda$" indicates spherical plasma;	Table II
S	As in "Case $PS\lambda$" indicates short ion free paths;	Table II
s	Dimensionless length parameter;	Eq. (10)
s'	Abbreviation for $ds/d\eta$.	
s''	Abbreviation for $d^2s/d\eta^2$.	
s_0 s_0''	Value of s Value of $d^2s/d\eta^2$ } for which $d\eta/ds = \infty$.	
s_t	Value of s;	Fig. 7
s_z	Dimensionless length parameter;	Eq. (11)
s_ϕ	Value of s at sheath edge;	preceding Eq. (62)
T_e	Electron temperature;	Eq. (3)
T_g	Gas temperature;	Eq. (27)
T_p	Positive ion temperature;	Eq. (1)
V	Space potential at r relative to plasma potential maximum.	
V_i	Ionization potential of gas;	Eq. (91)
V_w	Wall potential with respect to plasma potential maximum.	
V_z	Space potential at z;	Eq. (7)
v	$= v_z$ when v_z is independent of z;	Eq. (27)
$\bar{v}$	Average thermal velocity of gas molecules.	
v_e	Average thermal velocity of electrons;	Eq. (96)
v_f v_0	Component of average {final / initial} velocity of ion in free path;	preceding Eq. (35)
$\bar{v}_p$	Mean ion drift velocity;	preceding Eq. (57)
v_z	Velocity at r of ion generated at z;	Eq. (2)
W	Potential equivalent of electron velocity;	Eq. (91)
W_e	kT_e/e;	Eq. (71)
X	Potential gradient drawing ions toward sheath.	
x_p	Length unit;	defined by Eq. (71)
Z	Longitudinal potential gradient in positive column;	Eq. (97)
z	Distance in plasma;	Eq. (2)
α	Factor for transforming variables;	Eq. (10)
α_λ	Defined by;	Eq. (94)
β	Geometrical parameter;	Eq. (2)
β	Probability-of-ionization constant;	Eq. (91)
δs	$s - s_0$;	Eq. (63)

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$\delta\eta$	$\eta-\eta_0$;	Eq. (63)
δs_Φ	s_0-s_Φ;	Eq. (66)
$\delta\eta_\Phi$	$\eta_0-\eta_\Phi$;	Eq. (66)
ε	Base of natural logarithms.	
ζ	Potential parameter, $\eta^{1/2}/S$;	Eq. (83)
η	$-eV/kT_e$.	
η_0	Value of η for which $d\eta/ds = \infty$.	
$\overline{\eta}_0$	Mean value of $n_0-\eta$;	Footnote (19)
η_s	$\eta-\eta_\Phi+\overline{\eta}_\Phi$.	
η_z	$-eV_z/kT_e$.	Eq. (56)
η_w	$-eV_w/kT_e$;	Eq. (56)
η_Φ	Value of η at sheath edge;	preceding Eq. (62)
$\overline{\eta}_\Phi$	Mean value of $\eta_\Phi-\eta$;	Footnote (19)
θ	Integration parameter;	Eq. (15)
λ	Number of ions generated per electron per sec;	Eq. (8)
λ	As in "Case $CL\lambda$" indicates generation of ions proportional to electron density;	Table II
μ	$-eV/kT_p$;	Eq. (79)
μ_e	Electron mobility;	Eq. (96)
μ_z	$-eV_z/kT_p$;	Eq. (79)
ξ	Distance parameter in sheath;	Eq. (70)
ϱ	Fractional part of tube radius occupied by sheath;	Eq. (77)
ϱ	Integration parameter;	Eq. (15)
ϱ_e	Electron reflection coefficient;	Eq. (56)
ϱ_z	Integration parameter;	Eq. (15)
Σ	Constant slope of i_p^2vs. V or i_e^2 vs. V plot;	Eq. (88)
σ	Distance parameter, s/S;	Eq. (83)
τ	T_p/T_e (Section V only).	
τ	Mean free time of positive ions.	
Φ	Arbitrary fraction;	Eq. (61)

I. Introduction

THERE is a large amount of evidence to show that the vast majority of free electrons in the plasma of a gaseous discharge possess velocities distributed according to the Maxwell Distribution Law (hereafter "M. D."). The temperatures to which these velocity distributions correspond lie roughly in the range between 5000 and 70,000°K. The method for measuring these temperatures need not be gone into[1] beyond mentioning that it depends on the Boltzmann density distribution which the electrons assume in the sheath about a negatively charged collector. The validity of the method hinges, first, on the confinement of the electrode potential changes to a sheath about the electrode, and second, to the smallness of changes in sheath cross-section or volume compared to the whole cross-section or volume of the plasma.

No such direct measurements on the positive ions are possible for various reasons. The most direct evidence that the ions possess considerable velocities

[1] I. Langmuir and H. M. Mott-Smith, Jr., *Gen. Elect. Rev.* **27**, 449, 538, 616, 762, 810 (1924). Hereafter these articles will be referred to as "*L* and *M-S*, Part I, etc., to Part V", respectively.

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lies in the saturation current to a collector at negative voltages. It has been reasoned that since the potential difference between plasma and electrode is confined to the sheath, only those ions will be collected which are headed for the sheath edge anyway. The random ion current density I_p in the discharge can then be found by dividing the observed saturation current by the sheath area. The ion density is equal to the electron density n_e in the field-free plasma. On the assumption that the velocities are distributed according to the M. D. Law, but only over one hemisphere since no ions leave the collector, kinetic theory gives

$$T_p^{1/2} = (2\pi m_p/k)^{1/2} I_p/2en_e = 2.02\times 10^{13}(I_p/n_e)(m_p/m_e)^{1/2} \qquad (1)$$

A saturation electron current at small positive voltages on the same collector can be observed if the collector area is not too great. This can be treated in the same way, giving[2]

$$T_e^{1/2} = 4.03\times 10^{13} I_e/n_e.$$

Combining the two it is found that

$$T_p/T_e = (I_p/2I_e)^2 m_p/m_e$$

In a typical set of measurements made on a mercury arc[3] the average value of I_e/I_p was 405 ± 25. For mercury $m_p/m_e = 3.678\times 10^5$, whence it would follow that $T_p/T_e = 0.55$. It seems entirely unreasonable, however, that the ion energy should even approach the electron energy in view of the fact that it is the electrons primarily which supply energy to the rest of the plasma and the positive ions with their large relative mass and frequent impact with slow atoms are not adapted to acquiring large random kinetic energies.

But evidence in favor of a large random ion current is found in another experiment. When two equal plane electrodes, back to back and insulated from each other, are placed in the positive column of a discharge so that one electrode faces the anode, the other the cathode, the two electrodes receive comparable ion currents even though to reach one of the electrodes the ions must flow toward the anode. A similar conclusion offers itself in the related experiment in which the sheaths on a spherical electrode immersed in a positive column are seen to be of equal thickness on cathode and anode sides.[4]

There is other evidence also. Experiments in which a double electrode is used, the electrode facing the discharge being pierced with fine holes,[5] give ion temperatures of several thousand degrees. But how accurately Maxwellian these results show the velocity distribution to be is somewhat doubtful.

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[2] There appears to be some justification for using 4.03 in the equation pertaining to electrons as compared to half of this for the ions. See I. Langmuir, *Phys. Rev.* **33**, 96–45 (1929).

[3] *L* and *M-S*, Part II, Table III.

[4] *L* and *M-S*, Part V, p. 812.

[5] L. Tonks, H. Mott-Smith, Jr., and I. Langmuir, *Phys. Rev.* **28** 104 (1926).

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Although all this evidence points to the possession of considerable velocities by the positive ions, and the concept of a random velocity distribution among the positive ions has been a generally useful one in explaining a multitude of observations, there has been no convincing determination of the velocity distribution, and the hypothesis of large random velocities has grave theoretical difficulties to overcome.[6] A further difficulty crops up as soon as the attempt is made to join the sheath to the plasma, that is, to investigate theoretically the nature of the sheath edge.[7] In quite general terms the perplexity is this. Consider two nearby points in the discharge, one just inside the sheath about a negatively charged electrode and one just outside the sheath. At the outside point suppose that there is a one-sided M. D. of velocity among the ions — one-sided since no ions come out through the sheath. At this point the electron and ion densities are equal. The potential at the inside point is slightly less than in the plasma, with the result that both electron and ion densities are less there, the electrons according to the Boltzmann Law and the ions because of their greater average velocity. This can readily be seen by plotting the theoretical densities against the potential decrease. But these curves show an astonishing relation — for small negative potentials the electron density exceeds that of the positive ions because the electron density curve approaches the plasma potential with a finite, the ion curve with an infinite slope. By Poisson's Equation any such predominance of negative charge at the sheath edge requires positive curvature in the potential distribution curve there, thus making it impossible to merge the sheath into the plasma.

We have now reached a new point of view which seems in every way to be more satisfactory. We suppose as before that the electrons possess a M. D. of velocity and such a high mobility that they obey the Boltzmann Law irrespective of any drift in the plasma away from their points of origin. (This, is of course, not true of the longitudinal gradient in a positive column, but the arc current is so very much greater than the drift currents necessitated by the generation of electrons, that this gradient by its smallness justifies rather than invalidates our assumption.) But the positive ions are supposed to have negligible velocity when formed and to acquire only such velocities as correspond to the electric fields through which they pass. In the case of long mean free paths each ion will thus fall freely under the influence of the small plasma fields set up by the electrons and ions themselves until it strikes the tube wall or an electrode. For short free paths the ion will be impeded in its motion by collisions with atoms but still will be mainly guided by the electric field in which it finds itself.

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[6] In applying his quasi-neutral diffusion theory to the positive column of an arc W. Schottky, *Phys. Zeits.* **25**, 346 (1924), abandons the idea that the positive ions have a temperature comparable with the electron temperature.

[7] I. Langmuir, *Phys. Rev.* **33**, 976 (1929).

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This point of view, it will be seen, ascribes a less chaotic motion to the ions than they possessed according to the old concept. Thus in a sphere or cylinder at very low pressure the ions all move radially outward, each with a velocity corresponding to its point of origin, and for any geometrical configuration it becomes theoretically possible to associate each element of wall or electrode area with a tubular region of the plasma which alone contributes ions to that area.[8] Accompanying this picture is the idea that it is the presence of an electrode (or tube wall) in contact with the discharge which is responsible for the ion current flowing to that electrode by reason of its influence, as a boundary condition, on the potential distribution in the plasma.[9]

In order to handle this theory mathematically it is necessary to know the space distribution of the ion generation. Two cases will be considered below. If the ions are generated wholly by electrons which have acquired their velocity in the sheath surrounding an electron source, and the mean free path is long compared to the tube dimensions, the generation will, in many cases, be essentially uniform throughout the plasma. If, on the other hand, the ion generation is caused by the "ultimate" electrons themselves which are constantly renewing their energy, as in the positive column of a discharge, the rate of generation of ions will be proportional to the electron density. Other cases may suggest themselves, as for instance, generation by fast electrons at higher pressures where the mean free path of the electrons is small, and these cases can also be handled by the methods developed here.

For all geometrical configurations except those possessing the simplest symmetry, the mathematical difficulties become very great, and for this reason the quantitative treatment is confined in the following section to plasmas bounded externally by two infinite parallel planes, or by an infinite circular cylinder, or by a sphere, and in Sections VI and VII to plasmas bounded internally by a cylinder.

II. Potential Distribution in the Plasma. Part I

It is evident that if ions are to flow to electrodes and walls under the influence of the electric fields hypothecated, there must be a potential maximum in the plasma, and, in the simple cases to be discussed here, symmetry considerations place it at the center of the boundary structure and the ions thus move outward in straight lines. It is advantageous to select the origin of coordinates at the center (median plane, cylinder axis, or sphere-center) of the

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[8] This is, of course, an idealized representation. In any case the ions possess the random motions which they had as atoms just before ionization occurred, but this is usually small compared to the potentials through which the ion falls in the plasma. Cases in which these small velocities are not negligible appear later. At the higher pressures collisions with atoms introduce additional randomness.

[9] Later, in Section VI, it will be found that very small electrodes have no effect on the potential distribution through the body of the plasma and can, therefore, be used as true probes.

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structure and to denote distance in cm from this center by r. If the potential at $r = 0$ be taken as zero, it is negative elsewhere and the ions generated at any point z acquire a certain velocity v_z by the time they pass some further point r. In general v_z may be a complicated function of the potential distribution, and the high and low pressure cases are distinguished by the type of function assumed for v_z. If the number of ions generated per second per unit volume at z is N_z, their density when they pass r is, in each case, respectively,

Plane: $N_z\,dz/v_z$

Cylinder: $N_z z\,dz/rv_z$

Sphere: $N_z z^2\,dz/r^2 v_z$

Since ions generated at every value of z which is less than r contribute to n_p, the ion density at r, this density is given by

$$n_p = r^{-\beta}\int_0^r N_z z^\beta\,dz/v_z \tag{2}$$

where β assumes the values, 0, 1, 2, for plane, cylindrical, and spherical cases respectively.

The electron density at any point is given by

$$n_e = n_0 \exp\,(eV/kT_e) \tag{3}$$

where n_0 is the electron density at the origin, T_e is the electron temperature and V is the space potential. Poisson's Equation may be written

$$\nabla^2 V = -4\pi e(n_p - n_e).$$

Substituting Eqs. (3) and (2) in this equation we have

$$\nabla^2 V - 4\pi e n_0 \exp\,(eV/kT_e) + 4\pi e r^{-\beta}\int_0^r N_z z^\beta\,dz/v_z = 0. \tag{4}$$

This is the general integral equation for the potential distribution throughout plasma and sheath. In the form of Eq. (4) and in subsequent forms to which this equation may be transformed in various cases it will be known as the *complete plasma-sheath equation*. Throughout its various metamorphoses it will continue to consist of three terms, namely, one term corresponding to the Poisson differential coefficient, another corresponding to the electron density, and a third corresponding to the positive ion density. In its complete form this equation is far too complicated to handle, but it can, fortunately, be simplified in two important regions. First, in the plasma it will appear that the Poisson term is negligible. Dropping it from the complete equation leaves the *plasma equation*, the equation with which the present section deals. Second, in the sheath bounding the plasma other simplifications can be made leading to the *sheath solution* which will be discussed in Section IV.

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Eq. (4) can be simplified immediately by the substitution of a new dimensionless variable η for V,

$$\eta = -eV/kT_e. \tag{5}$$

The equation then becomes

$$(kT_e/4\pi e^2 n_0)\nabla^2\eta + \epsilon^{-\eta} - n_0^{-1} r^{-\beta} \int_0^r N_z z^\beta \, dz/v_z = 0. \tag{6}$$

This equation must now be adapted to the various particular cases.

Case 1. *Long mean free path and ion generation proportional to electron density*.—In this case each ion falls freely from the point at which it is generated. Accordingly,

$$v_z = [2e(V_z - V)/m_p]^{1/2} = [2kT_e(\eta - \eta_z)/m_p]^{1/2} \tag{7}$$

where V_z and V are the potentials at z and r respectively. Here, also

$$N_z = \lambda n_z = \lambda n_0 \epsilon^{-\eta_z} \tag{8}$$

where λ is the number of ions generated by an electron in one second. Substituting these expressions in Eq. (6), we have

$$(kT_e/4\pi e^2 n_0)\nabla^2\eta + \epsilon^{-\eta} - \lambda (m_p/2kT_e)^{1/2} r^{-\beta} \int_0^r z^\beta \epsilon^{-\eta_z} (\eta - \eta_z)^{-1/2} \, dz = 0. \tag{9}$$

In order to solve this equation the substitution

$$s = \alpha r \tag{10}$$

is necessary where s is the new variable and α is an adjustable constant. Remembering that $\Delta^2\eta$ has the dimensions of ηr^{-2}, it is seen that Eq. (9) takes on the form

$$(\alpha^2 kT_e/4\pi e^2 n_0)\nabla_s^2\eta + \epsilon^{-\eta} - \lambda (m_p/2kT_e)^{1/2} s^{-\beta} \alpha^{-1} \int_0^s s_z^\beta \epsilon^{-\eta_z} (\eta - \eta_z)^{-1/2} \, ds_z = 0 \tag{11}$$

where ∇_s^2 indicates that derivatives are taken with respect to s, and η_z is the same function of s_z that η is of s. At this point α is so chosen that the coefficients of the second and third terms are equal, that is

$$\alpha = \lambda (m_p/2kT_e)^{1/2} \tag{12}$$

and as α has the dimensions of a reciprocal length, s as well as η is dimensionless. This substitution puts the differential equation in the form

$$(m_p\lambda^2/8\pi e^2 n_0)\nabla_s^2\eta + \epsilon^{-\eta} - s^{-\beta} \int_0^s s_z^\beta \epsilon^{-\eta_z} (\eta - \eta_z)^{-1/2} \, ds_z = 0. \tag{13}$$

Later it will be shown that the introduction of typical values for the constants in the coefficient of $\nabla_s^2\eta$ renders that term negligible over practically the whole range of s in most cases. Thus the definition of the plasma as the region where ion and electron space charges are essentially equal receives addi-

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tional justification. If now the sheath edge is defined as the surface at which this essential equality fails, it is left somewhat indefinite, thus requiring a detailed treatment of the plasma-sheath transition, which will be touched on later.

Dropping the first term, then, we have to solve the integral equation

$$\epsilon^{-\eta} - s^{-\beta} \int_0^s s_z^{\beta} \epsilon^{-\eta_z} (\eta - \eta_z)^{-1/2} \, ds_z = 0. \tag{14}$$

The fact that the plasma equation can be reduced to this dimensionless form immediately enables us to draw the important conclusion that the potential distribution curve will be of the same shape irrespective of the particular values which the constants which originally entered the equation may have. To make the general curve fit any particular case it will simply be necessary to change the ordinate (η) scale according to the value of T_e and the abscissa (s) scale according to the tube dimensions and the thickness of the sheath on the wall when that is appreciable.

The solution of Eq. (14) is made possible by regarding η rather than s as the independent variable and putting successively

$$\eta = \varrho^2, \qquad \eta_z = \varrho_z^2, \qquad \varrho_z = \varrho \sin\theta. \tag{15}$$

This converts the equation to the form

$$s^{\beta} \epsilon^{-\varrho^2} - \int_0^{\pi/2} s_z^{\beta} \epsilon^{-\varrho^2 \sin^2\theta} (ds_z/d\varrho_z) \, d\theta = 0. \tag{16}$$

If now s be expressed as a power series in ϱ

$$s = a_0 + a_1\varrho + a_2\varrho^2 + \cdots \tag{17}$$

with s_z being expressed as the same function of ϱ_z, it is obvious that the integrand itself can be expressed in a series

$$b_0 + b_1\varrho_z + b_2\varrho_z^2 + \cdots$$

in which each b is a function of certain of the a's. The substitutions indicated in Eq. (15) give the $(p+1)$ term of this series the form $b_p\varrho^p \sin^p\theta$. The integration limits in terms of θ are 0 and $\pi/2$, and this integral of $\sin^p\theta d\theta$ being given in the tables, the integral term of Eq. (16) comes out as a power series in ϱ. The coefficients of this series can then be equated term by term with those in the expansion of the first term in Eq. (16) in order to evaluate a_0, a_1, a_2, etc. The boundary condition that $s = 0$, at $\varrho = 0$ requires that $a_0 = 0$. In the final solutions it also turns out that $0 = a_2 = a_4 = a_8$, etc., so, for simplicity, these values will be assumed immediately in obtaining the actual solutions. We shall, therefore, use,

$$s = a_1\varrho + a_3\varrho^3 + a_5\varrho^5 + \cdots. \tag{18}$$

The second boundary condition that the electric field at the origin be zero requires that $d\eta/ds = 0$, that is that $ds/d(\varrho^2) = \infty$ there, a condition which the above solution is seen to satisfy also.

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The cylindrical case may be used to illustrate the evaluation of the coefficients in detail. For this case $\beta = 1$ so that Eq. (16) becomes

$$s\epsilon^{-\varrho^2} - \int_0^{\pi/2} s_z(ds_z/d\varrho_z)\epsilon^{-\varrho_z^2}\, d\theta = 0. \tag{19}$$

From Eq. (18)

$$s_z\, ds_z/d\varrho_z = a_1^2\varrho_z + 2(2a_1a_3)\varrho_z^3 + 3(a_3^2+2a_1a_5)\varrho_z^5 + \cdots.$$

Also

$$\epsilon^{-\varrho_z^2} = 1-\varrho_z^2+\varrho_z^4/2!-\varrho_z^6/3!+\cdots.$$

Replacing ϱ_z^{2m+1} in the product by $\varrho^{2m+1}\sin^{2m+1}\theta$, the coefficient of ϱ^{2m+1} is seen to be

$$(-1)^m \sin^{2m+1}\theta\{a_1/m!-2(2a_1a_3)/(m-1)!+3(a_3^2+2a_1a_5)/(m-2)!-\cdots$$
$$\pm k[a_k^2+2(a_1a_{k-1}+a_3a_{2k-3}+\cdots)]/(m-k+1)!+\cdots \text{ to } k = m+1\} \tag{20}$$

Now

$$\int_0^{\pi/2} \sin^{2m+1}\theta d\theta = \frac{1\cdot 2\cdot 4\cdot 6\cdots(2m)}{1\cdot 3\cdot 5\cdot 7\cdots(2m+1)}$$

so that, denoting the bracketed expression in (20) by σ_m the coefficient of ϱ^{2m+1} in the integral is

$$(-1)^m\sigma_m \frac{1\cdot 2\cdot 4\cdot 6\cdots(2m)}{1\cdot 3\cdot 5\cdot 7\cdots(2m+1)}. \tag{21}$$

For the first term of Eq. (19) we have to form the product

$$(a_1\varrho+a_3\varrho^3+\cdots)(1-\varrho^2+\varrho^4/2!-\cdots).$$

The coefficient of ϱ^{2m+1} here is

$$-(-1)^m[a_1/m!-a_3/(m-1)!+a_5/(m-2)!-\cdots\pm a_{2m+1}].$$

Setting the sum of this expression and (21) equal to zero we find that the condition to be satisfied by the a's is

$$a_1/m!-a_3/(m-1)!+a_5/(m-2)!-\cdots = \frac{1\cdot 2\cdot 4\cdot 6\cdots(2m)}{1\cdot 3\cdot 5\cdot 7\cdots(2m+1)}$$
$$\times[a_1^2/m!-2(2a_1a_3)/(m-1)!+3(a_3^2+2a_1a_5)/(m-2)!-\cdots] \tag{22}$$

To evaluate the a's, m is set equal successively to 0, 1, 2, etc.
For $m = 0$, $a_1 = a_1^2$ whence $a_1 = 1$
For $m = 1$, $1-a_3 = (2/3)\ (1-4a_3)$, whence $a_3 = -0.2$
For $m = 2$, $1/2!+0.2+a_5 = (8/15)[1/2!+0.8+3(0.04+2a_5)]$ whence $a_5 = -0.026061$ and so on, giving for the solution

$$s = \eta^{1/2}(1-0.2\eta-0.026061\eta^2-0.0064894\eta^3$$
$$-0.0019840\eta^4-0.00067937\eta^5-0.000253\eta^6-0.000101\eta^7-\cdots) \tag{23}$$

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The coefficients of the last two terms are estimated from the trend of the preceding ones.

If this series be used to calculate the potential distribution, it is found that at a certain value of η, say η_0, $d\eta/ds$ passes through infinity and that thereafter s decreases with increasing η. We recognize in η_0 and in the corresponding value s_0 of s the extreme limit for the validity of the plasma solution since $d^2\eta/ds^2$ is infinite at s_0 and the $\nabla_s^2\eta$ term of Eq. (13) ceases to be negligible at some lesser value of s. By setting the derivative of Eq. (23) equal to zero, it is found by successive approximations that $\eta_0 = 1.155$, $s_0 = 0.7722$.

The results of a similar calculation for the plane case are given in Table IIa at the conclusion of this section.

Case 2. Long mean free path and ion generation constant throughout the plasma.—This case differs from the previous one in that here

$$N_z = J \tag{24}$$

J being the number of ions generated per cm³ per sec. throughout the plasma. Examination of the previous treatment will show that all the conversion factors and integral equations there used can be adapted to the new case by omitting the ϵ^{η_z} under the integral sign and substituting J/n_0 for λ everywhere. Thus, instead of Eqs. (12) and (13) we have

$$\alpha = J(m_p/2kT_e)^{1/2}/n_0 \tag{25}$$

$$(m_pJ^2/8\pi e^2n_0^3)\nabla_s^2\eta + \epsilon^{-\eta} - s^{-\beta}\int_0^s s_z^{\beta}(\eta-\eta_z)^{-1/2}\,ds_z = 0. \tag{26}$$

This equation can be solved by the identical method used before and the resulting solutions having the same general character as the earlier ones are given in Table IIb.

Case 3. Short mean free paths and uniform ion generation.—A. If the mean free paths are short compared to the dimensions of the plasma and the field strength is so small that the energy which an ion picks up between impacts with atoms is small compared to the thermal energy of the atoms, the ion drift velocity at r is made up of two components, one arising from diffusion, the other from the electric field,

$$v_z = -(D/n_p)\,dn_p/dr - (eD/kT_g)dV/dr$$

where T_g is the temperature of the gas and D is the diffusion constant for the ions into the gas. The fact that v_z has ceased to be a function of z will be indicated henceforward by dropping the subscript z. Since ion and electron densities are equal in the plasma, n_p can be substituted for N_e in Eq. (3), thus making it possible to eliminate n_p and dn_p and giving

$$v = -(eD/k)(T_e^{-1} + T_g^{-1})\,dV/dr$$

and since $T_g << T_e$, the diffusion component is negligible and

$$v = -(eD/kT_g)\,dV/dr. \tag{27}$$

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Assuming D to be the same as the interdiffusion constant for the gas, we have[10] $D = 0.561 l_g \bar{v}$ where l_g is the mean free path and $\bar{v}$ the average speed of the atoms in their random motion. Expressing $\bar{v}$ in terms of T_g we have

$$v = 2(2kT_g/\pi m_p)^{1/2} = 1.597(kT_g/m_p)^{1/2}$$

Thus Eq. (27) can be put in the form

$$\begin{aligned} v &= -0.895 l_g (kT_g m_p)^{-1/2} e dV/dr \\ &= -qedV/dr \end{aligned} \tag{28}$$

where

$$q = 0.895 l_g/(kT_g m_p)^{1/2}. \tag{29}$$

for convenience. Now $N_z = J$ can be brought from under the integral sign in Eq. (6) and since $v_z = v$ of Eq. (28) is independent of z this, too, comes out, giving

$$(kT_e/4\pi e^2 n_0)\nabla^2\eta + \epsilon^{-\eta} - (J/n_0 qkT_e) r^{-\beta}(d\eta/dr)^{-1} \int_0^r s_z^\beta \, ds_z = 0. \tag{30}$$

The substitutions used in Case 1 here lead to

$$\alpha^2 = J/n_0 qkT_e \tag{31}$$

and

$$(J/4\pi e^2 n_0^2 q)\nabla_s^2\eta + \epsilon^{-\eta} - s^{-\beta}(ds/d\eta) \int_0^s s_z^\beta \, ds_z = 0. \tag{32}$$

As before the first term can be neglected for the present and the solution comes out very simply as

$$\epsilon^{-\eta} = 1 - s^2/2(\beta+1). \tag{33}$$

A marked difference between this solution and those previously obtained is that here $n_0 = \infty$ although s_0 is finite, having the values

$$s_0 = 1.414, \quad 2.000, \quad 2.449; \quad \beta = 0, 1, 2. \tag{34}$$

In this case then, the potential at which the present solution fails to be a good approximation must be very far indeed from n_0 although the corresponding value of s may not be very different from s_0.

B.[11] At pressures intermediate between those just discussed and those contemplated in Cases 1 and 2 the ion temperature will be determined less by the gas temperature and more by the energy acquired in a free path. Thus T_g in

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[10] Jeans, *Dynamical Theory of Gases*, 2nd edition, § 440.

[11] Here, as later in Case 4*B*, the significance of the analysis is somewhat doubtful, first because of the small radial field strength near the potential maximum as mentioned below, and second, because in the very important class of cases pertaining to the positive column of an arc there is a uniform longitudinal field. Both in the free fall and the short path cases the ion motions can be simply resolved into components, leaving the theoretical results unaffected by this uniform field, but this resolution fails when, as here, the drift velocity is proportional to the square root of the field.

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Eq. (29) must be replaced by T_p, the ion temperature, and this becomes proportional to dV/dr so that the drift velocity is proportional to the square root of the field strength.[12] A rough value for the drift velocity can be calculated as follows. Let the ions be accelerated by a uniform field X and let the average components of velocity in the direction of the field at the beginning and the end of a mean free path be v_0 and v_f respectively. The gas atoms are moving so slowly compared to the ions that they can be assumed to be at rest. The persistence of velocity for the resulting type of collision[13] is 0.5 so that

$$v_f = 2v_0.$$

Denoting the mean free time of the ion between collisions by τ there are the additional relations

$$v_f = v_0 + (eX/m_p)\tau$$
$$l_p = v_0 + (eX/m_p)\tau^2$$

where l_p is the positive ion mean free path which is $2^{1/2}$ times the atom free path on account of the higher velocity. The last equation neglects the existence of a velocity component perpendicular to the field, but the effect of such a component in decreasing the progress made between collisions is small enough to be neglected in this rough calculation. Eliminating τ and v_f from these three equations, solving for v_0, and then finding $v = (v_0+v_f)/2$, the drift velocity, it is found that

$$v = 1.2(eXl_p/m_p)^{1/2}$$

If in the present case we put

$$q' = 1.2(l_p/m_p)^{1/2} \tag{35}$$

then

$$v = -q'(edV/dr)^{1/2}. \tag{36}$$

Substituting this expression for v and J for N_z in Eq. (6), and at the same time introducing s we have

$$a = (J^2 n_0^2 q'^2 kT)^{1/3} \tag{37}$$

$$(1/4\pi e^2)(kTJ^4/n_0^7 q'^4)^{1/3}\nabla_s^2\eta + \epsilon^{-\eta} - s^\beta (ds/d\eta)^{1/2}\int_0^s s_z^\beta\, ds_z = 0 \tag{38}$$

the solution for which, neglecting the $\nabla_s^2\eta$ term as usual, is

$$\epsilon^{-\eta} = [1-2s^3/3(1+\beta)^2]^{1/2}. \tag{39}$$

---

[12] P. M. Morse, *Phys. Rev.* **31,** 1003 (1928) uses a formula for ion mobility which is an adaptation of an expression for electron mobilities derived by K. T. Compton, *Phys. Rev.* **22,** 333 (1923). Morse does not, however, take into account the finite persistence of velocity of the ions, with the result that his consolidated numerical coefficient is 0.858 compared to 1.2 derived here, Eq. (35).

[13] *The Dynamical Theory of Gases*, J. H. Jeans, p. 279, 2nd edition.

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In this case, too, $\eta_0 = \infty$ while

$$s_0 = 1.145, \quad 1.816, \quad 2.378; \quad \text{for } \beta = 0, 1, 2. \tag{40}$$

A unique feature of this potential distribution is that η approaches zero at $s = 0$ along a cubic rather than along a parabola, as in all the previous cases. It is probable, however, that this is of little significance because the approach of the electric field to zero in the neighborhood of the origin makes Eq. (28) rather than Eq. (36) the more applicable there.

Case 4. Short mean free paths and ion generation proportional to electron density.—Here again two cases must be distinguished according to the length of the ion free paths relative to the electric field and gas temperature.

A. In the range of shorter free paths Eq. (30) is readily adapted by substituting $\lambda n_0 \epsilon^{-\eta_z}$ for J, leading to the equations

$$a = (\lambda/qkT)^{1/2} \tag{41}$$

$$(\lambda/4\pi e^2 n_0 q)\nabla_s^2 \eta + \epsilon^{-\eta} - s^{-\beta}(ds/d\eta)\int_0^s \epsilon^{-\eta_z} s_z^\beta \, ds_z = 0. \tag{42}$$

The solution, when the first term is neglected as usual, can be easily transformed into the Bessel Equation of order $(1-\beta)/2$

$$d^2w/ds^2 + (1/s)dw/ds + [1-(1-\beta)^2/4s^2)]w = 0$$

by putting $w = s^{(\beta-1)/2}\epsilon^{-\eta}$. The solutions are given in Table IId.

The solution for the cylindrical case,

$$\varepsilon^{-\eta} = J_0([\lambda/qkT_e]^{1/2}r),$$

is identical with Schottky's solution which he gives[14] as

$$n = n_0 J_0([a/D_a]^{1/2}r)$$

(the a being the present λ) if one assumes the ion temperature and mobility entering the expression for the ambipolar diffusion constant D_a to be small compared to the electron temperature and mobility, respectively. The new treatment is thought to have two advantages over the earlier treatment, first in that the present method, by including the Poisson term in the fundamental equation, will tell us when this term can no longer be neglected and how the problem can be handled beyond this point (see Section IV), and second, in that the radial ambipolarity of the diffusion is shown not to be essential to the solution. Schottky recognizes that his solution is, in fact, inconsistent with this idea, for its assumption that n is zero at the tube wall is equivalent to having the wall at an infinite negative potential, in which case the electron current would be zero. The essential requirement appears to be that the loss of electrons to the walls shall not be sufficient to disturb materially the M. D. of electrons in the plasma.

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[14] W. Schottky, *Phys. Zeits.* **25**, 635 (1924).

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B. In the range of longer free paths Eqs. (37) and (38) can be readily adapted by writing λn_0 for J and putting $\epsilon^{-\eta_z}$ under the integral sign, giving

$$a = (\lambda^2/q'^2kT)^{1/3} \tag{43}$$

$$(1/4\pi n_0 e^2)(kT\lambda^4/q'^4)^{1/3}\nabla_s^2\eta+\epsilon^{-\eta}-s^\beta(ds/d\eta)^{1/2}\int_0^s \epsilon^{-\eta_z}s_z\,ds_z = 0. \tag{44}$$

With the omission of the first term the equation can be integrated once to assume the form

$$\epsilon^{-\eta} = \{1-2\int_0^s y^{-2\beta}[\int_0^y \epsilon^{-\eta_z}s_z^\beta\,ds_z]^2\,dy\}^{1/2}.$$

Solutions of the form

$$\epsilon^{-\eta} = 1+a_1s+a_2s^2+\cdots$$

can then be obtained by substitution (see Table IId).

The theoretical potential distribution curves for a number of the cases so far discussed are shown in Figs. 1 and 2. Fig. 1 applies to cases where the ionization rate is proportional to electron density and thus covers Cases 1 and 4. Fig. 2, applying to a uniform ionization rate covers Cases 2 and 3. The abscissae are values of s/s_0 so that except for the small sheath thickness on the wall the radius of the tube is the unit of distance. The ordinates are values of η which can be converted readily to voltage by multiplication by $T_e/11{,}600$.

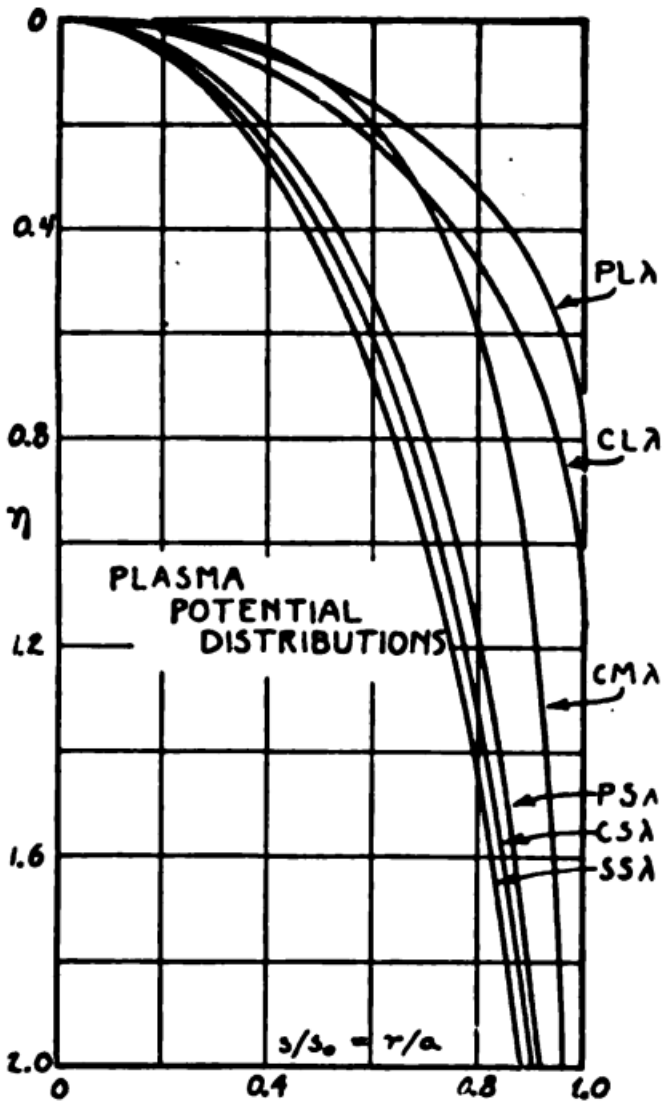

FIG. 1. Theoretical curves for rate of ionization proportional to electron density. For the meaning of curve designations see Table II.

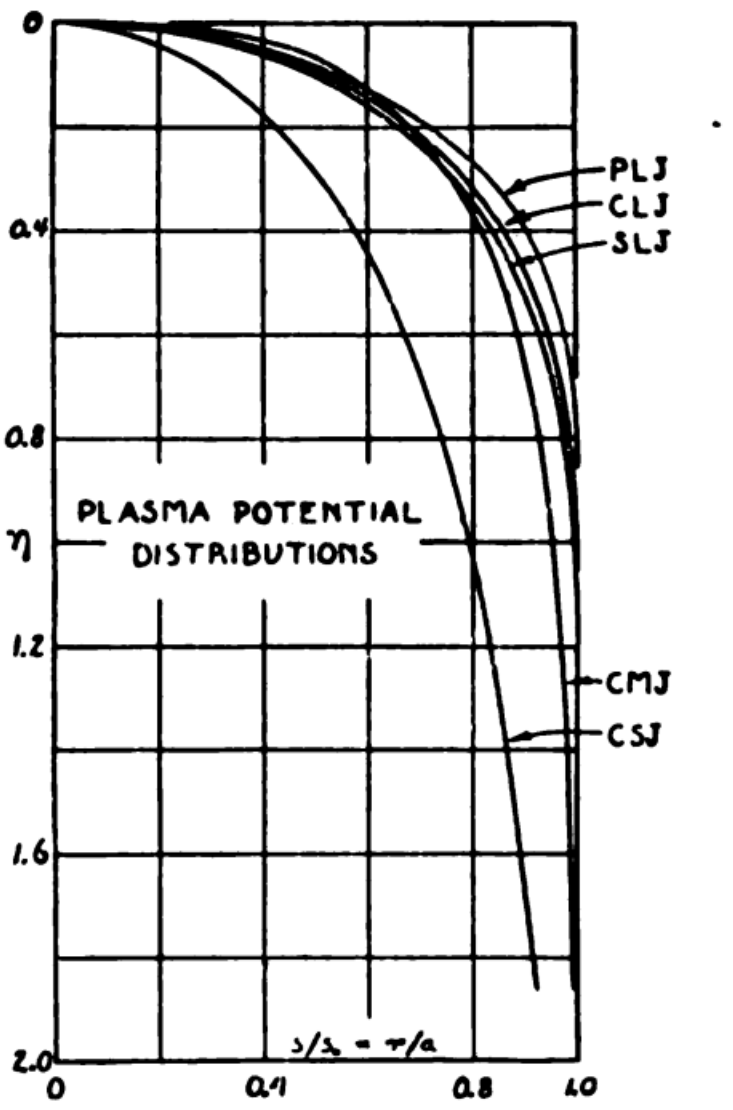

FIG. 2. Theoretical curves for rate of ionization constant. For the meaning of curve designations see Table II.

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Corresponding to the transition from long to short ionic free paths the respective potential distribution curves should show a progressive change. Curves for cylinder-long-path, cylinder-medium and cylinder-short-path in both Figs. 1 and 2 show this transition at larger values of s/s_0 but the cylinder-medium-path curve falls out of line at the lower values of s/s_0. This undoubtedly arises from the fact mentioned before that the mobility law underlying the medium-path curves is not valid in small fields, leaving the cases involving the long and the very short ionic free paths as those capable of the most rigorous treatment. involving the long and the very short ionic free paths as those capable of the most rigorous treatment.

Comparison with experiment.—Later, in Sections VI and VII, the orthodox use of a fine wire probe will be in the main justified on the basis of the new theory. Anticipating these conclusions, use can be made here of some measurements made by Mr. T. J. Killian[15] on the potential distribution along the diameter of the anode arm of a mercury arc for two different pressures of mercury vapor, corresponding respectively to 1.4° and 38.6°C. Killian measured also the electron temperature in each case, which gives all the information needed to check the observations against the theory. Actually, potentials were determined on both sides of the tube axis — but in view of the tube symmetry the

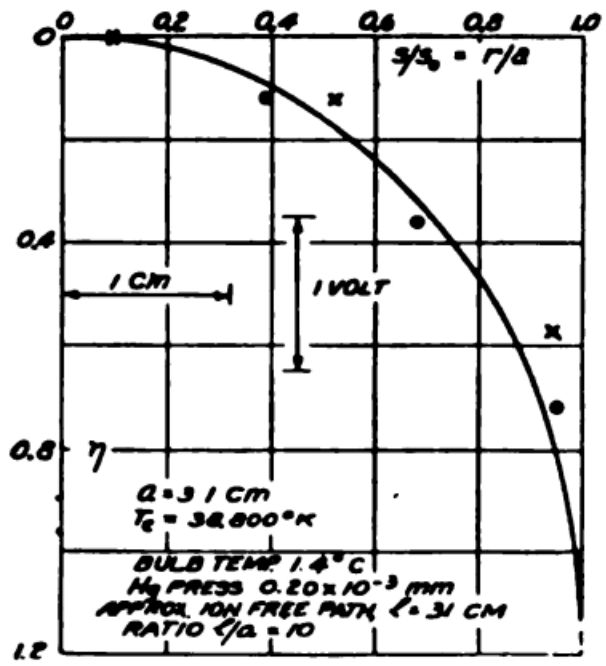

FIG. 3. Comparison of experimental and theoretical plasma potential distributions for long ion free paths.

points in Figs. 3 and 4 are plotted to one side only, those which have been transposed being indicated by crosses. The ordinates are, in each case, $11{,}600\ V/T_e = \eta$, V being the space potential in volts and 11,600 being e/k in degrees per volt. Along with the experimental points in each case is plotted one or two of the theoretical curves.

For the lowest gas pressure, Fig. 3, Curve $CL\lambda$ of Fig. 1 should give good agreement with the data, as it does. At some higher pressure, Curve $CS\lambda$ may be more nearly approached, as appears in Fig. 4. In judging of the agreement

[15] T. J. Killian, forthcoming paper in the *Physical Review*.

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between the theoretical and experimental results both the uncertainty of the potential measurements and also the absence of any adjustable constant in the theory must be considered. Since the transition from the $CL\lambda$ curve to the $CS\lambda$ curve should occur in the range where the free path length l_p is comparable with the radius a, the ratio l_p/a has been given on each figure. The value of l_p was calculated by multiplying Langmuir and Jones' value of 70 cm[16] for the electronic mean free path at 1 barye and 650°K by 1/4 and correcting to 300°K and the appropriate pressure. When the potential distribution curve assumes the short-free-path form as in Fig. 4 one should expect l_p/a to be rather

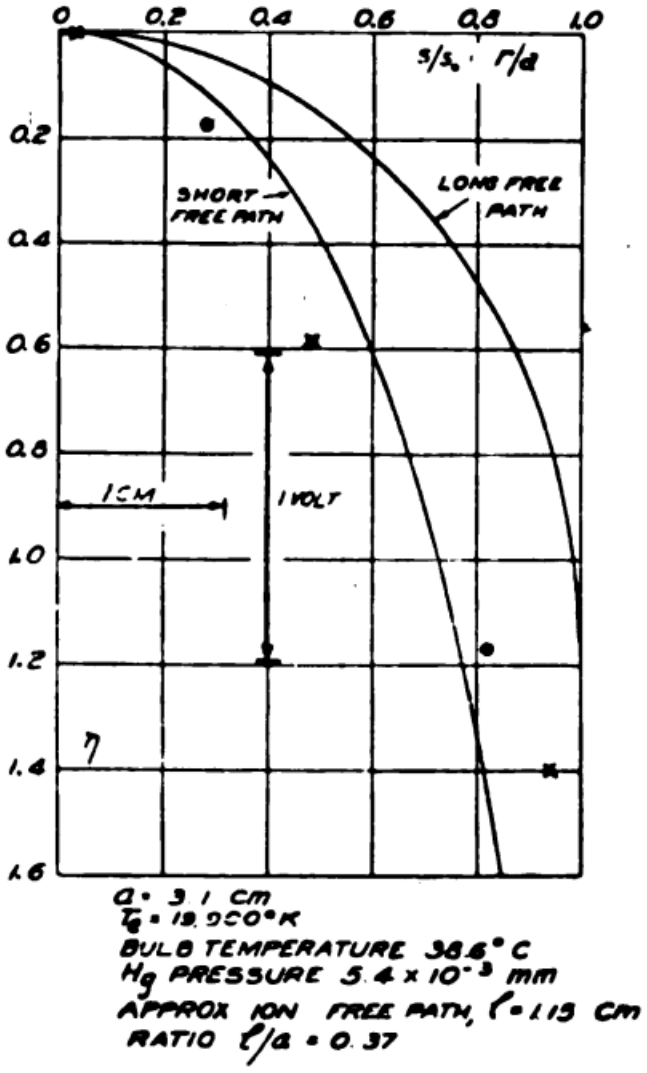

FIG. 4. Comparison of experimental and theoretical plasma potential distributions for short free paths.

less than 0.37, the value found. On the other hand free-path values — and even the concept itself — are somewhat uncertain. Probably the fact that the transition occurs in the general neighborhood expected is all that can be asked.

The plasma balance equation.—A digression may be permissible at this point to inquire into another consequence of the theory so far developed. In every case there is a fixed numerical value s_0 which is the upper limit of the values which s can assume. Now s_0 has already been identified with the tube radius in Figs. 3 and 4 and this is justifiable to the extent that the sheath thickness is negligible. Hence, using Eq. (10), we can write approximately for any case

$$a/s_0 = r/s = 1/\alpha. \tag{45}$$

[16] I. Langmuir and H. A. Jones, *Phys. Rev.* **31,** 357 (1928).

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To fix our ideas let us confine our attention to the low pressure λ cases covered by Table IIa. After introducing the value of a given by Eq. (12) this becomes

$$a\lambda = s_0(2k/m_p)^{1/2}T_e^{1/2} = 0.5522\times 10^6 s_0(T_e m_e/m_p)^{1/2}(\text{cm sec}^{-1}). \tag{46}$$

For a cylinder, $s_0 = 0.7722$ whence

$$a\lambda = 0.4264\times 10^6(T_e m_e/m_p)^{1/2}\text{cm sec}^{-1} \tag{47}$$

and in Hg $(m_p/m_e)^{1/2}$ is 605.5 so that for a cylindrical mercury discharge at low pressure

$$a\lambda = 703.1T_e^{1/2}. \tag{48}$$

We are thus enabled to calculate most easily the average rate at which each electron in the positive column of an arc ionizes atoms. Using some of the results given by Langmuir and Mott-Smith for a tube of 1.6 cm radius with bulb at 15.5°C corresponding to a vapor pressure of 1.05 baryes we find the values given in the 5th column of Table I.

TABLE I[17]

Run No.	Arc current, (amps)	T_e	I_p (ma cm^{-2})	$\lambda\times 10^{-4}$ calc. by	
				plasma balance equation	ion generation equation (See Section VIII)
34b	0.5	27,500	0.17	7.29	1.6
35a	1.0	29,000	0.25	7.48	2.1
35b	2.0	26,600	0.44	7.15	1.4
37a	8.0	19,500	2.29	6.13	0.21

Eq. (46) will be called the *plasma balance equation* for the low-pressure proportional-ionization cases because it states the adjustment of electron temperature to ion generation which just fits the plasma into the space available for it. It will be noted that Eq. (46) can be derived directly from the s vs. r equation appearing in Table IIa by inserting the limiting values s_0 and a. This same substitution when made in each of the other s vs. r equations of Table II yields the plasma balance equation appropriate to the particular case. Because of the finite thickness of the positive ion sheath on tube wall, a does not correspond exactly to s_0, and the necessary correction will be derived in Section IV.

Hitherto, the number of known relations in a positive column has been one less than the number of variables to be fixed. The plasma balance equation is important because it is the missing relation. This phase of the theory will be discussed in Section VIII.

---

[17] *L and M-S*, II, Table III. Examination of the original data reveals that the electron temperature in Run 34b is somewhat uncertain with the consequence that the value 1.6 for $\lambda\times 10^{-4}$ may, possibly, be as much as 30 per cent too low.

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The plasma balance equation can, obviously, say nothing concerning the actual causal dependence of λ on T_e and other quantities. This causal relationship might conceivably be incompatible with the plasma balance equation and in the case the formation of a plasma would be impossible. This relationship will be further discussed in Section VIII.

The first order correction to the plasma equation.—The total unimportance of the first order correction to the plasma solution throughout the greater part of the plasma becomes evident in Section IV but it will enter into any exact plasma-sheath transition calculation. The evaluation of this correction can be carried out by a method similar to that employed later in Section V. In long free path cases it will probably be advantageous to put $s_1+\delta s_1$ for s in the complete equation, s_1 being the solution already obtained and δs_1 being the correction desired. In short free path cases, on the other hand, it appears that the useful substitution will be $\varepsilon^{-\eta} = \varepsilon^{-\eta_1}+\delta(\varepsilon^{-\eta_1})$.

III. Positive Ion Currents in the Plasma

Pursuing the views advanced in the preceding sections one readily concludes that the positive ion current at the center of a discharge tube is zero, that at any other point it is radial (except for a longitudinal component in the positive column arising from the constant longitudinal gradient), and that it increases continually up to the sheath edge. If we abandon the approximation that each newly-created ion starts from rest, and make the more reasonable assumption that the newly-created ions have the same temperature as the gas — namely T_g, it is seen that there is a real, if small random ion current even at the center of the tube, but that does not concern us at present. The important thing to note here is that the ion currents en route to the walls are almost certain to remain unobservable by any direct measurements, for the introduction of any electrode which is apparently suitable for the purpose will itself so distort the plasma fields and ion motions as to destroy completely the effect sought. The result is that in seeking experimental agreement with the theory, we are limited to the observation of the ion current density at the tube wall. This current can be readily expressed in terms of the variables which have already been introduced. In all of the cases already discussed the number of ions reaching each unit of wall area in one second is the number generated per second in the volume subtended by that wall area. Thus

$$a^\beta I_p = \int_0^a eN_r r^\beta \, dr$$

where I_p is the positive ion current density at the wall. Introducing s here as before we have

$$I_p = aes_0^{-\beta-1} \int_0^{s_0} N_s s^\beta \, ds \tag{49}$$

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or where $\varrho(=\eta^{1/2})$ is the independent variable

$$I_p = aes_0^{-\beta-1}\int_0^{\eta_0^{1/2}} N_\varrho s^\beta (ds/d\varrho)\, d\varrho. \tag{50}$$

Comparison of this with the integrand appearing in the integral equations of Case 1 above shows an identity of form. The only difference lies in the variable of integration. Whereas for each series term a $\sin^m\theta\, d\theta$ was integrated before, here a $\varrho^m\, d\varrho$ is to be integrated. Thus with ionization proportional to electron density

$$I_p = aes_0^{-\beta-1}\lambda n_0 \sum_0^\infty (-1)^m \sigma_m \eta_0^{m+(\beta+1)/2}/(2m+\beta+1)$$

σ_m being defined for the cylindrical case just above (20) and for any case as the coefficient of the $(m+1)^{st}$ term of the series which occurs as integrand in the course of the original solution. In this way I_p is seen to

TABLE II

Plasma Solutions and Values of s_0, η_0, h_0
Designation of Cases

Form of plasma	Ion free paths	Ion generation
P plane	*L* long	*J* constant throughout ($N_z = J$)
C cylindrical	*M* medium	λ proportional to electron density ($N_z = \lambda n_z$)
S spherical	*S* short	

TABLE IIa[18]

Plasma Solutions for Case 1 in Text

Plasma equation: $s = G\eta^{1/2}(1+g_1\eta+g_2\eta^2+\cdots)$, $\quad a = s/r = \lambda(m_p/2kT_e)^{1/2}$

Constant	Case: *PLλ*	*CLλ*[19]	*SLλ*
G	$2/\pi$	1	$4/\pi$
g_1	-0.333333	-0.200000	-0.142857
g_2	-0.0333333	-0.0260260	-0.017722
g_3	-0.00476190	-0.00648941	—
g_4	$-0.0_3661376$	-0.0019840	—
g_5	-0.0_484181	-0.0_36794	—
g_6	-0.0_59715	*-0.0_3253*	—
g_7	—	*-0.0_310*	—
s_0	0.4046	0.7722	—
η_0	0.8540	1.155	—
h_0	0.8513	0.3500	—

[18] In the numerical coefficients the subscripts have the meaning indicated by $0.0_47 = 0.00007$.

[19] The coefficients in italics were obtained by extrapolation from the previous ones.

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TABLE IIb[20]

Plasma Solutions for Case 2 in Text

Plasma equation: $s = G\eta^{1/2}(1+g_1\eta+g_2\eta^2+\cdots)$, $\quad a = s/r = (J/n_0)\ (m_p/2kT_e)^{1/2}$

Constant	Case: *PLJ*	*CLJ*	*SLJ*
G	$2/\pi$	1	$4/\pi$
g_1	$-2/3$	-0.600000	-0.571429
g_2	$+4/15$	$+0.238182$	$+0.227712$
g_3	$-8/105$	-0.068573	-0.0661527
g_4	—	$+0.015303$	$+0.0147939$
g_5	—	-0.0027721	-0.00265902
g_6	$[\pm 2^p/1.3.5\cdots\cdots(2p+1)]$	$+0.0_34242$	$+0.0_3395$
g_7	—	-0.0_456	-0.0_450
g_8	—	$+0.0_565$	$+0.0_555$
s_0	0.3443	0.5828	0.7707
η_0	0.9244	1.0542	1.1950
h_0	1.0000	0.5000	0.3333
s_0[20]	0.38	0.638	0.818
η_0[20]	0.943	1.26	1.50
h_0[20]	1.000	0.500	0.333

TABLE IIc

Plasma Solutions for Case 3 in Text

Cases (*P*, *C*, *S*) *SJ*:

Plasma equation: $s^2 = 2(1+\beta)\ (1-e^{-\eta})$

$q = 0.895\, l_p(kT_g m_p)^{-1/2}$ $\qquad$ $a = s/r = (J/n_0 qkT_e)^{1/2}$

Constant	Case: *PSJ*	*CSJ*	*SSJ*
β	0	1	2
s_0	1.414	2.000	2.449
η_0	∞	∞	∞
h_0	1.0000	0.5000	0.3333

Cases (*P*, *C*, *S*) *MJ*:

Plasma equation: $s^3 = (3/2)\ (1+\beta^2)\ (1-e^{-2\eta})$

$q' = 1.2\,(l_p/m_p)^{1/2}$ $\qquad$ $a = s/r = (J^2/n_0^2 q'^2 kT_e)^{1/2}$

Constant	Case: *PMJ*	*CMJ*	*SMJ*
β	0	1	2
s_0	1.145	1.816	2.378
η_0	∞	∞	∞
h_0	1.000	0.500	0.333

[20] These solutions were obtained by an approximate method. (See Section VI.)

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TABLE IId

Plasma Solutions for Case 4 in Text

Cases (P, C, S) $S\lambda$:

$$q = 0.895\, l_p(kT_g m_p)^{-1/2} \qquad a = s/r = (\lambda/qkT_e)^{1/2}$$

	Case: $PS\lambda$	$CS\lambda$	$SS\lambda$
Plasma equation	$\epsilon^{-\eta} = \cos s$	$\epsilon^{-\eta} = J_0(s)$	$\epsilon^{-\eta} = (\sin s)/s$
s_0	1.571	2.405	3.142
η_0	∞	∞	∞
h_0	0.6366	0.2159	0.1013

Cases (P, C, S) $M\lambda$;

$$q' = 1.2\ (l_p/m_p)^{1/2} \qquad a = s/r = (\lambda^2/q'^2 kT_e)^{1/3}$$

Plasma solution for case $CM\lambda$ only:

$$\epsilon^{-\eta} = 1-(1/12)s^3-0.0_3 69444 s^6-0.0_4 79089 s^9-0.0_6 32511 s^{12} -0.0_6 16513 s^{15}-\cdots$$

$$s_0 = 2.154 \qquad \eta_0 = \infty \qquad h_0 = \cdots$$

involve a new dimensionless constant which can be chosen to correspond to current density just as s_0 corresponds to distance and η_0 to voltage

$$h_0 = s_0^{-\beta-1} \sum_0^\infty (-1)^m \sigma_m \eta_0^{m+(\beta+1)1/2}/(2m+\beta+1)$$

so that

$$I_p = h_0 e a n_0 \lambda \tag{51}$$

which will be called an *ion current equation.* In Cases 2 and 3 ($N_s = J$, constant) it is readily seen that Eq. (49) can be integrated directly giving the ion current equation

$$I_p = h_0 e a J \tag{52}$$

where $h_0 = 1/(1+\beta)$. In Case 4 when the mean free paths are short Eq. (49) again applies. The values of h_0 for this and other cases are given in Table II. Another significance may be attributed to h_0 in Cases 1 and 4, namely that

$$(1+\beta)h_0 = \text{average value of } \epsilon^{-\eta} \tag{53}$$

taken throughout the plasma. Thus in the cylindrical case the total number of electrons N_e per unit length of tube is given by

$$N_e = 2h_0 n_0 \pi a^2 \tag{54}$$

whence the total ion current

$$2\pi a I_p = N_e e\lambda = 2\pi a^2 h_0 n_0 e\lambda$$

which checks Eq. (51). The plane and spherical cases work out similarly.

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Both when the ionization rate is uniform and when it is proportional to electron density, a simultaneous solution of the plasma balance equation and the equations involving ion current gives in long free path cases

$$I_p = s_0 h_0 e n_0 (2kT_e/m_p)^{1/2} \tag{55A}$$

$$= 8.787 \times 10^{-14} s_0 h_0 n_0 (T_e m_e/m_p)^{1/2} \text{ amp cm}^{-2} \tag{55B}$$

and in short free path cases

$$I_p = s_0^2 h_0 e n_0 q k T_e/a \tag{55.5}$$

These types of equation will also be called *ion current equations.*

It will be noted that the ion current equation affords a method by which the electron density at the potential maximum in the plasma may be determined. Solving for n_0 in Case $CL\lambda$ Eq. (55B) becomes

$$n_0 = 4.21 \times 10^{13} (m_p/m_e)^{1/2} I_p T_e^{-1/2}.$$

This method was checked experimentally against the electron and ion densities determined from the positive and negative branches of the volt-ampere characteristic of a fine probe as discussed in Sections VI and VII. For this purpose three runs were made with the tube which Killian used. The results are shown in column 5 of Table III.

TABLE III

Arc current, (amp)	Bulb temp.	Electron temp. (°K)	$I_p \times 10^4$ (amps/cm²)	$n_0 \times 10^{-10}$		
				by Eq. (55)	by $i^2 - V$ plot	
					for ions	for electrons
5.0	15.5°C	20,600	4.52	8.04	16.3	8.07
5.0	0°C	27,800	4.57	7.0	16.4	6.2
1.0	15.5°C	23,300	0.84	1.40	3.12	1.54

The value of I_p for the 1.0 amp case has been corrected for the wall sheath thickness in accordance with Section VI so that I_p refers to the sheath edge as it should rather than the tube wall. The correction amounts to 4.5 per cent in this case.

The potential which a non-conducting tube wall bounding the discharge assumes can now be calculated. Such a wall becomes sufficiently negative so that all the electrons are turned back in the sheath except the small number required to neutralize the positive ion current. By kinetic theory the electron current density reaching the surface of the wall is

$$I_n = e n_0 \epsilon^{-\eta_w} (kT_e/2\pi m_e)^{1/2}$$

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where $n_w = -11{,}600\ V_w/T_e$, V_w being the wall voltage with respect to the plasma potential maximum. But if the reflection coefficient of the wall is ϱ_e the current actually collected[21] will be only

$$I_n' = (1-\varrho_e) I_n.$$

Equating I_n' to I_p as given by Eq. (55) it is found that

$$n_w = \ln\left[(m_p/m_e)^{1/2}/2\pi^{1/2} h_0 s_0\right] + \ln(1-\varrho_e) \tag{56}$$

which in Case $CL\lambda$ and Hg gives

$$\eta_w = 6.45 + \ln(1-\varrho_e). \tag{57}$$

Both the voltage on the axis of positive column and T_e can be determined from the semi-log plot of the volt-ampere characteristic of a fine axial wire. Also, the voltage at which a collector on the tube wall opposite the wire takes zero current is readily measured. The difference of the two voltages multiplied by $11{,}600/T_e$ gives η_w. Referring to the original data for collectors F[22] and H upon which Tables III and XIV of L and M–S Parts II and IV are based it is found that $\eta_w = 5.9 \pm 0.2$ for Runs 34b to 37a. In a special test with a positive column of twice the diameter used there it was found that η_w was 6.13 for one pair and 5.73 for another pair of electrodes, again giving an average of 5.9. A reasonable value for ϱ_e is thought to be 0.15 leading to the theoretical value 6.3 for η_w. The agreement with experiment is considered to be good particularly in view of the effect of the collector support.[22]

This is, perhaps, as good a place as any to point out that in many cases the plasma theory as so far developed applies to the plasma in the neighborhood of an anode almost as well as to electrodes drawing less electron current. The only necessary condition is that the electron current density reaching the anode shall be small compared to the random current density at the sheath edge. It has already been pointed out[23] that without violating this condition anodes of reasonable area are capable of collecting the full arc current.

We are now in a position to discuss the magnitude of the ion currents received by two equal plane collectors arranged back to back in the positive column of a discharge so that one electrode J faces the cathode, the other K, the anode.

---

[21] It might reasonably be expected that the constant drain of fast electrons by the walls would cause a deficiency of high velocity electrons. No such effect has been found at small distances from a wall or from an electrode considerably less negative than the wall. The explanation of this phenomenon is not known. Were it not for this mechanism which rapidly re-establishes a M. D., slow electrons would accumulate indefinitely at a potential maximum and build up the ionization density to a high value, escaping finally either by recombination or by setting up oscillations. Perhaps the unknown mechanism involves just such oscillations.

[22] The ideal collector would be axial, but the fact that F is not axial is unimportant compared to the errors introduced by the lead-in structure in lowering the plasma potential at the supported end of the collector.

[23] *L and M–S*, Part IV, pp. 766, 767.

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Fig. 5 gives the volt-ampere characteristics[24] in Hg vapor saturated at 16°C of two such electrodes. Each was allowed to float while the characteristic of the other was being taken. Both were square plates 0.95 cm on a side spaced 0.08 cm apart in a tube of 3.2 cm diameter carrying an arc current of 0.60 amps. The arc gradient was 0.24 v cm^{-1}. It is to be noted that not only did J receive

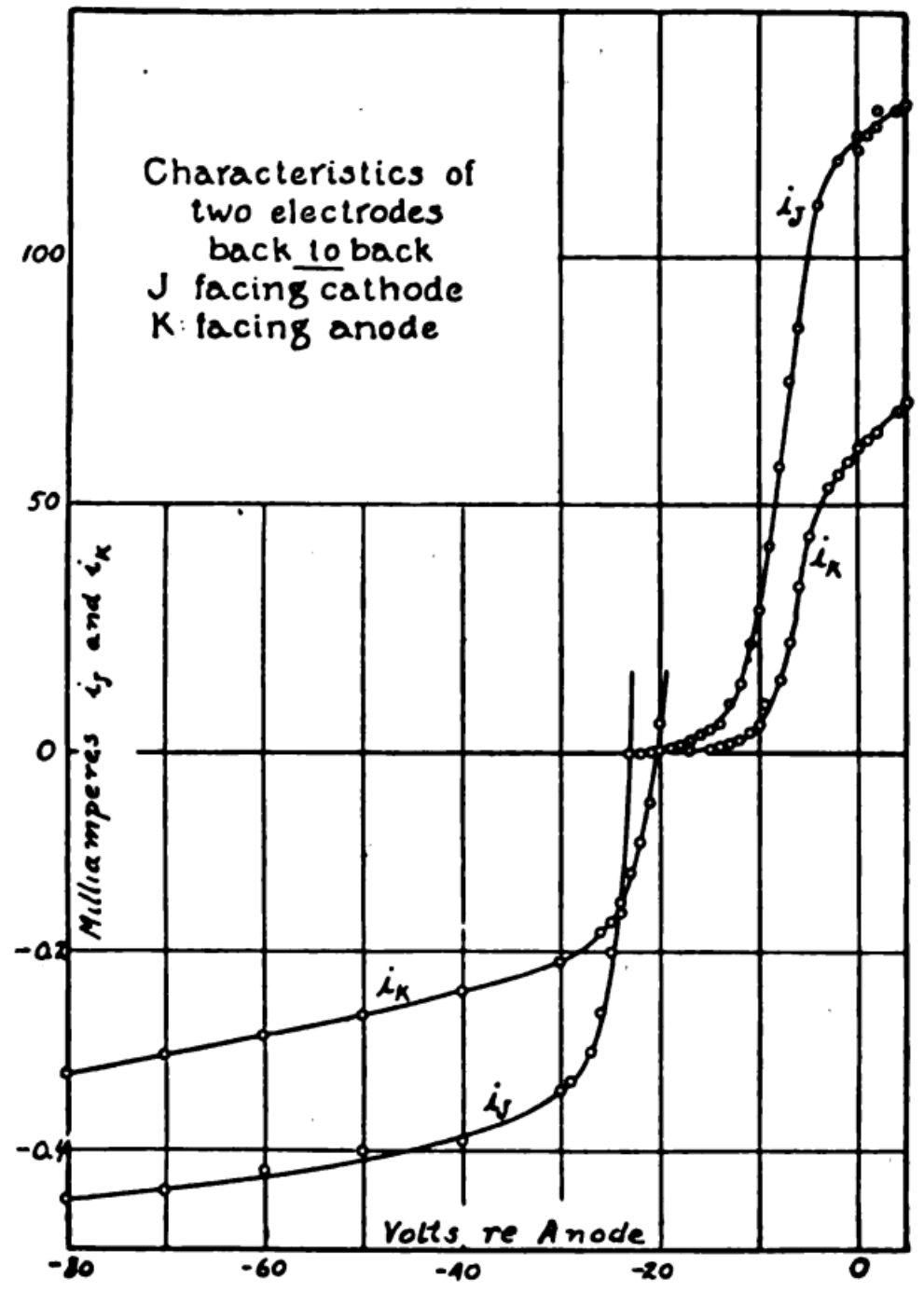

FIG. 5. Note change in current scale at zero.

the larger electron current at positive voltages but also the larger ion current at negative voltages. K, which was completely exposed to any longitudinal drift of the ions toward the cathode captured fewer ions than J which, on the random ion current theory, could only receive the random component of the ion current. The present theory, however, explains this quite readily. We observe that the larger electron current to J arises from the very appreciable ratio of drift to random electron current density. This not only increases the density of electrons in the plasma near J above normal but decreases the density near K below normal. Positive space charge arising from an excess of ions if present would set up potential differences tending to wipe this out, but such an excess has little tendency to occur because the generation of the positive ions is it-

[24] From unpublished data taken by I. Langmuir and H. M. Mott-Smith, Jr.

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self proportional to the electron density. Thus the density of ionization as a whole is less near K than near J, and the smaller ion current to K than to J at negative electrode voltages is to be expected. The relative difference between the two ion currents is smaller than between the two electron currents and this probably arises from the effect of the longitudinal field in the column which tends to increase the volume from which ions can reach K and to decrease the volume contributing ions to J. (This longitudinal field of 0.24 v cm^{-1} is small compared to the average radial plasma field. The electron temperature was approximately 30,000°K. In case $CL\lambda$ $\eta_0 = 1.155$ whence the potential difference between tube axis and plasma edge is $1.155(30{,}000/11{,}600) = 3$v, a difference equal to that found in 12.5 cm along the axis. It is interesting to note that the displacement of the curves shows that the presence of the dual electrode causes a potential difference of some 3 v in less than a millimeter distance.)

The ratio of random electron current to drift current.—The quotient of the density of the ion current to the wall in a positive column and the arc current itself leads directly to the ratio of random electron current density I_e, to drift current density I_x. From kinetic theory

$$I_e = en_0(kT_e/2\pi m_e)^{1/2}$$

at the tube axis. Combining this with Eq. (55A) we have

$$I_e/I_p = (m_p/m_e)^{1/2}/2s_0h_0\pi^{1/2}.$$

The average drift current over the tube cross-section is $i_B/\pi a^2$ and taking account of the actual distribution by using Eq. (53) we have

$$I_x = i_B/2h_0\pi a^2$$

at the tube axis. It follows then, that

$$I_e/I_x = (\pi^{1/2}/s_0)(m_p/m_e)^{1/2}a^2I_pi_B/. \tag{57.5}$$

The approximate equality of I_p for electrodes of various sizes and in various positions.—One of the experimental facts which favored the random ion current belief was the observation that at low gas pressures electrodes variously disposed in a tube excited from a hot cathode received ion current densities (corrected to sheath area) which differed usually less than 2 to 1 in ratio even though one electrode might be in the center, the other on the wall of the bulb. Experiment thus shows that the shape of the plasma boundary at a certain place has no great effect on the ion current density to that place. The theory indicates the same result, for Eq. (55A) shows that for plasmas of different sizes and varying in shape from the plane to the spherical but all having the same maximum ionization intensity, I_p is proportional to s_0h_0. From Table IIb it is seen that the variation involved is only from 0.34 to 0.26.

The average ion velocity at s_0.—Later it will be convenient to know the average velocity of the ions as they pass from the plasma into the sheath. The

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η equivalent to this velocity will be denoted by $\bar{\eta}_0$ and can be calculated readily. We have

$$I_p = ne\bar{v}_p, \quad n = n_0\epsilon^{-\eta_0}, \quad \bar{v}_p = (2kT_e\bar{\eta}_0/m_p)^{1/2}$$

where $\bar{v}_p$ is the mean ion velocity. Combining these, we find

$$I_p = en_0\epsilon^{-\eta_0}(2kT_e\bar{\eta}_0/m_p)^{1/2}$$

and eliminating I_p with Eq. (55A) it is found that

$$\bar{\eta}_0 = h_0^2 s_0^2 \epsilon^{2\eta_0} \tag{58}$$

in long free path cases. In Case $CL\lambda$ $\bar{\eta}_0 = 0.7359$.

The magnitude of the neglected term in the plasma equation.—Eq. (55) makes it possible to calculate the coefficient of the neglected term in Eq. (11) or (13) which will be denoted by A, for any given case. Using Eqs. (45) and (55) to eliminate α and n_0 respectively, we have

$$A = h_0 s_0^3 (2kT_e)^{3/2}/8\pi e a^2 m_p^{1/2} I_p \tag{59A}$$

for long free path cases. Introducing the numerical values of known constants and expressing I_p in amps cm^{-2} this becomes

$$A = 4.210\times 10^{-12} h_0 s_0^3 (m_e/m_p)^{1/2} T_e^{3/2}/a^2 I_p \tag{59B}$$

and for Case $CL\lambda$ in mercury

$$A = 1.119\times 10^{-15} T_e^{3/2}/a^2 I_p. \tag{60}$$

Thus in Run 37a[25] where the arc current was 8.0 amps, $T_e = 19{,}500°$, $a = 1.6$ cm, and $I_p = 2.29\times 10^{-3}$, and we find $A = 5.19\times 10^{-7}$. In Run 34b at the other extreme of this group of runs the arc current was 0.5 amp. In this case $T_e = 27{,}500°$ and $I_p = 0.17\times 10^{-3}$, whence $A = 1.17\times 10^{-5}$. From Table IIa it is readily found that $\nabla_s^2\eta$ at the potential maximum has its largest value 4.94 in the plane case. We thus confirm the smallness of $A\nabla_s^2\eta$ at the origin relative to the other terms of Eq. (13). How far out this term may be neglected will be discussed in the next section.

IV. The Sheath Edge and Sheath

The limit of validity of the plasma equation.—We have already noted that the approximation which gives the plasma equation fails at some value of s less than s_0 because of the fact that $\nabla_s^2\eta$ becomes infinite at s_0. Physically, of course, this is just the type of development that is necessary to give a sheath. Accordingly, the problem of carrying the solution up to and past s_0 is the problem of the sheath edge and sheath.

Only the low pressure case will be analyzed in this section and that comparatively roughly because the plasma-sheath transition is inherently more complicated than either plasma or sheath alone. The high pressure case might

[25] *L. and M-S.* Table III, Part II.

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be covered by the assumption that the ion velocity was proportional to the electric field, but there is a wide range of pressures for which the ion may drift in the plasma yet fall freely through the greater part of a thin sheath.

The first question which arises concerns the point at which the plasma solution should be abandoned. We may, without definitely committing ourselves for the present, say that we shall have to do this when the Poisson term, neglected in the plasma solution, becomes equal to a certain fractional part, Φ, of either of the other two terms, that is when

$$Ad^2\eta/ds^2 = \Phi\epsilon^{-\eta}. \tag{61}$$

Here $\nabla_s^2\eta$ has been replaced by $d^2\eta/ds^2$ since the only sheaths which will be considered are those which are thin compared to the radius of the tube.

The point on the plasma solution at which this relation is satisfied is designated on Fig. 6 by n_Φ, s_Φ. The coordinates s_Φ and η_Φ are most easily expressed

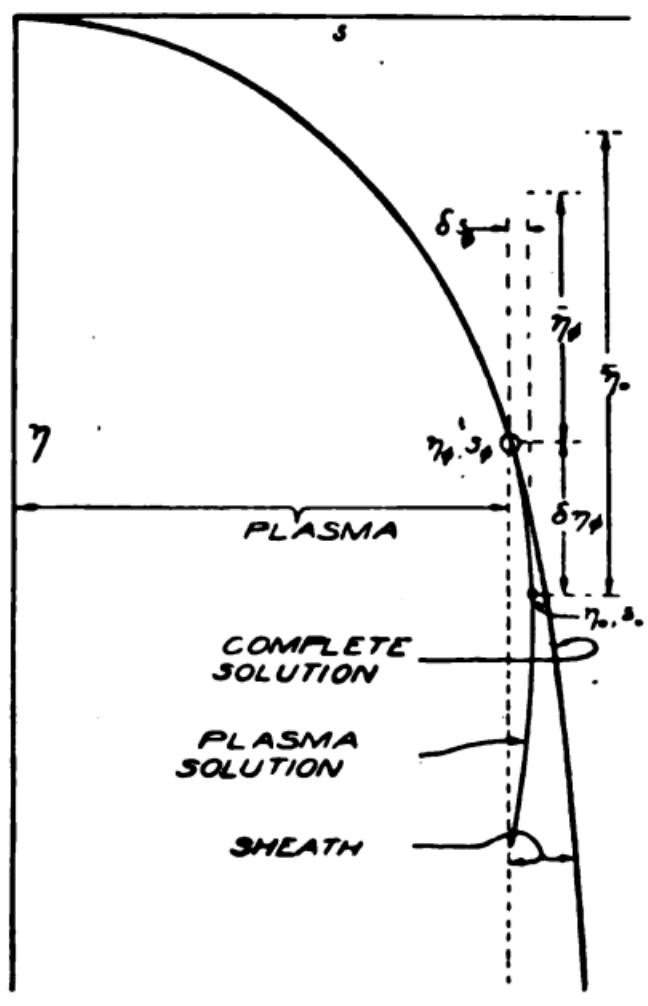

FIG. 6. The relation of the plasma and complete solutions of the plasma-sheath integral equation.

by their differences δs_Φ and $\delta\eta_\Phi$ from s_0 and η_0 respectively. In order to evaluate δs_Φ and $\delta\eta$, $d^2\eta/ds^2$ must first be expressed with η as the independent variable. We have

$$d\eta/ds = (ds/d\eta)^{-1}, \quad d^2\eta/ds^2 = -(d^2s/d\eta^2)(ds/d\eta)^{-3} \tag{62}$$

A Taylor Expansion gives

$$\delta s = \cdot\,(\tfrac{1}{2})s_0''\delta\eta^2[1+\delta\eta\,(\cdots)] \tag{63}$$

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where $\delta s = s - s_0$, $\delta\eta = \eta - \eta_0$, s_0'' denotes the value of $d^2s/d\eta^2$ at s_0, and the term involving s_0' is absent since $s_0' = 0$ by definition of s_0. Neglecting all but the lowest order terms in $\delta\eta$ we also have

$$ds/d\eta = s' = s_0''\delta\eta$$

$$d^2s/d\eta^2 = s'' = s_0''.$$

Substitution in Eq. (62) gives

$$d\eta/ds = s_0''^{-1}\delta\eta^{-1} \tag{64}$$

$$\delta^2\eta/ds^2 = -s_0''^{-2}\delta\eta^{-3} \tag{65}$$

In addition,

$$\epsilon^{-\eta} = \epsilon^{-\eta_0}$$

to the same degree of approximation.

Then $\delta\eta_\Phi$, the value of $-\delta\eta$ for which Eq. (61) is satisfied, (the minus sign being inserted simply to make $\delta\eta_\Phi$ intrinsically positive), is found through substituting the last two equations into Eq. (61). It is found to be given by

$$\delta\eta_0^3 = A\epsilon^{\eta_0}/s_0''^2\Phi \tag{66}$$

The corresponding value, δs_Φ, of $-\delta s$ can be found from Eq. (63). In case $CL\lambda$

$$s_0'' = -0.635$$

so that using A from Eq. (60).

$$\delta\eta_\Phi = 2.064\times10^{-5}T_e^{1/2}a^{-2/3}I_p^{-1/3}\Phi^{-1/3}$$

and

$$\delta s_\Phi = 1.35\times10^{-10}T_e a^{-4/3}I_p^{-2/3}\Phi^{-2/3}.$$

for mercury.

As example we may take the two runs already used. Noting that $\epsilon^{\eta_0}/s_0''^2 = 7.88$ in Case $CL\lambda$, the values of A already found substituted in Eq. (66) give immediately

$$\delta\eta_\Phi = 0.0160/\Phi^{1/3} \quad \text{and} \quad \delta\eta_\Phi = 0.045/\Phi^{1/3}$$

for the 8 amp and 0.5 amp arcs respectively. Thus for $\Phi = 0.05$ we have

$$\left.\begin{aligned}\delta\eta_\Phi &= 0.045\\ \delta s_\Phi &= 6.4\times10^{-4}\end{aligned}\right\} \text{ and } \left\{\begin{aligned}\delta\eta_\Phi &= 0.122\\ \delta s_\Phi &= 47\times10^{-4}.\end{aligned}\right.$$

The extension of the general plasma-sheath equation, Eq. (4), past n_Φ, s_Φ is very much more complicated than the solution of the plasma itself. In addition, the only theoretical result which finds application at present is the sheath thickness, and that occurs only as the correction to the discharge tube radius which gives the plasma radius. Accordingly, the present treatment of the problem will be only approximate. The solution involves three simplifications of the general plasma-sheath equation. The first consists in the replacement of $\nabla^2 V$ by d^2V/dr^2 as already noted in connection with Eq. (61). The

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second concerns the ion charge term of the general equation and results in the elimination of the integral. The third consists in dropping the electron charge term when it becomes negligible.

Regarding the second simplification it is to be noted that in λ cases the low and rapidly decreasing electron density in the wall sheath suppresses the generation of ions there, and in any case, the thinness of the sheath will render its ion contribution negligible compared to that from the plasma. Hence the ion current density through the sheath can be assumed to be constant and to have the value calculated for s_Φ. This value is equal to I_p, the value at s_0, to within a quantity of the order of δs_Φ or $\delta\eta_\Phi^2$. To calculate the ion space charge the velocities of the ions must also be known. The plasma equation enables us to find the single velocity which is equivalent as regards space charge to the actual distribution of ion velocities at s_Φ. The same steps that yield Eq. (58) lead to

$$\bar{\eta}_\Phi = s_0^2/h_0^2\epsilon^{2\eta_\Phi} \tag{67}$$

where η_Φ is the η corresponding to the equivalent single velocity. A serious difficulty enters here, for it immediately appears that $\bar{\eta}_\Phi$ is characteristic of η_Φ, s_Φ only. For example, $\bar{\eta}_0$, corresponding to the single equivalent velocity of the stream at η_0, s_0, is not $\bar{\eta}_\Phi+\delta\eta_\Phi$, but by combining Eqs. (57) and (67) it is found that

$$\bar{\eta}_0 = \bar{\eta}_\Phi+2s_0^2h_0^2\epsilon^{2\eta_\Phi}\delta\eta_\Phi = \bar{\eta}_\Phi+1.47\delta\eta_\Phi \tag{68}$$

in Case *CL*λ. This is indicated qualitatively in Fig. 6. If $\bar{\eta}_\Phi$ and $\bar{\eta}_0$ represented mean potentials of origin of the ions, the expected relation would have been correct. Actually the average here encountered is the reciprocal square of the mean reciprocal square root of $\eta_\Phi-\eta$ and $\eta_0-\eta$ respectively. In the present solution, however, we shall treat η_Φ like a simple mean value rather than the complicated average which it really represents. The positive ion space charge in the sheath then becomes $(m_p/2kT_e\eta_s)^{1/2}I_p$ where $\eta_s = \eta-\eta_\Phi+\bar{\eta}_\Phi$. By Eq. (55A) this becomes $en_0s_0h_0\eta_s^{-1/2}$ and using this in place of the integral term in Eq. (4) we have

$$d^2V/dr^2-4\pi en_0\epsilon^{-\eta}+4\pi en_0s_0h_0\eta_s^{-1/2} = 0$$

which can be reduced to

$$Ad^2\eta/ds^2+\epsilon^{-\eta}-s_0h_0\eta_s^{-1/2} = 0 \tag{69}$$

A being given by Eq. (59A).

In order to bring this equation into a form which has already been treated by Langmuir[26] it is convenient to substitute for $s(=\alpha r)$ a new independent variable ξ, which is defined by the following series of equations

$$d\xi = dr/x_p \tag{70}$$

$$I_p = ((2)^{1/2}/9\pi)(e/m_p)^{1/2}W_e^{3/2}/x_p^2 \tag{71}$$

$$W_e = kT_e/e = T_e/11{,}600 \tag{72}$$

[26] I. Langmuir, *Phys. Rev.* **33**, 976–980 (1929) Eq. (68).

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$d\xi$ rather than ξ being defined since the origin of ξ's is better left indefinite for the present. The physical significance of W_e and x_p are evident from their relation to electron temperature and the simple space charge equation respectively. As the new variable appears in the first term only of Eq. (69), the substitution is most easily carried out by means of the relations

$$ds^2 = \alpha^2 dr^2 = \alpha^2 x_p^2 d\xi^2$$

followed by substitutions for x_p^2 and W_e from Eqs. (71) and (72). It is found that

$$A/ds^2 = 9s_0h_0/4d\xi^2 \tag{73}$$

so that

$$d^2\eta/d\xi^2 = (4/9)(\eta_s^{-1/2} - \epsilon^{-\eta}/s_0h_0)$$

Integrating once, it is found that

$$(d\eta/d\xi)^2 - (d\eta/d\xi)_\varphi^2 = (16/9)\eta_s^{1/2} - \bar{\eta}_\varphi^{1/2} - (1-\epsilon^{-\eta_s+\bar{\eta}_\Phi})\epsilon^{-\eta_\Phi}/2s_0h_0].$$

Evaluating $(d\eta/d\xi)_\Phi$ by means of Eqs. (73), (61), (64), and (65) we find

$$(d\eta/d\xi)_\Phi^2 = (4\epsilon^{-\eta_\Phi}/9s_0h_0)\Phi\delta\eta_\Phi.$$

In order to proceed, a further approximation now becomes necessary. The quantities η_Φ and $\bar{\eta}_\Phi$ must be replaced by η_0 and $\bar{\eta}_0$ throughout both the above equations. Thus η_s which was equivalent to $\eta - \eta_\Phi + \bar{\eta}_\Phi$ is redefined as $\eta - \eta_0 + \eta_0$. Naturally, $d\eta_s$ can be written for $d\eta$ in the derivatives. Making use of Eq. (58) to eliminate s_0h_0, the differential equation then becomes

$$(d\eta_s/d\xi)^2 - (4/9\bar{\eta}_0^{1/2})\Phi\delta\eta_\Phi = (16/9)[\eta_s^{1/2} - \bar{\eta}_0^{1/2} - (1-\epsilon^{-\eta_s+\bar{\eta}_0})/2\bar{\eta}_0]^{1/2} \tag{74}$$

The right member now vanishes when $\eta = \eta_0$ (i.e. $\eta_s = \bar{\eta}_0$). The left member then tells us that the electric field at η_0, s_0 has the value which we had supposed it to have at η_Φ, s_Φ. Thus the effect of the last approximation has been to transfer the beginning of the sheath solution to η_0, s_0 as regards ion and electron space charge concentrations and ion velocities but to retain the correct value of initial electric field.

The next integration has to be performed in three steps, namely; *A*. By expansion in a Taylor Series at $\eta_s = \bar{\eta}_0$, giving

$$\xi = (3/2)\bar{\eta}_0^{3/4}(\bar{\eta}_0 - \tfrac{1}{2})^{-1/2}\left\{\ln\frac{\eta_s - \bar{\eta}_0}{\bar{\eta}_0} - (1/2)\ln\frac{\Phi\delta\eta_\Phi}{4\bar{\eta}_0(\bar{\eta} - \tfrac{1}{2})}\right\}$$

for the range $\bar{\eta}_0 < \eta_s \leqslant 2\bar{\eta}_0$. Putting $\eta_0 = 0.736$ for Case $CL\,\lambda$ the value of ξ for $\eta_s = 2\bar{\eta}_0$ becomes

$$\xi]_{1.472} = -0.449 - 1.23\ln(\Phi\delta\eta_\Phi).$$

B. By quadrature of Eq. (74) neglecting $(4/9\bar{\eta}_0^{1/2})\Phi\delta\eta_\Phi$, giving for Case $CL\lambda$

η_s	1.472	2.944	4.416
$\xi - \xi]_{1.472}$	0	2.82	4.40

in the range $2\bar{\eta}_0 \leqslant \eta_s \leqslant 6\bar{\eta}_0$.

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C. By integration of Eq. (74) neglecting the exponential (electron space charge) giving, finally

$$\xi = (\eta_s^{1/2}+2.88)(\eta_s^{1/2}-1.44)^{1/2}-0.10-1.23 \ln (\Phi\delta\eta_\Phi) \tag{75}$$

for Case $CL\lambda$ when $\eta_s \geqq 6\bar{\eta}_0$, ξ being measured from s_0.

In Run 37a already cited, it was found that $\delta\eta_\Phi = 0.045$ when $\Phi = 0.05$. We have also found that the potential of the tube wall is given by $\eta_w = 6.30 = \eta_s+\eta_0-\bar{\eta}_0$. whence $\eta_s = 5.88$ and

$$\xi_w = 5.16-1.23 \ln (\Phi\delta\eta_\Phi) \tag{76}$$

where ξ_w measures the sheath thickness on a non-conducting wall. Thus $\xi_w = 12.7$ whereas the same ion current space charge limited in the absence of electrons would give

$$\xi_w = (5.88)^{3/4} = 3.78.$$

This sheath, then, is 3.3 times what might be called the normal thickness. In Run 346 (0.5 amp arc) $\delta\eta_\Phi = 0.12$ for $\Phi = 0.05$ and $\xi_w = 11.4$.

The exact sheath solution would not involve Φ which is a measure of the error tolerated in the plasma solution before it is abandoned, and consequently the presence of Φ can be used to estimate the degree of approximation involved in Eq. (75). Since $\delta\eta_\Phi$ varies with $\Phi^{-1/3}$, the term which is variable in Φ becomes $-0.81 \ln\Phi$. Thus a two-fold change in Φ causes a change in ξ of only ± 0.56 which is less than errors introduced in the integration of Eq. (74).

The relation between ξ_w and the tube radius is most readily obtained from Eq. (73). Putting ϱ for $\Delta s/s_0$, the fraction of the tube radius occupied by the sheath, this gives

$$\varrho = (2/3s_0)(A/s_0h_0)^{1/2}\xi_w. \tag{77}$$

In Case $CL\lambda$ this becomes

$$\varrho = 1.662A^{1/2}\xi. \tag{78}$$

Whence, in Run 37a, $\varrho = 0.015$ and in Run 34b, $\varrho = 0.065$.

V. Effect on the Plasma of an Ion Temperature

Up to this point the theoretical treatment of the plasma has been based on the assumption that newly formed ions start from rest, whereas it is most reasonable to suppose that they actually posses the velocity distribution characteristic of the gas atoms from which they have just been formed. W. Schottky's treatment[27] of the short mean free path case includes consideration of such an ion temperature. Here the long free path case is handled. In view of the result that the difference in voltage between the potential maximum and the sheath edge is of the order of $T_e/11{,}600$ in the cases so far discussed, whereas any ion

[27] Schottky, *Phys. Zeits.* **25,** 342 (1924).

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temperature effects would probably be confined to a voltage difference of $T_p/11{,}600$, ($T_p = T_g$) this assumption appears to be justified in the main. Only in the neighborhood of the potential maximum could the ion temperature conceivably influence the result appreciably, and it is possible to evaluate there the first and second order corrections from this cause.

Case *PLJ* will serve as the example. As before, η denotes $-eV/kT_e$, but in addition it is convenient to introduce μ to denote $-eV/kT_p$. The plasma-sheath equation now takes the form

$$A'\nabla_q^2\mu+\epsilon^{-\eta}-\pi^{1/2}\left\{\int_0^q \exp(\mu-\mu_z)\,[1-P((\mu-\mu_z)^{1/2})]\,dq_z + \int_q^\infty \exp(\mu-\mu_z)\,dq_z\right\} = 0 \tag{79}$$

where $P(x) = 2\pi^{-1/2}\int_0^x \epsilon^{-t2}\,dt$, and $q = s(T_e/T_p)^{1/2}$ takes the place of s in the previous analyses. We shall not give the derivation of this equation. Let it suffice to point out that the first integral gives the density of the accelerated ions originating at values of η_z (or μ_z) less than η(or μ) and the second integral gives the density of the retarded ions originating at values of η_z (or μ_z) greater than η(or μ) and that these expressions include the flow of ions across the potential maximum. The two integrals can be rearranged to give

$$\pi^{1/2}\int_0^\infty \exp(\mu-\mu_z)\,dq_z-\pi^{1/2}\int_0^q \exp(\mu-\mu_z)P((\mu-\mu_z)^{1/2})\,dq_z.$$

Assuming the solution

$$\left.\begin{aligned}\mu &= Bq^2+Cq^4+\cdots\\ \mu_z &= Bq_z^2+Cq_z^4+\cdots\end{aligned}\right\}, \tag{80}$$

the first integral immediately above, which will be denoted by H_1, becomes

$$H_1 = \pi^{1/2}\epsilon^\mu\int_0^\infty \exp(-Bq_z^2-Cq_z^4-\cdots)\,dq_z.$$

Now, to the present degree of approximation

$$H_1 = H_1\Big|_{C=0}+C(dH_1/dC)_{C=0}$$

by a Taylor Expansion, so that

$$H_1 = \pi^{1/2}\epsilon^\mu\left[\int_0^\infty \epsilon^{-Bq_z^2}\,dq_z+C\int_0^\infty -q_z^4\epsilon^{-Bq_z^2}\,dq_z\right]$$

and

$$H_1 = (\pi/2B^{1/2})\epsilon^\mu[1-3C/4B^2] = (\pi/2B^{1/2})(1+\mu)[1-3C/4B^2].$$

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The second integral expands to

$$H_2 = 2\int_0^q [(\mu-\mu_z)^{1/2}+(2/3)(\mu-\mu_z)^{3/2}+\cdots]\,dq_z.$$

Using the assumed solution we have

$$H_2 = 2\int_0^q [(Bq^2+Cq^4-Bq_z^2-Bq_z^4)^{1/2}+(2/3)(Bq^2+Cq^4-Bq_z^2-Bq_z^4)^{3/2}]\,dq_z$$

and applying the same method as before

$$H_2 = (\pi B^{1/2}q^2/2)(1+Bq^2/2+5q^2C/8B).$$

Eq. (80) can be written

$$q^2 = (\mu/B)(1-\mu C/B^2+\cdots). \tag{81}$$

Using this to express H_2 in terms of μ we have

$$H_2 = (\pi\mu/2B^{1/2})(1+\mu/2-3C\mu/8B^2).$$

If T_p/T_e be noted by τ then $\eta = \tau\mu$ and Eq. (79) can now be written

$$A'\nabla_q^2\mu+\epsilon^{-\tau\mu}-H_1+H_2 = 0.$$

Neglecting $A'\nabla_q^2\mu$, expanding $\epsilon^{-\tau\mu}$, substituting for H_1 and H_2, and equating sums of coefficients of like powers of μ to zero, it is found that

$$1-(\pi/2B^{1/2})(1-3C/4B^2) = 0$$

$$-\tau-(\pi/2B^{1/2})(1-3C/4B^2)+\pi/2B^{1/2} = 0$$

whence

$$B = \pi^2/4(1+\tau)^2, \quad C = \pi^4\tau/12(1+r)^5.$$

Noting that $q^2 = s^2/\tau$ and $\mu = \eta/\tau$ we thus find for Eq. (81)

$$s = [2(1+\tau)\eta^{1/2}/\pi][1-2\eta/3(1+\tau)+\cdots]$$

for the plasma solution in the neighborhood of $s = 0$, at which place, as has been pointed out, the ion temperature will have the maximum effect. Comparison with the solution originally obtained (Table II*b*) shows that the distortion introduced by the finite ion temperature is only of the order of T_p/T_e, a very small quantity in most cases.

VI. Potential Distribution in the Plasma
Part II — Internal Plasma

The cases hitherto treated have all dealt with outward motion (in the extreme case parallel motion) of the ions toward an external collector. If, however, a negatively charged electrode is placed in the midst of the plasma this electrode becomes surrounded by a potential maximum and an entirely new condition

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arises in which the ions generated inside that maximum flow inward while those generated outside flow outward. Among the three plasma shapes already investigated it is evident that in the plane case the introduction of an additional infinite plane electrode parallel to the walls leaves the problem formally unchanged. On the other hand, introducing an axial cylindrical electrode in a cylindrical tube, or a central spherical electrode in a spherical bulb does change the problem. It is readily seen that the lower limit of the integral expressing the ion concentration in the plasma equation, Eq. (14) for instance, must lie at the potential maximum. Thus, to deal with the new cases, the zero limit of the integral has to be replaced by the finite value of s, say S, which corresponds to the radial distance of the potential maximum. Hence, the equation becomes,

$$\epsilon^{-\eta} - s^{-\eta} \int_S^s s_z^{\beta} \epsilon^{-\eta_z} (\eta - \eta_z)^{-1/2} \, ds_z = 0. \tag{82}$$

As before, an expansion in series can be attempted,

$$s - S = (2/\pi)\eta^{1/2}(1 + a_1\eta + \cdots)$$

outside the potential maximum and

$$s - S = -(2/\pi)\eta^{1/2}(1 + b_1\eta + \cdots)$$

inside the potential maximum. The coefficients a_1, a_2,..., b_1, b_2,..., are functions of S. Unfortunately, for the smaller values of S these series do not converge for all values of η less than η_0 with the result that rigorous solutions would be most complicated even if at all possible. It is questionable whether any involved mathematical investigation which is intended to cover electrodes of all sizes is justifiable.[28]

But the case of a very small cylindrical collector is of particular importance, because fine wires are often used experimentally as probes. In this instance the external plasma, namely that part of the plasma outside the potential maximum, very soon becomes indistinguishable from the plasma about an axial potential maximum and equations already derived can be applied. Inside the potential maximum an approximate mathematical method can be used. This method was employed before the rigorous solution of the long free path plasma equation had been found and applies strictly only when ionization is uniform. It gives the comparative results shown in Table II*b*.

The approximation is made by supposing that the ion density at A, Fig. (7), is caused by ions starting from the potentials lying on the broken line As_tS

[28] In the very short mean free path cases rigorous solutions are readily obtained. These solutions involve the zero order Bessel Functions of both the first and second kind in the cylindrical case and $(\cos s)/s$ as well as $(\sin s)/s$ in the spherical case.

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rather than from the potentials lying on the actual potential curve ABS. As_t is tangent to ABS at A. On this basis

$$\eta_z = \eta + (s_z - s)\eta' \quad \text{when } s < s_z < s_t$$
$$\eta_z = 0 \quad \text{when } s_t < s_z < S$$
$$s_t - s = -\eta/\eta'.$$

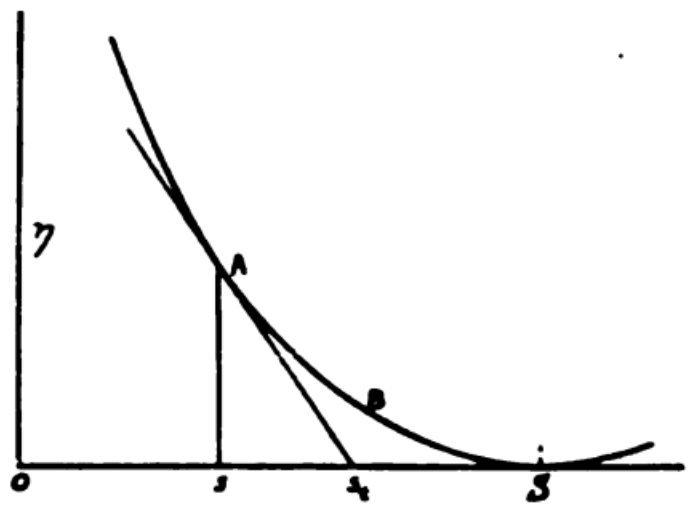

FIG. 7. Illustrating the approximation made in solving the internal plasma case.

These are the quantities now to be introduced in the integral of Eq. (82) after making that equation apply to uniform ionization ($\epsilon^{-\eta_z} = 1$) and to the cylindrical case ($\beta = 1$). The integral breaks up into two parts

$$(-\eta')^{-1/2} \int_s^{s_t} s_z (s_z - s)^{-1/2} ds_z + \eta^{-1/2} \int_{s_t}^{S} s_z \, ds_z$$

which combine to

$$\eta^{-1/2}[(S^2 - s^2)/2 - s\eta/\eta' + (\eta/\eta')^2/6].$$

Dropping $(\eta/\eta')^2/6$ because it is always negligible we have the approximate differential equation for an internal cylindrical plasma

$$\epsilon^{-\eta} - \eta^{-1/2}[(S^2 - s^2)/2s - \eta/\eta'] = 0.$$

The substitutions $\sigma = s/S$ and $\zeta = \eta^{1/2}/S$ convert this equation to

$$2\sigma\zeta\epsilon^{-S^2\zeta^2} + \sigma^2 + \sigma\zeta \, d\sigma/d\zeta - 1 = 0. \tag{83}$$

In the limit, when S is zero, $\epsilon^{-S^2\zeta^2} = 1$ and

$$2\sigma\zeta + \sigma^2 + \sigma\zeta \, d\sigma/d\zeta - 1 = 0. \tag{84}$$

The solution of this equation which satisfies the boundary conditions at $s = S$, (i.e., at $\sigma = 1$) namely, that $\zeta = 0$ and $d\zeta/d\sigma = 0$ there, is

$$2\zeta(\sigma + \zeta) + \ln(1 - 2\sigma\zeta) = 0 \tag{85}$$

or in terms of s and η,

$$(2/S^2)(\eta + s\eta^{1/2}) + \ln(1 - 2s\eta^{1/2}/S^2) = 0.$$

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Fig. 8 is a plot of this limiting form of the plasma equation. With increasing η Eq. (85) rapidly approaches the limiting form

$$2\sigma\zeta = 1-\epsilon^{-2\zeta^2-1} = 1.$$

Thus when $\eta^{1/2}/S > 1$ the internal plasma solution may be written

$$\eta = S^4/4s^2. \tag{86}$$

If as before, Eq. (61) be used as the criterion for the failure of the plasma solution as the sheath is approached, the limit of validity is again given by that

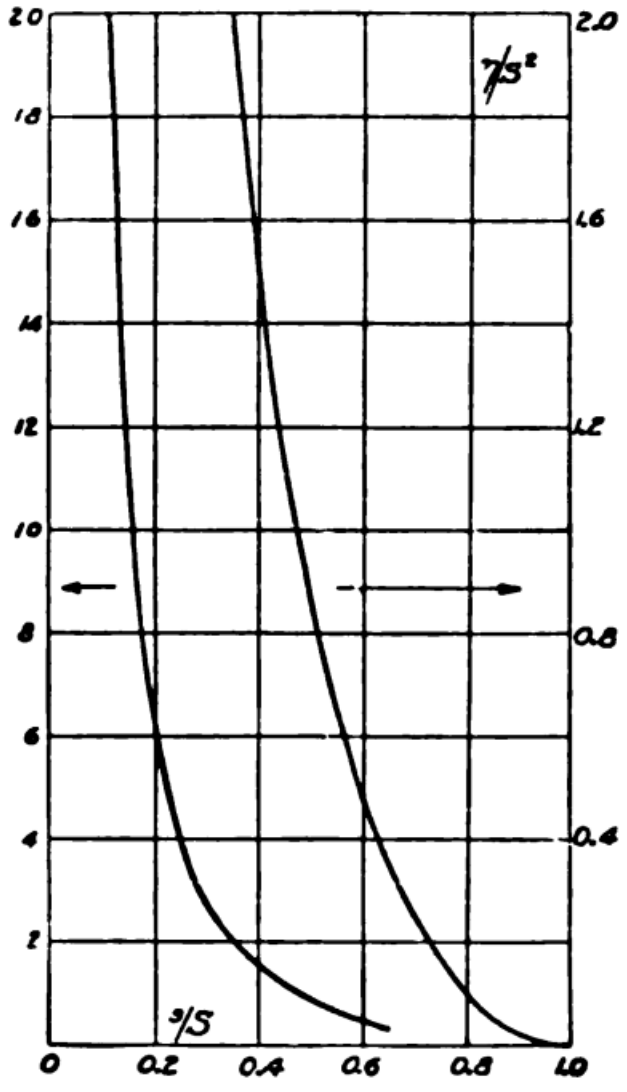

FIG. 8. Limiting form of potential distribution in plasma.

equation. In our present approximations, however, $\epsilon^{-\eta}$ may be taken as unity and it is found that

$$S^4/s_\Phi^4 = \Phi/A, \quad s_\Phi = S(A/\Phi)^{1/4}, \quad \eta_\Phi = S^2\Phi^{1/2}/4A^{1/2}.$$

Thus, choosing $\Phi = 0.05$ it is found in the two cases previously cited that
$s_\Phi = 0.0571\,S$, $\eta_\Phi = 78.8S^2$ in the 8.0 amp arc of Run 37a
$s_\Phi = 0.123S$, $\eta_\Phi = 16.3S^2$ in the 0.5 amp arc of Run 34b.
In order that errors arising from putting $\epsilon^{-\eta} = 1$ should not be serious, it may be insisted that for the present approximation $\eta_\Phi < 0.25$. If follows that in Run 37a, $S < 0.056$ whence $s_\Phi < 3.2\times10^{-3}$, or in terms of fractional tube radius $s_\Phi(\text{int})/s_\Phi(\text{ext}) < 4.1\times10^{-3}$. In Run 34b these quantities come out $S < 0.124$, $s_\Phi < 0.0153$, $s_\Phi(\text{int})/s_\Phi(\text{ext}) < 0.0197$. Thus, for the present formulae to hold within the approximations stated, the radius of the probe used

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in Run 37a would have to be somewhat less than 1/250th of the tube radius and in Run 34b somewhat less than 1/50th. Further quantitative relations depend on the possession of an adequate sheath theory for this case.

A striking feature of the internal plasma is the very uniform potential throughout all but a small portion near the internal electrode. Thus Eq. (86) shows that when $s = S^2/2$ then $\eta = S^2$. Accordingly, at least three-quarters of the ions generated in the internal plasma are formed at less than $\eta = S^2$ below the maximum. Even though S be as large as 0.1, the majority of these ions will possess enough thermal energy to cross the potential maximum and escape from the probe vicinity, for the ratio $\tau = T_p/I_e$ is rarely less than 0.01. This means that the trapping of ions by the potential maximum is relatively unimportant in the case of a fine wire probe. Of the ions reaching the probe only a small fraction may originate within the internal plasma; the vast majority have crossed the maximum from the outside. Thus the negatively charged probe can be treated on the present theory just as it was on the old.[29,30]

The important case appears to be that in which the collected current is limited by orbital motion. The equation for the resulting volt-ampere characteristic may be written[30]

$$i_p^2 = (4A^2I_p^2/\pi)(-eV_c/kT_p+1) \tag{87}$$

where i_p is the positive ion current to the probe, A is the probe area, I_p is the positive ion current density in the plasma about the probe, and V_c is the negative probe voltage measured with respect to the potential maximum in the plasma. Thus if the square of the observed current i_p be plotted against the collector voltage V_c a straight line will be obtained whose negative slope Σ is given by

$$I_p/T_p^{1/2} = (\pi k/4e)^{1/2}\Sigma^{1/2}/A. \tag{88}$$

In the previous applications of this theory it has been assumed that I_p at the tube axis was substantially equal to its value at the wall, and on this basis values of T_p were calculated which were comparable with T_e. It will be observed, however, that in view of the general kinetic theory relation

$$(I/e)T^{1/2} = n(k/2\pi m)^{1/2}. \tag{89}$$

Eq. (88) can say nothing regarding I_p and T_p individually. It can yield only the ion density in the plasma near the probe

$$n_p = (\pi m_p/2e)^{1/2}\Sigma^{1/2}/Ae = 3.32\times10^{11}(m_p/m_e)^{1/2}\Sigma^{1/2}/A \text{ cm}^{-3}$$

if Σ is expressed in amp volt$^{-1/2}$. It might appear possible to use the addition of unity to $-eV_c/kT_p$ in the second factor in the right member of Eq. (87) for determining T_p and I_p, but the extrapolation required and the uncertainty of the zero point of V_c among other factors make such an attempt futile.

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[29] *L and M-S*, Part I. The next few equations are taken directly from this article.

[30] H. M. Mott-Smith and I. Langmuir, *Phys. Rev.* **28,** 727 (1926).

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Although Langmuir and Mott-Smith have apparently used this method for measuring ion density with success, two difficulties appeared in the tests already tabulated in Table III. The first is evident in the table, namely, that the apparent density (Column 6) obtained for the ions in this way has approximately twice the value of the density found either by Eq. (55) or from the positive branch of the volt-ampere characteristic of the probe, as treated in Section

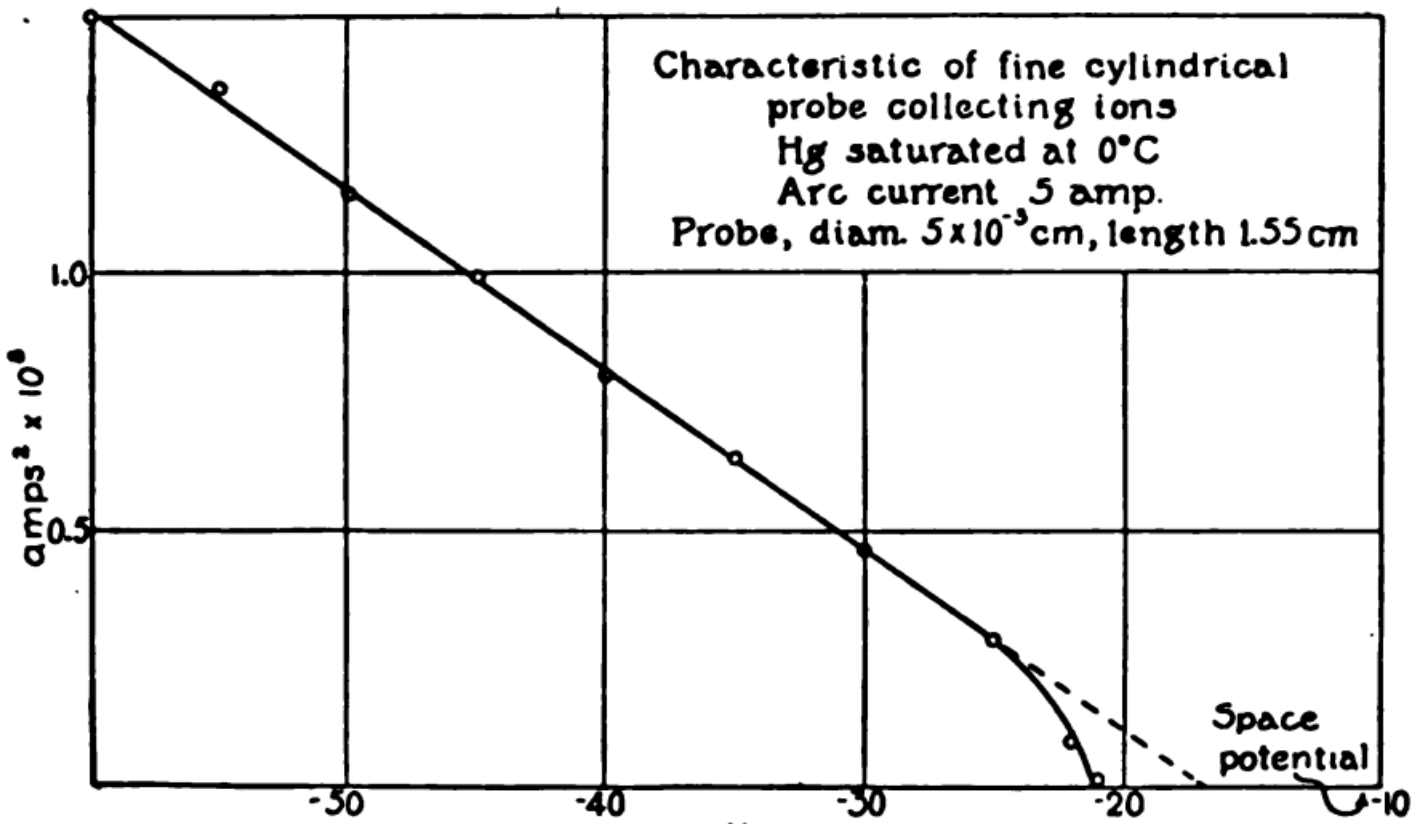

FIG. 9.

VII. The second difficulty is illustrated by Fig. 9, where it is seen that the straight line coinciding with the upper portion of the curve cuts the axis not at the space potential as theoretically required, but some 6v negative to it. Only the first run was normal in this respect. Neither of these difficulties casts serious doubt on the proposed theory; because the questions raised pertain rather to the theory of the collector. Quite apart from any plasma theory the ionization densities as determined from the two ends of the volt-ampere characteristic should agree, and yet there is the two-fold difference between Columns 6 and 7 of Table III.

If probes of greater and greater diameter be employed there are two important effects. The diameter of the potential maximum increases and as a first result the number of ions generated in the internal plasma increase more rapidly than the number crossing inward over the potential maximum. This occurs because the former number is roughly proportional to the cross-sectional area of the internal plasma, while the latter is proportional to the perimeter of the cross-section. As a second result the average potential within the internal plasma decreases with respect to the maximum so that a smaller proportion of the ions generated inside the maximum escape. The progression of these two factors thus accomplishes the transition from the condition where the small electrode

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only intercepts a part of the low temperature random ion current which is flowing around it to the condition where the large electrode determines a volume in the plasma from which it drains all the ions generated there.

VII. Potential Distribution in the Plasma
Part III — The Vicinity of a Positively Charged Electrode

It has been pointed out that if an electrode whose exposed area is just sufficient to receive a random electron current equal to the arc current be used as anode there is zero anode drop.[31] If the electrode area is greater than this critical area the anode drop is negative, if less than this positive. A collector having an area equal to or greater than this critical area in a discharge tube carrying a fixed arc current can be maintained at any desired potential negative to the plasma without any material effect on the plasma potential, as this is fixed by the anode. If, now, the collector potential is raised past the anode potential, the collector becomes anode and the plasma potential rises with it, the original anode thus taking the role of negative collector. Only when the electrode area is but a small fraction of the critical area can it be maintained at a voltage considerably positive with respect to the plasma.

The simplest case to consider is that of a small wire in a plasma of large dimensions relative to it. This case has not, as yet, been treated quantitatively since qualitative considerations seem to suffice for the present. The potential distribution is of the type shown in Fig. 10. The space charge in the sheath is made up of orbital electrons and out-going ions which have been generated in the sheath and whose space charge contribution may be very small except near the sheath edge. The electric field decreases to zero (or at least a very small value) at the sheath edge or plasma potential maximum, and from that point out the potential distribution is approximately normal. There are two disturbing factors, the drain of electrons from the plasma and the ions flowing outward from the sheath. Over wide ranges of potential on a fine wire the former results in only a very small electron deficiency and even when this becomes considerable it is compensated by the proportional decrease in ion generation. The second factor can be compensated by a slight increase in curvature of the plasma potential curve which causes a decrease in the space charge contribution from ions generated immediately outside the sheath edge. Thus a positively charged electrode of this type does not affect more than a small region in the plasma and the theory of electron collection which calls for a straight line $i_e^2 - V$ plot as previously outlined applies.[32] Ionization densities calculated in this way have already been given in comparison with the results of other methods in Table III.

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[31] *L and M-S*, Part IV, p. 766.

[32] *L and M-S*, Part I, p. 455, and Part III, Fig. 10.

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It is probable that the thickness of the electron sheaths in such cases is considerably greater than the value obtained from the ordinary space charge equation because of neutralization of electron space charge by ions generated within the sheath. They are generated there at a greater rate per electron than at the plasma potential maximum because of the higher electron velocities.[33] But this greater rate of generation does not lead to a net positive charge as the large electric field rapidly gives the ions considerable velocities. At the plasma potential maximum, however, the space charge neutralization is practically complete. Accordingly, near the sheath edge (plasma potential maximum) and

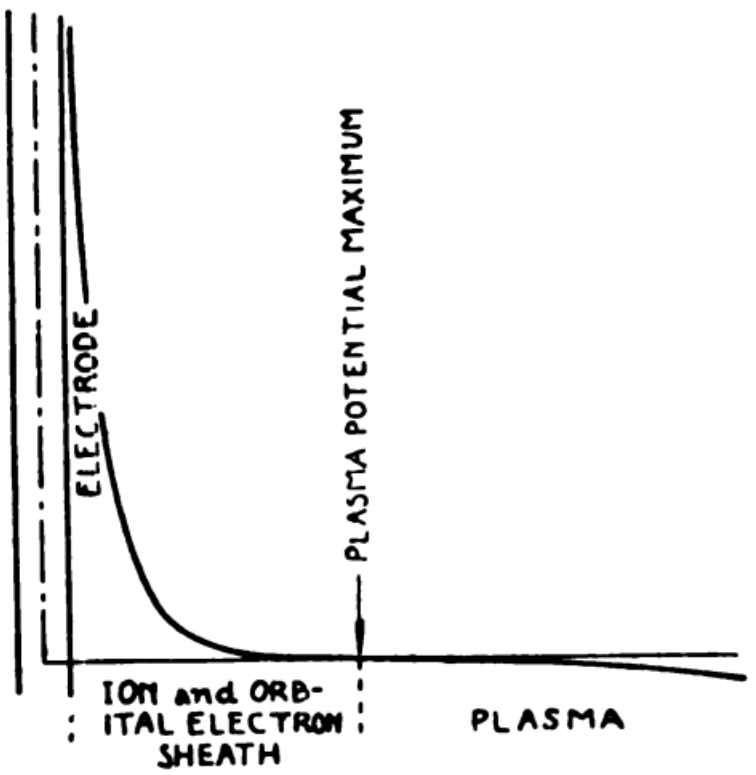

FIG. 10. Potential near a positively charged cylinder.

for some distance within it there will be appreciable neutralization of electron space charge by the ions. The resulting increase in the sheath thickness may be sufficient to make orbital motion the factor limiting the current to the electrode even with electrodes of comparatively large diameter. Such a collector has actually been found to give an approximately linear $i_e^2 - V$ plot at small positive potentials.[34]

A more complicated case is that of a small positively charged electrode on the tube wall. Collector *H*,[35] a square plate 1.9 cm on a side, bent to fit the wall of the 3.2 cm diameter discharge is such an electrode. Fig. 4[35] shows that with 2 amps leaving the cathode, a constant current of 1 amp reached this collector in the voltage range −10 to −5 v measured with respect to the anode. The present theory makes a rough calculation of the magnitude of this current possible.[36] As the potential of *H* was increased up to −10 v, the maximum plasma potential opposite it in the discharge tube remained fixed at −10 v. (Strictly

[33] See Section VIII (3).
[34] *L and M-S*, Part III, Fig. 11.
[35] *L and M-S*, Part II, Figs. 3 and 4.
[36] See also the treatment of this given by I. Langmuir, *Phys. Rev.* **33,** (1929).

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speaking, of course, there is no potential maximum in the plasma near H because of the arc gradient, but the resulting differences of potential over the width of H are so small compared to the radial plasma potential differences that the arc gradient is neglected in the present discussion). If, as the potential of H with respect to anode was still further raised (the total arc current being maintained constant at 2 amp), the plasma had tended to rise with it, the whole arc current would have flowed to it immediately. This is what would occur with a collector somewhat larger than H, as was pointed out at the beginning of this section. But as H varies from -10 to -5 v the maximum plasma potential opposite remains practically constant at -10 v. The existence of the saturation current is then accounted for in the usual way by the formation of an electron sheath over H so that the whole random electron current crossing the sheath edge is captured by H, whereas no ions can reach it.

A fairly good approximation to the conditions in the adjacent plasma when H is drawing its saturation current would be obtained if H were replaced by an orifice in the tube wall opening into a second discharge tube identical with the first in size as well as excitation. The most important difference is that in this hypothetical case electrons travel both ways through the orifice, but this is not thought to be an essential difference since the actual one-way flow results to a first approximation only in lowering the electron density to one-half, the ion density dropping in proportion due to the proportional ionization. Our hypothetical picture shows that for small openings there is a saddle-like potential distribution in the hole, the pommel lying in one tube, the cantle in the other. As the size of the hole is increased, the plasma maxima in the two tubes approach each other along the hole axis, finally merging into a single maximum at the center of the hole. This size undoubtedly corresponds to the size at which H would become anode as soon as it reached plasma potential. Since this did not happen, the plasma maximum lies some distance from the edge of the sheath on H. Thus we may adopt the plasma maximum as the upper limit of the plasma potential in contact with the sheath edge at the center of H.

The radial motion of the ions over the cross-section of the hypothetical hole is so similar to their motion in a cylinder that we are led to use the same mean value of $\epsilon^{-\eta}$ over the sheath edge of H as over the cross-section of a cylinder, namely, $2h_0$ (Eq. (53)). Then applying the Boltzmann Equation to the electrons at the sheath edge the current density is found to be

$$e^{-\eta} n_0 e (kT_e/2\pi m_e)^{1/2}$$

at any point of H and to average

$$I_e = 2h_0 n_0 (kT_e/2\pi m_e)^{1/2}$$

over the whole of H.

The use of the Boltzmann Equation in this case where accelerating fields are involved needs some justification. The hole analogy indicates that the plasma equipotentials are perpendicular to H at the sheath edge, so that with respect

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to tube radii the plasma potential at H is a minimum. In addition the accelerating field from the edge of H toward its center is similar in nature to the accelerating field toward the axis of the discharge tube which does not lead to any apparent discrepancies, and the flow of electrons toward H across the sheath edge is analogous to their longitudinal drift in the discharge tube. The chief effect of the accelerating field at H is to reduce the electron density at the sheath edge everywhere to one-half normal, the effect of which has already been dealt with.

Combining the equation for I_e with Eq. (55) the ratio of I_e to I_p is found to be

$$\begin{aligned} I_e/I_p &= (m_p/m_e)^{1/2}/\pi^{1/2}s_0 \\ &= 0.731(m_p/m_e)^{1/2} \end{aligned} \tag{90}$$

which, in the case of Hg gives $I_e/I_p = 444$, as an upper limit. This is to be compared with the average experimental value 380 of six low pressure runs.[37]

VIII. General Arc Relations

It will be shown in this section that the plasma balance equation completes the number of relations necessary to determine all the variables of the positive column of an arc as a function of one of them. Although all these relations are recognized qualitatively, the complete quantitative formulation of certain ones is lacking. For this reason the discussion undertaken here is to be regarded as suggestive of the possibilities offered by the theory and also as serving, perhaps, to define the problems still awaiting solution.

The variable quantities involved in the positive column of an arc may be divided into two classes, the independent and the dependent. Among the former belong the gas used, the tube radius a, the gas pressure p_g, and the wall temperature, which, in case the atomic mean free path is comparable with a, may be used for the gas temperature T_g. One of the arc variables proper must also be included in this category — experimentally it is usually the total arc current i_B. The dependent variables are, therefore, the axial electric field Z, the electron density in the axis n_0, the electron temperature T_e, the positive ion current density at the wall I_p, and the number of ions generated per electron per second, λ. These variables are five in number and five equations will be required for their complete determination. These equations, involving various more or less accurately known relations and constants will be discussed individually for the low pressure (Case $CL\lambda$) arc.

(1) ***The Plasma Balance Equation.*** Eq. (46).—This is the essentially new equation given by the present theory. When the wall sheath is not thin the tube radius a must be corrected by using Eq. (77). Further, this equation only

[37] *L and M-S,* Part II, Table III, Runs 34b to 37a.

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applies strictly when ions are formed by a one-stage process, for only in that case is the ion generation at a point proportional to electron density. In the hypothetical case that ion generation is entirely a two-stage process, the rate would tend toward proportionality to the square of the electron density. Now the transition from uniform ion generation to ion generation proportional to the first power of electron density causes s_0 to go from 0.5828 to 0.7722. Hence the further complete transition to proportionality to the square of the electron density would probably cause a further comparable increase in s_0. It seems probable, however, that not until high arc current densities and high gas pressures as well are reached will this effect on s_0 become considerable, for s_0 depends on the distribution of the ion production through the tube cross-section and not on its magnitude. Thus the ionization of excited atoms may be contributing effectively to the total ionization, thereby causing the "constant" λ to increase with i_B, but as long as these excited atoms are distributed with fair uniformity over the tube cross-section (as in the case at low pressure) s_0 will be but slightly affected.

(2) *The Ion-Current Equation*, Eq. (55A).—Deviations from the one-stage ionization process will also affect the accuracy of this equation. But since $s_0 h_0$ changes only from 0.2914 to 0.2703 in passing from uniform to proportional ion generation this equation is much less dependent on ionization mechanism than is the plasma balance equation.

(3) *The Ion Generation Equation.*—In accounting for the ion generation Killian[38] has assumed that the ionizing is done by those electrons in a normal M. D. which possess velocities greater than the equivalent of the ionization potential V_i of the gas. As Killian points out, it is sufficient for this purpose to represent the ionization probabilities at the lower voltages only and these are given accurately enough by the relation

$$P = \beta(p_0/T_0)(W - V_i) \tag{91}$$

where P is the number of ions generated per electron per cm of path at gas pressure p_0 and temperature T_0 (the really significant variable is density) and where W is the equivalent voltage of the ionizing electrons, V_i is the gas ionization potential, and β is an experimentally determined constant. It is the slope of the P vs. W curve reduced to unit pressure and temperature. Using W_e for $T_e/11{,}600$ the calculation of λ on the basis stated gives

$$\lambda = 6.70 \times 10^7 \beta(p_0/T_0) W_e^{3/2}(2 + V_i/W_e)\epsilon^{-Vi/We} \tag{92}$$

Judging from the shape of the experimental probability curves[39] it seems more reasonable to assume that initially P is proportional to the excess velocity of

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[38] Forthcoming article in *Phys. Rev.*

[39] K. T. Compton and C. C. Van Voorhis, Fig 6, *Phys. Rev.* **26,** 436 (1925) and T. J. Jones, Fig. 2, *Phys. Rev.* **29,** 822 (1927).

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the electron rather than its excess energy. The only effect of this assumption is to change the 2 in the parenthesis of Eq. (92) to 3/2. Thus, putting

$$f_\lambda = 6.70\times 10^7 (W_e/V_i)^{3/2}(3/2+V_i/W_e)\epsilon^{-V_i/W_e} \tag{93}$$

and

$$a_\lambda = \beta V_i^{3/2}(p_0/T_0) \tag{94}$$

we have

$$\lambda = a_\lambda f_\lambda. \tag{95}$$

A plot of f_λ facilitating calculations of λ is given in Fig. 11. From Compton and Van Voorhis $\beta p_0/T_0$ is 1.4 where $p_0 = 1$ mm $= 1330$ baryes and T_0 is room temperature, that is about 300°K.[40] Thus $\beta = 0.31$ for mercury vapor.

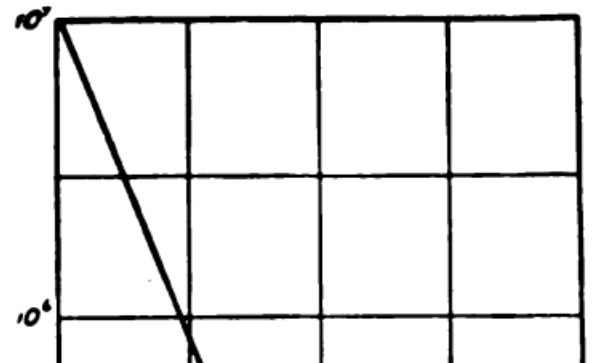

FIG. 11. Variation of relative ionizing power of electrons with their equivalent voltage.

Also, $V_i = 10.4$. For the case illustrated in Fig. 3 we may assume $T_0 = 400$°K roughly. Correcting 0.27 baryes, the vapor pressure of Hg which is saturated at 1.4°C, for thermal effusion it is found that $p_0 = 0.33$ baryes, whence $a_\lambda = 0.0086$. Since $T_e = 38{,}800$°K, $V_i/W_e = 3.11$ and from Fig. (11) $f_\lambda = 2.5 \times 10^6$ giving $\lambda = 2.1\times 10^4$. Eq. (48) can also give a value of λ. Using $a = 3.1$ it

[40] By letter K. T. Compton has explained that the pressure of 1 mm, given in Fig. 6 (*loc. cit.*) corresponds to the initial temperature of the ionization compartment before any heating occurred there.

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is found that $\lambda = 4.5 \times 10^4$ which is rather satisfactory agreement in view of (a), large uncertainties in β, and (b), the large errors in f_λ which arise from small errors in T_e.

Thus the plasma balance equation and the ion generation equation together constitute a pair of simultaneous equations in the variables λ and T_e only, which should fix these two arc variables irrespective of the others. The simultaneous graphical solution of the two equations is given by Fig. 12. The relatively small changes in T_e for large changes in a and α_λ is evident.

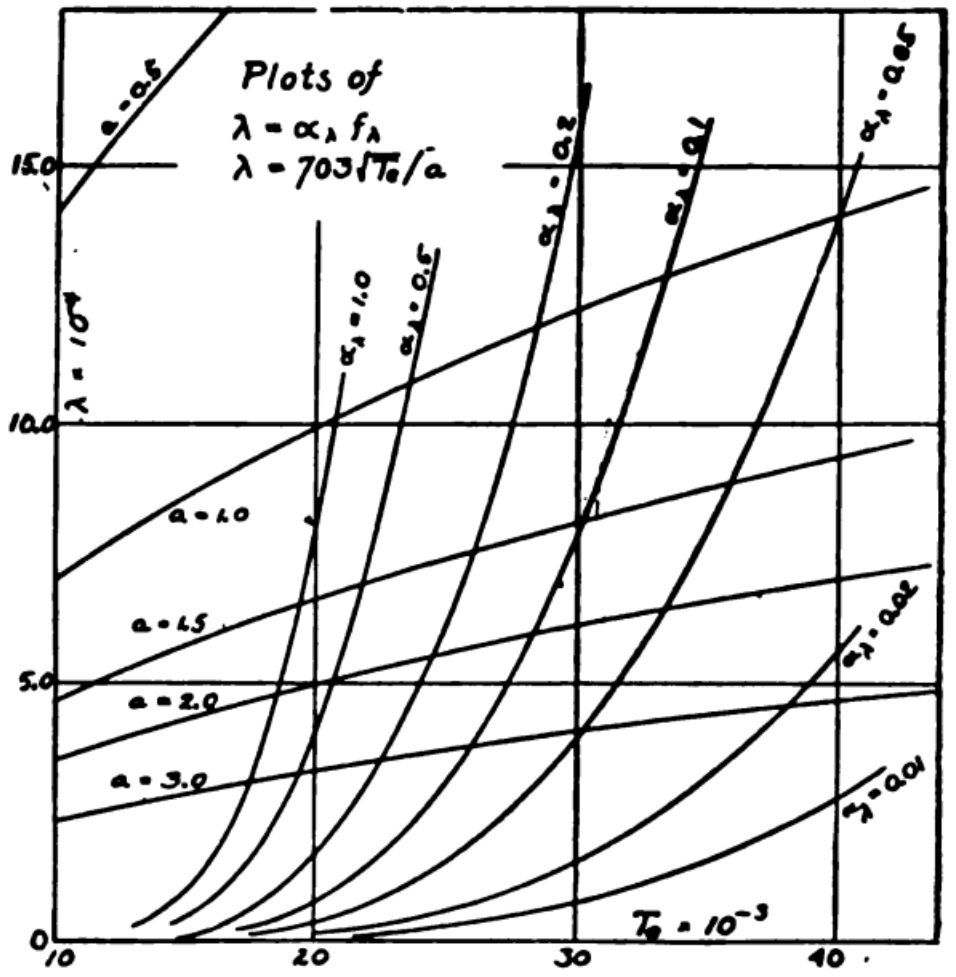

FIG. 12. Simultaneous solution of plasma-balance and one-stage ionization equations.

The fact, however, that T_e does vary with arc current, decreasing in general with increasing current, indicates that two-stage ionization processes contribute appreciably to the total ionization. How important such processes may be is shown by a comparison of the values of λ given in the last two columns of Table I. These values were calculated in the same way as the values of λ for Killian's results. The fifth column gives the rate of ion generation necessary to maintain the plasma, the last column the rate at which the one-stage process can supply ions. The rapid failure of this source of ionization with increasing arc current is evident.

Unfortunately a new element of uncertainty has been introduced in the attempt to confirm Eq. (95) further by the additional experiments already mentioned in connection with Table III. It is to be noted that the electron temperatures there listed are considerably lower than those found by Killian, who for instance found 38,800° under apparently the same conditions as those which gave us 27,800° as listed in the second row of Table III.

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(4) *Mobility Equation.*—Killian has used[38] Langevin's Mobility Equation

$$\mu_e = 0.75 e l_e / m_e \bar{v}_e \tag{96}$$

where μ_e is the electron mobility, l_e the electron mean free path and $\bar{v}_e$ the average thermal velocity, to calculate the electron mean free path from the arc current i_B, arc gradient Z, and N the number of electrons per cm length of column through the relation

$$i_B = N e \mu_e Z \tag{97}$$

The value so obtained checks well with the accepted value. Combining these two equations and noting at the same time that $\bar{v}_e = 2(2kT_e/\pi m_e)^{1/2}$ and that N_e is given by Eq. (54), it is found that

$$i_B = 0.75 \pi^{3/2} h_0 e^2 a^2 n_0 l_e Z / (m_e k T_e)^{1/2} \tag{98}$$

$$= 8.7 \times 10^{-10} a^2 n_0 l_e Z / T_e^{1/2} \tag{99}$$

for Hg in practical units.

(5) *Energy Balance Equation.*—Although the types of energy loss in the positive column are probably known, their quantitative formulation is not possible as yet. The power input is, of course, $i_B Z$ per cm of tube. Of this, the kinetic energy of the ions striking the wall accounts for approximately $2\pi a I_p(\eta_w - 0.3) kT_e/e$ ergs sec^{-1}, the 0.3 being estimated as the average potential drop in the plasma. The electrons striking the wall account for $2\pi a I_p 2kT_e/e$ and the heat of recombination for $2\pi a I_p V_i$ watts. In addition, the ions will, on the average, have fallen through various distances parallel to the axis before striking the wall and will therefore dissipate additional energy there. The energy which the electrons lose in their elastic collisions is probably negligible,[38] but this cannot be true of their inelastic collisions which do not lead to ionization, those which do lead to ionization having already been counted at the wall. Certain unknown probabilities are involved in these processes but no new arc variables.

The power immediately accounted for is thus

$$2\pi a I_p [8.0 T_e/11{,}600 + V_i] \text{ watts.}$$

Applying this to two typical cases, a 1 amp and an 8 amp arc at low pressure[41] it is found that only 0.31 to 0.28 of the energy is accounted for in this way, necessitating a detailed investigation of the radiation loss and of the other more obscure factors.

Schottky's treatment of the positive column.—In his arc theory Schottky[42] combines his plasma solution directly with the plausible assumption that $N_e \lambda$ is proportional to $i_B Z$ for different tube diameters to obtain the important result that arc gradient is inversely proportional to tube diameter if electron and ion mobilities remain constant. Certain experiments of Claude in neon in which

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[41] Runs 35a and 37a of *L and M-S*, Part II, Table III.

[42] W. Schottky, *Phys. Zeits.* **25,** 635 (1924).

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the product Za [Schottky's $(\partial V/\partial Z)\cdot R$] was found to be constant appear to confirm the original assumption. The practical value of this treatment cannot be denied, but from a more fundamental point of view such a short cut evades some of the basic relations in an arc, all of which must be woven into any comprehensive theory.

Stability and oscillations.—The plasma balance equation represents an equilibrium but it is not obvious that it is a stable equilibrium. Instability of one type in a mercury cathode arc is certainly shown by its negative resistance. But there seem to be other possibilities also. Thus if we suppose that due to statistical fluctuations in the electron velocities the rate of generation of ions in a certain cross section of a positive column is momentarily $\lambda+\delta\lambda$ what will ensue? The plasma balance equation tells us that $\lambda+\delta\lambda$ corresponds to a smaller tube diameter than λ, that is, that the plasma field is stronger and that the ions will, consequently, flow out faster. But this increased field and increased positive ion density result in an increased potential at the tube axis in this region, at least while the excess ions are flowing away. This, in turn, by unduly accelerating electrons causes further excess ionization in this cross-section. At the same time, to the anode side of this region there will be a deficiency of ionization, for there the arc gradient will be less than normal. Will such a lump of excess ionization be dissipated more rapidly than renewed? Will the general drift of ions toward the cathode carry it as a wave in that direction?

If a positive column is maintained by an anode of such size that it has a negative drop, there is an absolute potential maximum in the plasma near the anode. At that place one should expect to find more and more electrons trapped because of inelastic collisions made nearby. This would mean a progressive decrease in their effective temperature. Does the same mechanism which accomplishes the rapid recovery of a disturbed M. D. elsewhere in the positive column operate in this region also, preventing the average electron energy from decreasing? Or do the electrons accumulate to a degree where new forces predominate which allow them to dissipate once more?

A study along the lines indicated by these and possibly other similar questions may lead to further insight into the oscillation behavior of arcs.

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METASTABLE ATOMS AND ELECTRONS PRODUCED BY RESONANCE RADIATION IN NEON

With CLIFTON G. FOUND as co-author

Physical Review
Vol. XXXVI, No. 3, August (1930).

IN STUDIES of arcs in pure neon at a few millimeters pressure, we have observed some phenomena which indicate that the resonance radiation from the arc can travel through the un-ionized neon beyond the end of the arc for distances of 20 to 30 centimeters, or more, and the absorption of this radiation produces, by excitation of the gas and by subsequent collisions of the second kind, metastable atoms at the rate of at least 10^{13} cm^{-3} sec^{-1}. In these experiments the neon arc carrying currents of approximately one ampere was passed through a long tube between a hot cathode at one end of the tube and an anode placed near the center. One or more collecting electrodes were placed in the tube beyond the end of the arc, and, in some experiments, movable collectors were used whose distance from the end of the arc could be varied.

The metastable atoms thus produced beyond the end of the arc diffuse into contact with the walls or with metallic electrodes and liberate electrons from the glass or metal surfaces which have velocities of the order of 5 or 10 volts. The walls thus become positively charged and within the space there is a definite concentration of electrons. If a collector is maintained at, say, 70 volts negative with respect to the anode, the electrons emitted by it serve as a measure of the concentration of the metastable atoms. The concentration of metastable atoms diffusing out of the arc should fall to $1/e^{\text{th}}$ of its value every time the distance is increased by an amount equal to 0.4 of the radius of the tube. Within a few centimeters of the end of the arc this theoretical decrement is actually observed, but at greater distances the concentration of metastable atoms decreases far more slowly.

The width of the line corresponding to the resonance radiation in the arc is far greater than that of the absorption line in the unexcited neon, which is determined mainly by the Doppler effect. Assuming this cause of broadening of the absorption line, it can be calculated that between parallel planes the energy absorbed per unit volume in the gas is inversely proportional to the

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[EDITOR'S NOTE: This paper appears as Chapter Thirteen in the author's book, *Phenomena, Atoms and Molecules*, Philosophical Library, 1950.]

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distance from the source and thus, as the end of the arc is approximately a point source of this radiation, the rate of production of metastable atoms should vary approximately inversely as the cube of the distance from the end of the arc. The observed values are in accord with this theory.

The electrons in the tube generated at the walls by the metastable atoms give to the neon a conductivity whose value can be measured, and from the known mobility of the electrons in neon gas, the concentration of electrons can be calculated. For low voltages on the collectors, the currents are determined solely by this mobility of the electrons, and the volt-ampere characteristics for small currents are thus linear on both sides of the zero point. For larger negative voltages, the current becomes limited by the number of electrons that can escape from the collector. For sufficient positive voltages, the currents increase more rapidly than corresponds to the linear relationships, this being due to ions produced within the tube by the acceleration of the electrons. The experiments prove that these ions are produced by the ionization of metastable atoms rather than normal atoms, for the rate of production of ions varies approximately with the square of the light intensity. These conclusions are confirmed by experiments with a shutter placed beyond the end of the arc, which can cut off the radiation without interfering with the diffusion of atoms, ions, etc., past the shutter. It is thus proved that the production of metastable atoms is produced by light which travels only in straight lines. The fact that when the collector is positive the electron currents are also proportional to the light intensity proves that the electrons owe their origin not to photoelectric effect from the electrodes, but to the production of metastable atoms or excited atoms. Small disk-shaped collectors have also been used in which one side, which faces the source of resonance radiation, is covered with mica so that the exposed surface cannot receive radiation, and yet the electron current was nearly as great as if the mica were placed on the back side of the collector, proving that the electron currents are generated by metastable atoms which diffuse short distances to the electrode even when the resonance radiation does not reach the electrode itself.

Since the electrons emitted by the collectors possess considerable initial velocities, and since these electrons make elastic collisions with the neon atoms, it is impossible to obtain a saturation current from these electrodes at these gas pressures, for electrons which diffuse back to the electrodes have sufficient energy to be reabsorbed by it. A theoretical investigation shows that in this case the current i_x that flows between any two electrodes is given by the equation,

$$i_x = \frac{16\pi}{3} I_0 \lambda C (V/V_0) \frac{1}{\log (1 + V/V_0)}$$

where V is the potential difference between the electrodes, V_0 is the volt equivalent of the velocity with which the electrons are emitted from the cathode,

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I_0 is the theoretical saturation current density of the electrons emitted from the cathode, λ is the mean free path of electrons, and C is the electrostatic capacitance between the two electrodes (expressed in centimeters). Experiments show that this equation expresses well the volt-ampere characteristics of neighboring electrodes in neon exposed to resonance radiation.

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STUDY OF A NEON DISCHARGE BY USE OF COLLECTORS

With CLIFTON G. FOUND as co-author

Physical Review
Vol. XXXIX, No. 2, January (1932).

ABSTRACT

Experiments are described from which it is concluded that electrons are emitted from a negatively charged collector in the positive column of a neon discharge. The influences of conditions in the plasma and conditions in the sheath are separated and their influence on the formation of a cold cathode discharge considered. Measurements of currents to a negatively charged collector in un-ionized neon beyond the end of the positive column, with variation of position of the collector discharge current, and external conditions, support the theory that resonance radiation travels in straight lines over large distances beyond the end of the positive column. The absorption of this radiation and collisions of the second kind result in the formation of metastable atoms, which diffuse into contact with the walls or with metallic electrodes and there liberate electrons.

Introduction

A NUMBER of observers have found that currents to negatively charged electrodes in the positive column of a neon discharge exceed those predicted from the collector theory of Langmuir and Mott-Smith.[1] Measurements of Uyterhoeven[2] show that the observed current to a collector, in neon, at 150 volts negative to the surrounding space, is twice that calculated from the collector voltage and observed sheath thickness. The large observed current can be accounted for on the basis of a secondary electron emission from the collector. More recent results of Uyterhoeven and Harrington[3] show that at low pressures of neon from 15–50 per cent of the current to a negatively charged collector is due to secondary electron emission, the greater part of which is caused by uncharged particles which are probably metastable atoms. Oliphant[4] accounts for the high secondary electron emission by neutral particles, which he obtained with a discharge in helium, as being due to metastable atoms striking the electrode and thereby releasing electrons.

Results of investigations with discharges in neon at 1 to 2 mm pressure indicate that electron currents of considerable magnitude leave a negatively charged electrode not only when the electrode is in the main discharge,[5] but

[1] Langmuir and Mott-Smith, *Gen. Elect. Rev.* **27,** 449, 538, 616, 762, 810 (1924).
[2] Uyterhoeven, *Proc. Natl. Acad. Sci.* **13,** 32 (1929).
[3] Uyterhoeven and Harrington, *Phys. Rev.* **36,** 709 (1930).
[4] Oliphant, *Proc. Roy. Soc.* May, (1929).
[5] Langmuir and Found, *Phys. Rev.* **36,** 604 (1930).

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also when the electrode is located in un-ionized neon at a distance as great as 30 cm from the end of the positive column. The present paper gives the experimental evidence for the conclusion that these currents are actually electron currents and also gives results which indicate that the secondary electrons originate from bombardment by metastable atoms.

Electrode in Positive Column

Studies with the rectifier type of hot cathode neon tube which has been described by Found and Forney[6] showed that a cold cathode discharge between the anodes occurs when the voltage between the anodes is much lower in neon than for similar tubes filled with mercury. In order to investigate the processes involved in the break-down between the anodes, a d.c. discharge was passed through neon at 2 mm pressure between the hot cathode located at one end of the cylindrical tube of 2.5 cm diameter and 45 cm long, and one of the anodes. The anodes were iron cups 1.3 cm in diameter and 1.3 cm deep and were placed in side arms which extended at right angles to the main tube

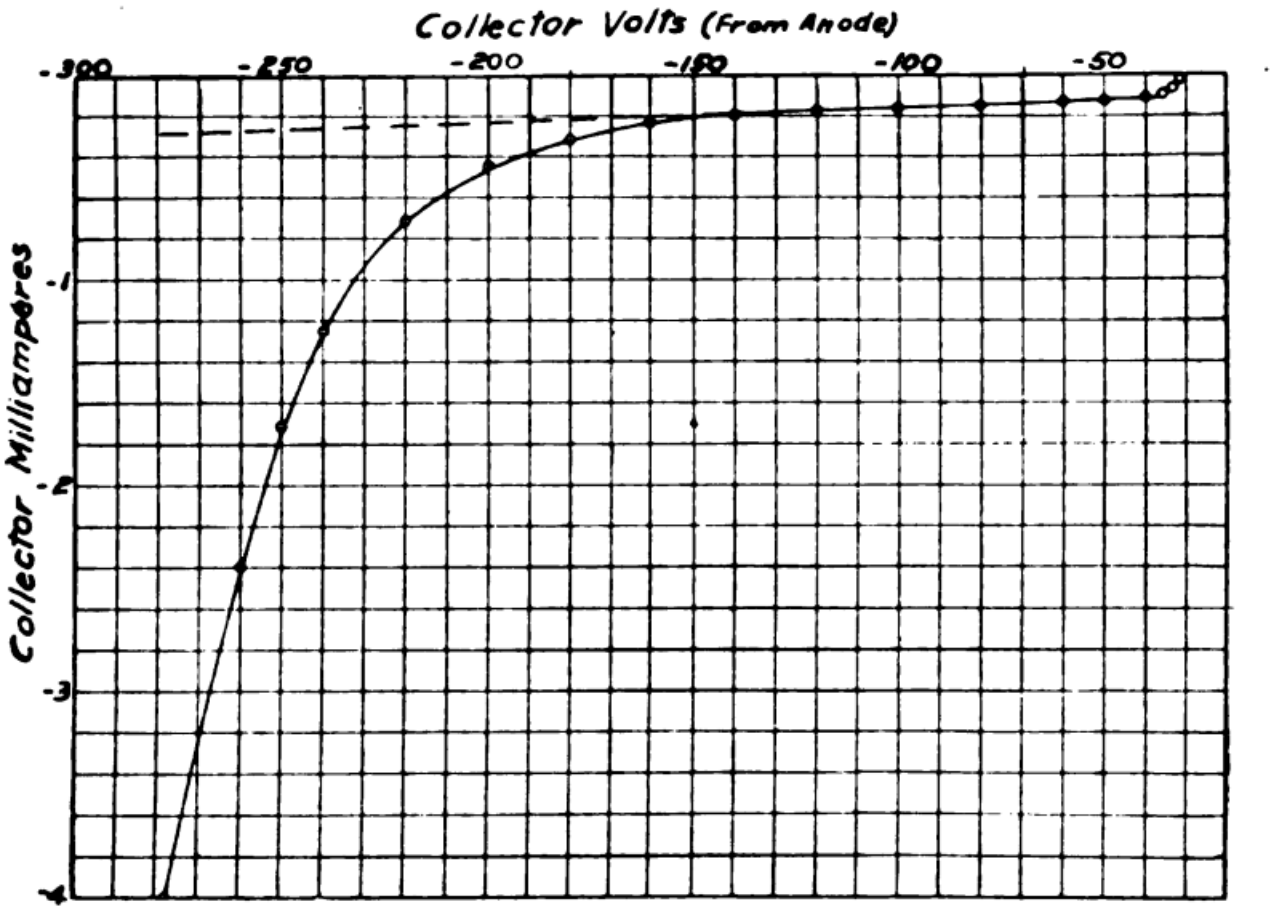

FIG. 1. Neon tube No. 381. Discharge current = 3.0 amps. Tube volts = 69. Neon pressure = 2 mm. Second anode as collector.

at the end opposite the cathode. The fronts of the anodes were 2.8 cm from the center of the main discharge. The second anode was used as a collector and the current to it measured as its potential was made increasingly negative with respect to the anode carrying the main discharge. The volt-ampere characteristics, given in Fig. 1, show that the current is not independent of the collector

[6] Found and Forney, *Trans. A. I. E. E.* **47**, 751 (1928).

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voltage, as predicted from the theory of sheaths, but increases as the potential of the electrode is made increasingly negative.

At voltages more negative than about —180 volts, the rate of increase of the negative current[7] is much more rapid, approximating an exponential increase. In the voltage range, where the current increases rapidly, if the main cathode is disconnected, a cold cathode discharge is maintained between the collector and the anode. In order to maintain such a discharge, there must be an electron emission from the collector. That such an electron emission does exist is shown by the fact that when negative voltage on a collector within the positive column reaches the value at which the current increases rapidly, a dark space, resembling a Faraday dark space, is formed in the positive column on the anode side of the electrode.

Effect of Electrons Injected into the Positive Column

In order to connect the formation of a dark space with the emission of electrons, it is necessary to describe a phenomenon observed by the writers several years ago while measuring the potential distribution in a glow discharge.

The tube in which the observations were made was a cylindrical glass tube 6.5 cm in diameter. At each end of the tube was placed a flat disk-shaped aluminum electrode which filled the cross section of the tube. The distance between the electrodes was about 40 cm. The tube contained nitrogen at a pressure of 0.34 mm in which a d.c. discharge of 5 ma current passed between the electrodes. The cathode dark space of about 0.6 cm thickness was followed by the blueish cathode glow which extended about 1 cm. Beyond the cathode glow was the Faraday dark space for a distance of 13 cm. The remainder of the tube was filled with the reddish positive column which was striated, the distance between striations being about 3 cm. The voltage drop across the tube was about 750 volts, approximately half of which was cathode drop. A V-shaped tungsten filament of 1 cm length and 0.17 mm diameter was placed within the positive column about 5 cm from its cathode end (i.e., between the second and third striation).

When the filament is heated to a temperature at which it emits electrons, if the potential of the filament is negative to the space surrounding it, electrons are injected into the discharge with a velocity corresponding to the difference of potential between the filament and the space. The number of electrons injected can be controlled by the filament temperature and their velocity by the voltage on the filament. It was found that when electrons with a velocity greater than that corresponding to about 100 volts were injected into the positive column of the nitrogen discharge, a dark space, similar to the Faraday dark space, was observed on the anode side of the point where the

[7] A negative current is defined as one in the direction of electrons leaving an electrode and positive current, one in the direction of electrons to an electrode.

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electrons were injected. No change could be seen in the discharge on the cathode side of the filament. When the filament was connected directly to the main cathode, that is, the electron velocity corresponded to about 400 volts, it was found that an injected electron current of only 20 microamperes, or '0.4 per cent of the discharge current, caused the formation of a dark space, extending from the filament a distance of 8 cm toward the anode. The length of the newly formed dark space increased almost linearly with the number of injected electrons, approaching a limiting length equal to the Faraday dark space between the cathode glow and the positive column. The value of the injected electron, current, at which the limiting length of dark space was reached, was lower the higher the velocity of the injected electrons. For example, with 400 volt electrons the limit was reached with 20 microamperes of injected current, while with 100 volt electrons it required about 400 microamperes.

The formation of the dark space, produced in the present experiment with neon, took place when the collector was about 150 volts negative to the space and increased in length as the collector voltage was increased. The close analogy between the two cases indicates that electrons of high velocity were being injected into the neon discharge when the collector was at high negative voltages.

Ionization by Collision

When electrons are emitted from an electrode into a gas the current from the electrode increases as the applied potential is increased. Dr. K. B. Blodgett[8] has made observations on the increase of current in argon and finds when the electronic mean free path is small compared to the sheath thickness, that the current increases according to the relation

$$i = i_0 2^n$$

where i_0 is the electron emission from the cathode and n the number of ionizing collisions within the sheath. When the sheath is thin compared to the mean free path the maximum increase of current with voltage corresponds to an arithmetic progression.

The current from a collector in a neon discharge increases with voltage in a manner similar to that observed by Dr. Blodgett for argon. When the collecting electrode is in a region of low ionization, as is the case when the second anode was used as collector for curve of Fig. 1, the collector current increases exponentially with increase in voltage. This is shown in Fig. 3 in which the logarithm of the difference between the observed current of Fig. 1 and the positive ion current expected from space charge considerations is plotted against the voltage. The equation of this curve is

$$i = 0.94 \times 2^{(V-21)/21} \times 10^{-6} \text{ amps}$$

[8] K. B. Blodgett, *Proc. Natl. Acad. Sci.* **15,** 233 (1929).

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which, from analogy with Dr. Blodgett's results, corresponds to an electron emission from the collector of 0.94 microamperes.

Moreover, when the collector is in a region of intense ionization (i.e., when the sheath thickness is thin) the rate of increase of current decreases with in-

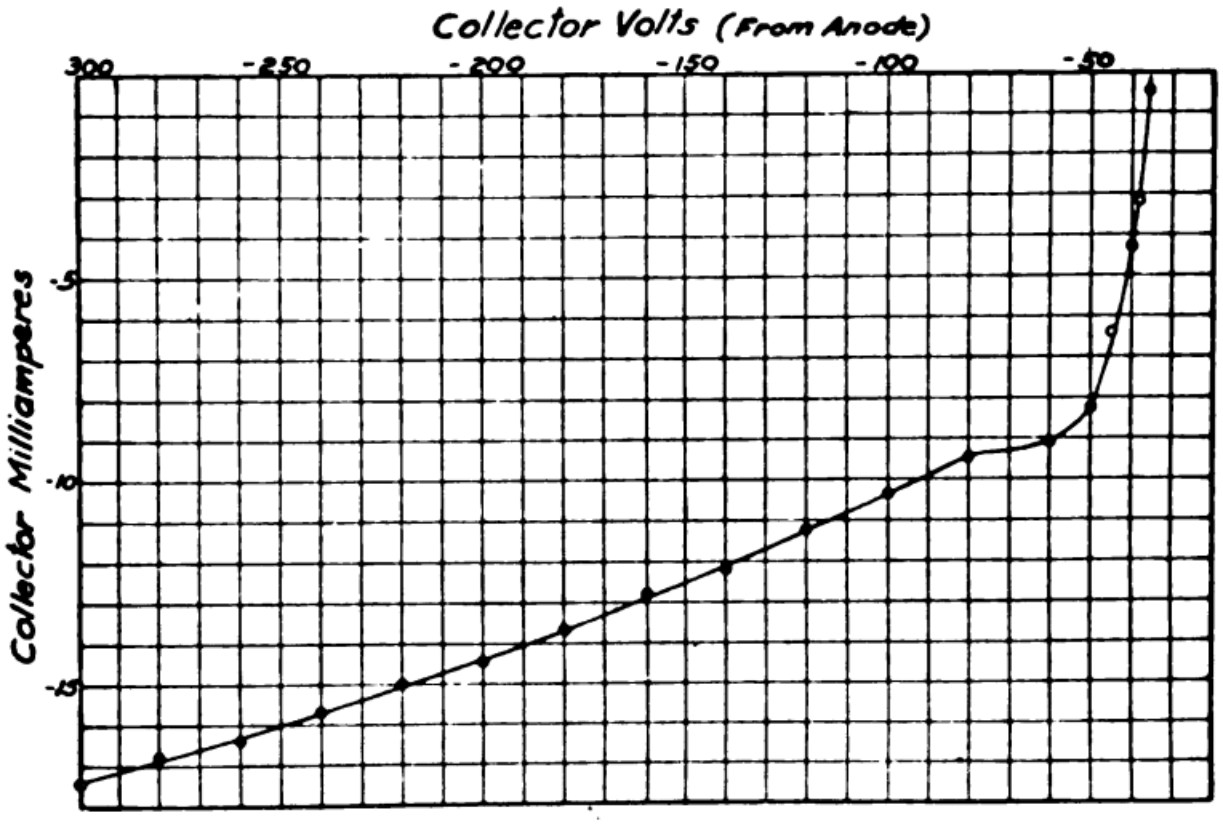

FIG. 2. Neon tube No. 19,800. Discharge current = 2.0 amps. Neon pressure = 0.7 mm. Collector is tungsten disk in positive column.

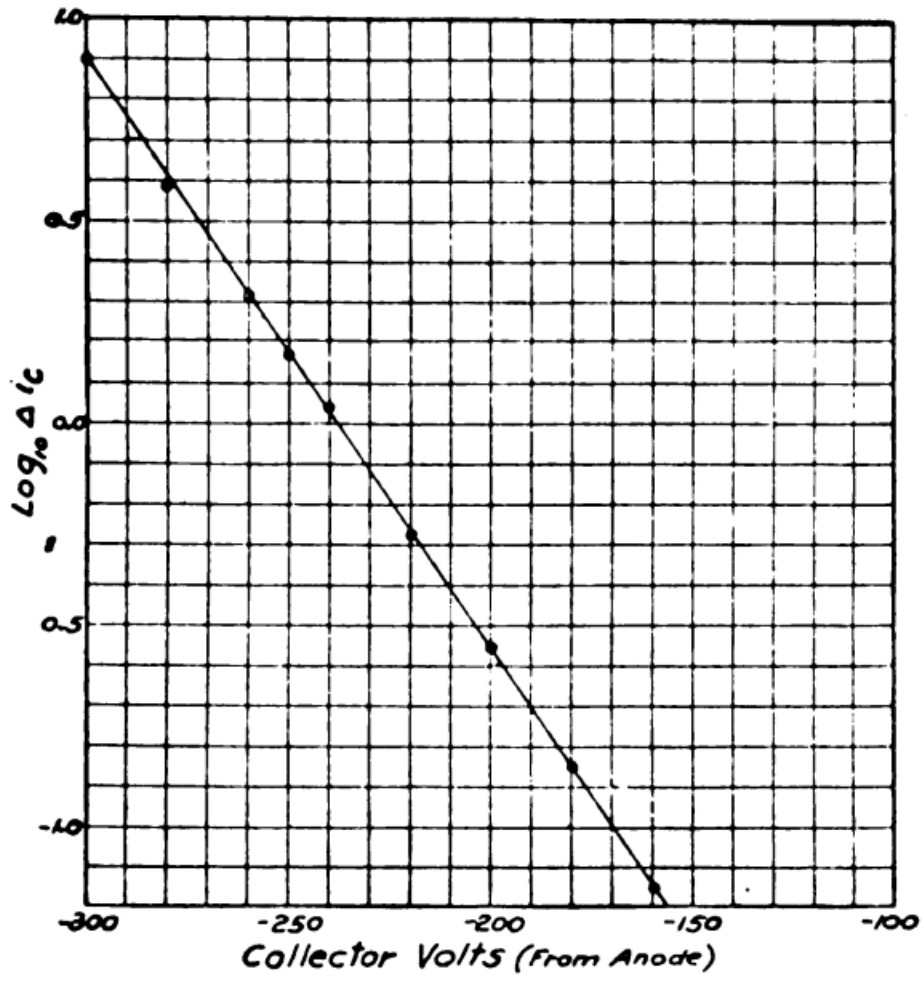

FIG. 3. Increase of negative collector current with voltage. Plotted from Fig. 1.

creasingly negative voltages, approaching a limiting value as shown in the curve of Fig. 2, which is for a tungsten disk in the positive column of a discharge of 2 amps in neon at 0.7 mm pressure.

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Analysis of Current-Voltage Curve

The shape of the volt-ampere characteristic of a collecting electrode depends greatly on the intensity of the discharge and also on the pressure of neon. The electron emission from the collector is due to conditions in the plasma and the increase of current is governed by the conditions in the sheath. This is demonstrated in the curves of Fig. 4, which give the volt-ampere characteri-

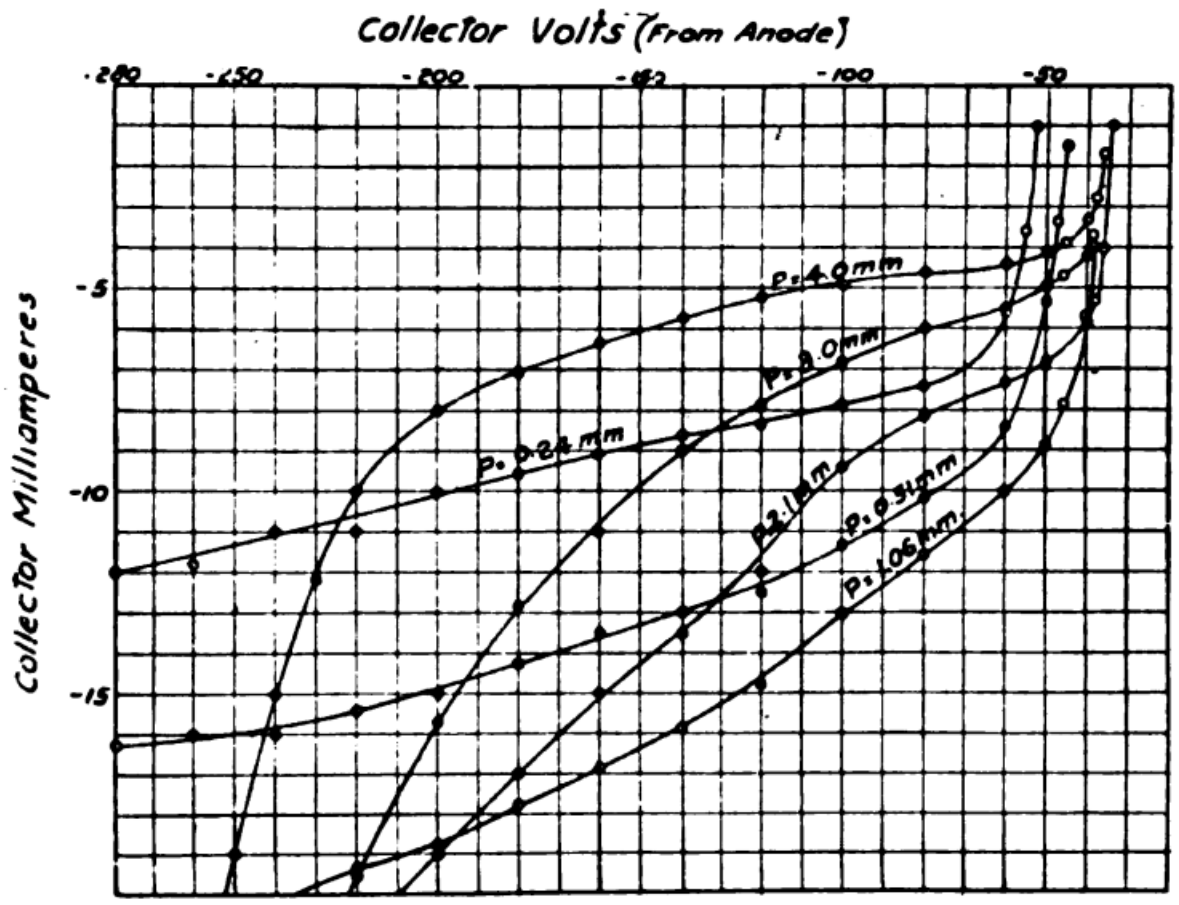

FIG. 4. Collector characteristics as function of pressure. Neon tube No. 19,800. Discharge current 2.0 amps. Collector is tungsten disk in positive column.

stics of a plane collector of 1.2 cm diameter at the center of a 5 cm diameter tube in which is a neon discharge of 2 amperes. Although at low voltages the collector current increases with pressure up to about 1 mm, and then decreases again at higher pressures, the rate of increase of current at higher voltages continues to increase as the pressure is increased. At low pressures the m.f.p. of the electrons is so high that ionization by collision cannot take place within the sheath. Since at high pressures the thickness of the sheath increases due to the decrease in collector current, and the m.f.p. of the electron is less, ionization by collision within the sheath causes the collector current to increase rapidly.

The following experiment shows that the shape of the current-voltage curve at low negative voltages is determined by conditions in the plasma, and at high voltages by conditions in the sheath. The collecting electrode was placed in a side arm at a distance of about 2.5 cm from the edge of the main discharge of 2.0 amps in neon at 2 mm pressure. Curve *ABC* of Fig. 5 gives the current-voltage curve for the collector. The side arm was then covered by a cloth which was cooled by means of liquid air, and a second voltage curve *ABD* was

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obtained. The curve remained unaltered at low voltages (AB), showing that this portion of the curve was controlled by conditions in the plasma, which were not altered by cooling the region around the collecting electrode. Since a similar current increase at the higher voltages can be obtained by increasing the pressure of neon 50 per cent, as shown by the curve EFD for 3 mm pressure, we account for the rapid rise of current in this range of voltages by the increased density of gas in the region of the sheath, which permits more efficient ionization by collision and hence a faster increase in collector current with voltage.

Effect of Negative Currents on Formation of an Arc

The results just described are important in the design of full wave rectifiers in which there are two anodes and a cathode. In this type of tube, the maximum voltage at which the tube can operate is determined by the voltage at which a cold electrode discharge takes place between the two anodes.

Whether or not a cold cathode discharge will take place is determined not by the current to the negative electrode, but by the rate of increase of the current with voltage. This is illustrated in the curves of Fig. 4 which give the current as a function of negative voltage on a collector in a neon discharge.

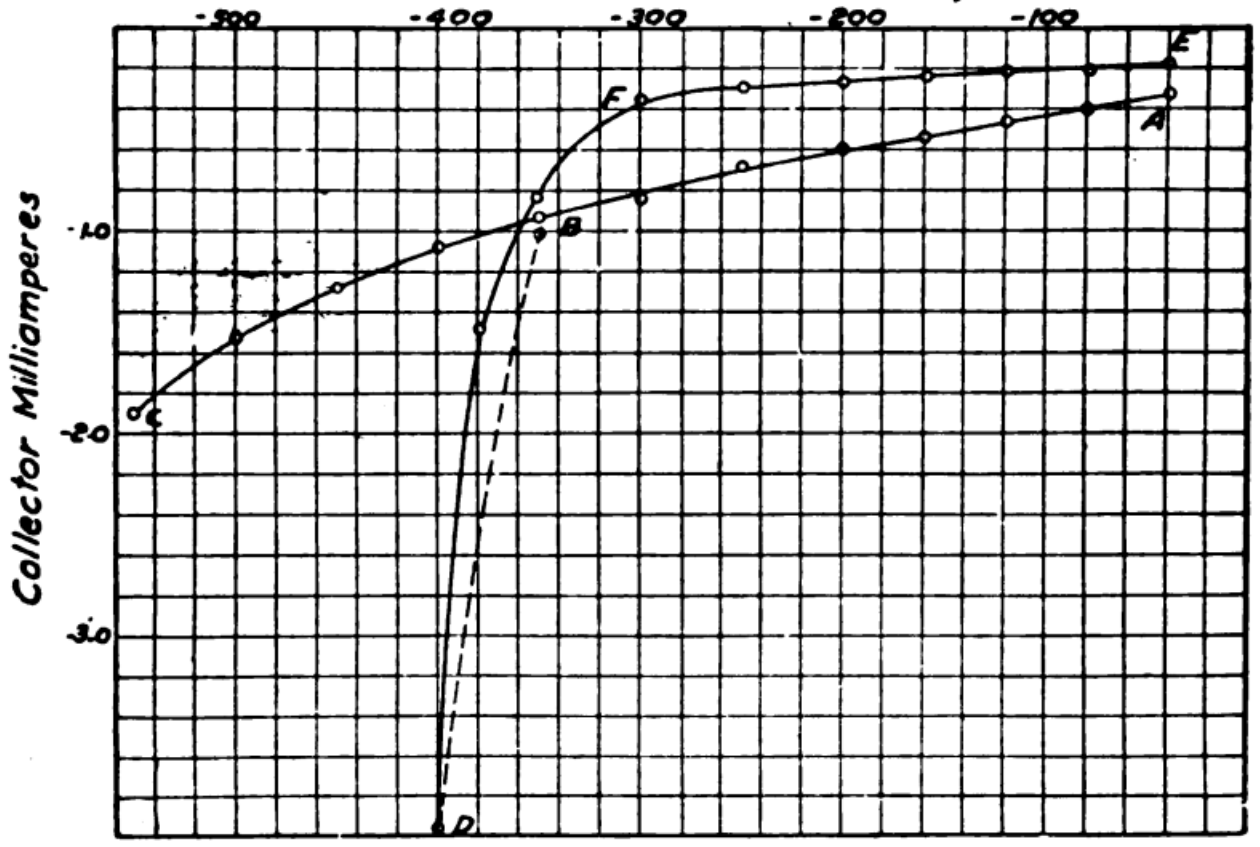

FIG. 5. Effect of pressure change on collector characteristics. Discharge current = 2.0 amps.

Although the currents at 1.06 mm pressure are many times greater than those at 0.4 mm pressure, the curve at the former pressure is concave upwards over the total voltage range, while the curve at 4.0 mm pressure is concave downward, showing a very sharp rise of current with voltage at about —200 volts, where the tendency to form a self-sustaining discharge is quite evident. When the anode is in a region of low ionization during the negative half cycle,

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the positive ion sheath about it is large permitting multiple ionizing collisions and thus favorable conditions are produced for the initiation of a discharge between the anodes, as illustrated by the curve for 4.0 mm pressure. On the other hand, when the anode is in a region of such dense ionization that the sheath thickness is of the order of an electronic m.f.p., a secondary electron can produce only one electron within the sheath, and although the positive ion current to the anode may be high during the negative half cycle, no discharge will take place between the anodes as no cascading process of ionization can occur.

Thus in a discharge tube containing a multiplicity of anodes, there is less tendency for an arc to form between the anodes when they are so placed that the electrodes, when negative, are in a region of intense ionization. However, in such position a great deal of bombardment is caused, and, consequently, a loss of electrode material by sputtering.

Studies of Secondary Electron Emission

The above experiments indicate that a negatively charged collector in the positive column of a neon discharge emits electrons, and it has been shown that these secondary electrons cause ionization by collision at relatively large negative voltages. In order to avoid the complications produced by ionization by collision, the following experiments to determine the source of secondary emission were performed with low negative voltages at which ionization by collision was negligible.

Variation of Currents with Distance of Electrode from Discharge

In order to study the variation of the currents to a negatively charged collector as a function of the concentration of ions, a tube of 2.5 cm diameter was used in which the collecting electrode could be moved along the tube from a position in the discharge to a distance of several cm beyond the anode end of the positive column. The curve in Fig. 6 is a plot of the log of the current in ma to a collector at a potential 75 volts negative to the anode, as a function of the distance between the electrode and the anode end of the positive column. As the electrode is moved back from the discharge, the collector current decreases exponentially at first, but at distances greater than 5 cm from the discharge, the current decreases less rapidly. At short distances from the discharge where the collector current decreases exponentially, the same processes are involved in the production of current as are responsible for the current when the collector is in the main discharge. For, whether the current is due to reception of positive ions, secondary electron emission by metastable or excited atoms, or photoelectrons produced by resonance radiation (of short

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free path) from the positive column, it will fall off exponentially according to the laws of diffusion. While most of the resonance radiation has a short free path, and will follow the laws of diffusion, there is, as will be pointed out in a later section, a small fraction which is not so strongly absorbed and, therefore, will decrease according to some other law. The theory of diffusion down

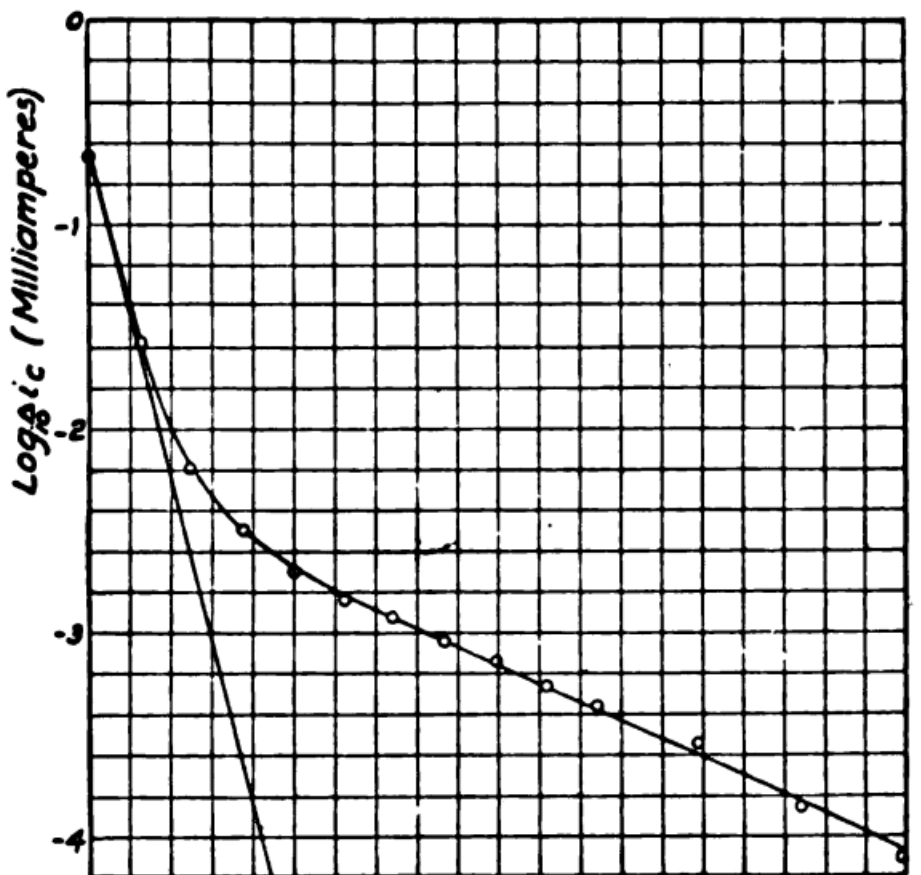

FIG. 6. Variation of negative collector current with distance from end of positive column. Discharge current = 0.05 amps. Neon pressure = 1 mm.

a long cylinder has been worked out by Schottky.[9] Assuming that all diffusing particles are destroyed when they reach the walls, the number of particles which diffuse per second across a cross-section of the tube at a distance Z from the origin as derived on the basis of Schottky's theory by Langmuir and Compton[10] is

$$i = 3.26 D n_0 a e^{-(2.405\, Z/a)}$$

where n_0 is the number of particles per cm^3 at the origin; D the diffusion coefficient; and a the radius of the tube. From the equation given above, it is seen that the exponential term alone determines the rate of decrease of intensity of the diffusing substance along the tube, and, since the attenuation constant, $2.405\, Z/a$, involves only the tube dimensions and is independent of the absolute value of the diffusion coefficient, the rate of decrease is the same for all substances which are destroyed when they reach the walls. Thus, due to diffusion, the collector current will be reduced to 1/10th value when the collector is moved away from the plasma a distance equal to the radius of the tube. The solid

[9] Schottky, *Phys. Zeits.* **25,** 635 (1924).

[10] Langmuir and Compton, *Rev. Mod. Phys.* **3,** 212 (1931).

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straight line in Fig. 6 represents such a decrease for this tube, and it is evident that the large currents at distances greater than 5 cm cannot be accounted for by diffusion from the discharge.

Currents not Caused by Diffusion

Experimental proof, that the large currents at great distances from the discharge are not due to diffusion, is supplied by the following experiment. In the tube used, the main discharge current of about 1 amp took place in neon at about 1 mm pressure between a hot cathode and anode. One collector *b* is placed in the main tube 15 cm beyond the anode end of the discharge; a second collector *d* of the same area as *b* is placed in a side tube which is attached to the main tube at a point about 5 cm from the anode end of the discharge. The position of *d* in the side tube is such that it is the same distance from the discharge as *b*. Since the mean free path of electrons, positive ions, atoms and most of the resonance radiation is small compared to the dimensions of the tube, diffusion should be the same in the side tube as in the main tube. However, with the collector 75 volts negative to the anode, the current to *d* in the side tube is less than 1/1000th of that to *b* in the main tube. Thus, the larger collector currents to *b* cannot be due to diffusion of ions, metastable atoms or resonance radiation, but are caused by something, having a long free path, and which, therefore cannot diffuse around the corner into the side tube.

That these large currents to *b* are observed only when light from the discharge can reach the neighborhood of this collector is indicated by a second experiment arranged as follows. At a distance of about 6 cm from the end of the neon positive column, the main tube, which had a diameter of 2.5 cm, was expanded into a bulb of 8 cm diameter, and beyond the bulb the tube diameter was again 2.5 cm. At a distance of 5 cm beyond the bulb, a collector of 1.9 cm diameter was placed in the axis of the tube. The side of the collector away from the discharge was backed with mica so that only the collector surface was facing the discharge. In the center of the bulb was a nickel disk of 3.75 cm diameter, which could be rotated about an axis perpendicular to the axis of the main tube. The disk was so mounted that when it was at right angles to the axis of the tube, it shadowed the collector completely from the main discharge. Since the shutter area was less than 1/4 the cross section of the bulb, it is apparent that the orientation of the shutter could have little or no effect on diffusion from the discharge to the collector. With the collector 75 volts negative to the anode, it was found that when the collector was shadowed from the discharge by the shutter (the shutter at right angles to the axis of the tube) the collector current was reduced to an extremely small value compared to that observed when the shutter was parallel to the axis and the collector could "see" the discharge. Also, as the angle of the shutter was changed, the effect varied as the direct rays from the positive column were intercepted by the

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shutter. Since positive ions, metastable atoms and electrons all have a short m.f.p., one is forced to conclude from the above observations that the secondary electron currents from a collector facing the discharge, but situated at a distance of several cm from it, are caused by some kind of radiation which originates in the discharge and which can travel long distances without change of direction.

Variation of Collector Currents with Discharge Currents

The variation of negative collector currents with change of discharge current is shown in Fig. 7, from which it is seen that when the collector is at the edge of the discharge, the current to it increases almost linearly with the discharge current. When the collector is at a distance of 7.5 cm from the plasma,

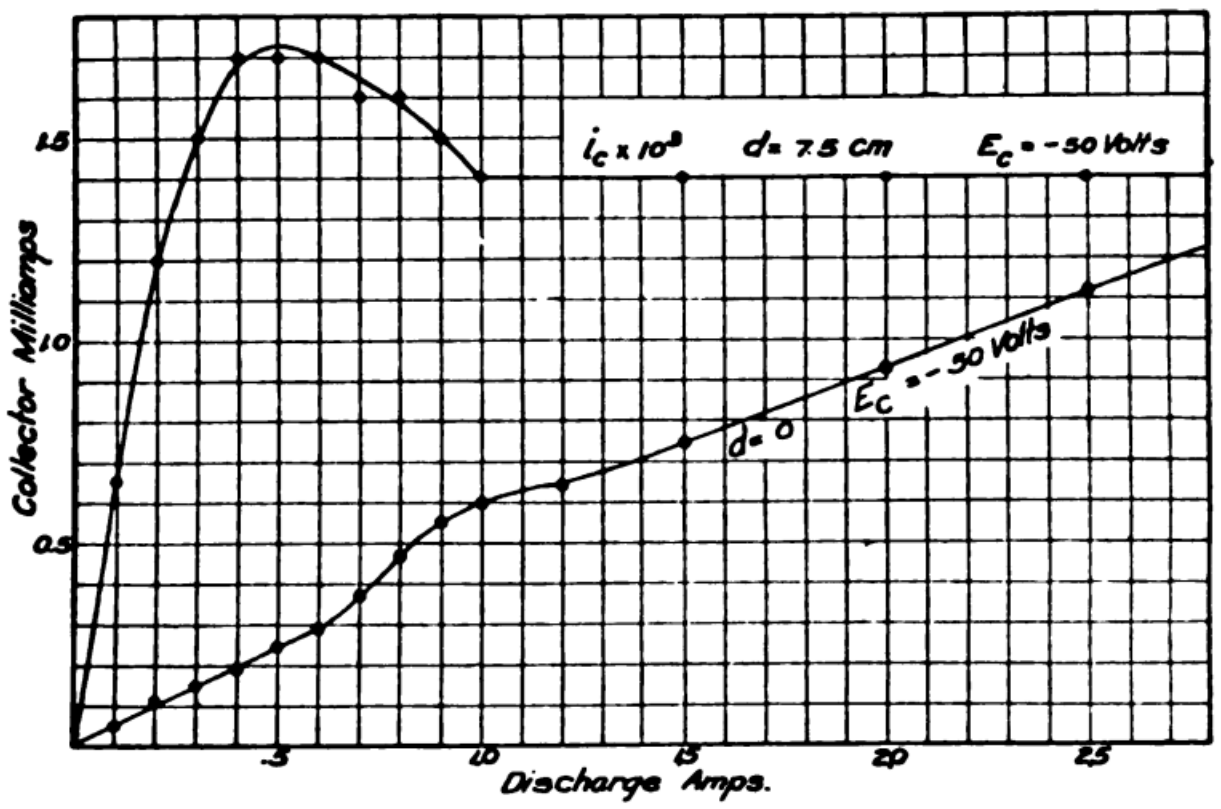

FIG. 7. Variation of negative collector current with discharge current. Neon pressure = 1 mm.

the current to it increases to a maximum value at a discharge current of 0.5 amps. With still higher discharge currents the collector current decreases, approaching a constant value when the discharge reaches 1 amp. Since the results of the previous section show that diffusion from the plasma can contribute less than 1 per cent of the current observed when the collector is at a distance of 7.5 cm, it is obvious from the curves of Fig. 7 that the negative currents to a collector in un-ionized neon several cm from the plasma do not originate from the same source as the negative currents due to diffusion of particles from the plasma. The shape of the curve of Fig. 7 for 7.5 cm is very similar to the curve of Fig. 8, which gives the variation with discharge current of the concentration of metastable atoms in the positive column of a neon discharge as measured by Kopferman and Ladenburg[11] by anomalous dispersion.

[11] Kopferman and Ladenburg, *Zeits. f. Physik* **48,** 26 (1928); *Zeits. f. physik Chem.* Haber Band 375 (1928).

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This similarity suggests that the currents at large distances beyond the plasma are due to metastable neon atoms, or that their production is governed by the same conditions which govern the production of metastable atoms in the po-

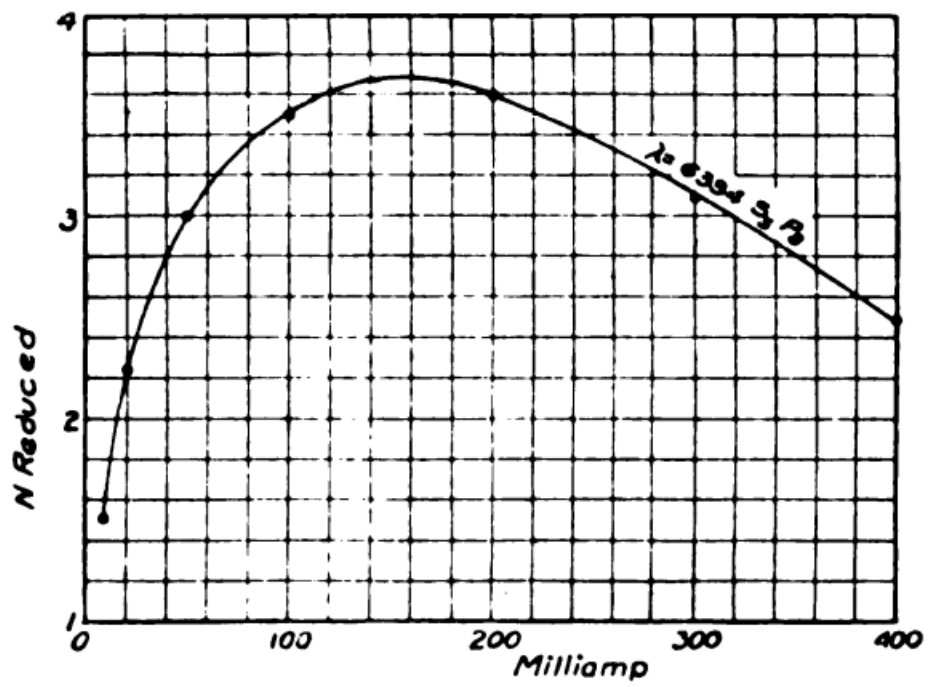

FIG. 8. Concentration of metastable atoms as function of discharge current (Kopferman and Ladenburg).

sitive column. Moreover, in the present results, as well as in those of Kopferman and Ladenburg, the position of the maximum shifts to lower discharge current densities as the gas pressure and the tube diameter are increased.

Discussion of Possible Sources of Secondary Electrons from a Collector in Un-ionized Neon

We have seen that the currents to a negatively charged collector at relatively long distances from the plasma are much greater than can be accounted for by diffusion of positive ions from the plasma. Also, these currents cannot be due to secondary emission by metastable atoms from the discharge as the number diffusing down the tube would fall off according to Schottky's diffusion law since they have a short m.f.p. and are destroyed when they come in contact with the walls. Since these collector currents appear to be so closely connected with the concentration of metastable atoms in the discharge, it may prove helpful to consider the processes responsible for the formation of these metastable atoms.

A normal neon atom is excited to resonance when an electron with a velocity corresponding to 16.6 volts collides with it. Because of the very short m.f.p., or high absorption coefficient of resonance radiation in normal gas,[12] this radia-

[12] According to classical theory (R. Ladenburg, *Ber. d. D. Phys. Gesell.* **16,** 770 (1914)) the absorption coefficient of a gas for resonance radiation is given by

$$a_0 = \frac{Ne^2}{\nu_0 m v}$$

where N is the number of atoms in the normal state, ν_0 the frequency of the resonance radiation, v the average velocity of the gas atoms, e the charge and m the mass of an electron. Substituting the numerical values for neon resonance, we find $a_0 = 1.55 \times 10^5$ or the m.f.p. for resonance radiation in neon at 1 mm pressure equals 6.45×10^{-6} cm.

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tion is absorbed and re-emitted millions of times before it reaches the walls, and as practically all of the time is spent in the excited state, a collision of the second kind is almost certain to take place which will result in a transition from the resonance to an adjacent metastable level. This is more probable since the energy involved in such a transition corresponds to only 0.05 volts. Thus, if the metastable atoms in the plasma and the collector currents at large distances from the plasma have a common origin, we must look for the origin in the resonance radiation from the main discharge. If the m.f.p. of the resonance radiation is as short as given by classical theory, it cannot travel great distances, and since it is absorbed by the walls it will have the same attenuation constant as electrons, positive ions and metastable atoms. However, although little is known of the properties of the neon resonance lines (about 740A) it is safe to assume from experiments with mercury of R. W. Wood[13] that the emission line is much broader than the absorption line. The absorption coefficient mentioned above applies only to the center of the resonance line, and the radiation from the edges of the broadened resonance line will have a much lower coefficient and will be able to travel large distances. When this broadened resonance radiation, which travels down the tube, is finally absorbed, most of it will be re-emitted as a narrow line which is strongly absorbed. Consequently, such a process consists in the transormation of radiation having a long free path to radiation which has a short free path or high absorption coefficient. Due to the fact that the re-emitted radiation has a high absorption coefficient, only that radiation re-emitted very close to the electrode will be incident on it. Also, as described above, the production of metastable atoms from the re-emitted, strongly absorbed resonance radiation, by collisions of the second kind, will be high. The metastable atoms, so formed, will strike the electrodes and glass walls of the vessel, and, as shown by Oliphant and Uyterhoeven, wilt cause a secondary electron emission from the electrode which would accounl for the large currents observed when the collector is at great distances from the positive column. The mathematical derivation of this theory, which will be presented in detail in a subsequent paper, leads to the following expression for the energy E absorbed per unit volume per second at a distance x from the source of resonance radiation.

$$E = 5.9\times 10^{-7}\, \nu_0 I_\nu \frac{r^2}{x^3}\left(\frac{T}{M \ln (a_0 x)}\right)^{1/2}$$

where a_0 is the absorption coefficient of the resonance radiation of frequency ν_0; I_ν is the density of resonance radiation per unit volume per unit frequency at end of positive column, M the molecular weight, and T the temperature of the gas, and r the radius of the tube.

Since $a_0 x$ is an extremely large number, the expression under the square root sign varies only slightly with change in x. It is seen that the energy ab-

[13] R. W. Wood, *Phil. Mag.* **44,** 1111 (1922).

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sorbed, and hence the number of metastable atoms formed, decreases inversely as the third power of the distance from the positive column. Experimental results are in good agreement with this relation.

Of course, the electron current from the collector could be attributed directly to photoelectron emission by the broadened resonance line. While such photoelectrons are undoubtedly emitted, experiments which will now be described indicate that the chief cause of the currents at large distances is the formation of metastable atoms by the resonance radiation rather than the resonance radiation itself.

Positive Currents to a Collector

When a collecting electrode, 10 cm or more from the end of the positive column, is made positive, it is found that there is a current in the direction of electrons to the electrode (positive current) of the same order of magnitude as the negative current. The fact that these positive currents vary with the discharge current in exactly the same way as the negative currents indicates that both originate from a common cause. It is very improbable that photoelectric currents of any measurable magnitude come from the glass. It seems more plausible to assume a secondary emission from the walls because of bombardment by metastable atoms which establishes a definite concentration of electrons in the space between the collector and the discharge and thereby gives it a definite conductivity.

Effect of Radiation from an External Neon Light

Penning[14] has shown that the addition of a slight trace of gas, such as argon, which has an ionization potential less than the energy of the metastable state of neon, lowers the arcing voltage of neon. This he attributes to ionization of the added gas by the metastable atoms of neon. Furthermore, he has demonstrated[15] that the arcing voltage of such a tube containing a trace of argon is increased when subjected to radiation from an external neon tube. This increase is accounted for by the destruction of neon metastable atoms by the radiation, and hence a decrease in the amount of ionization of the argon. Penning's curves show that these changes in break-down voltage occur only in the pressure range in which the break-down voltage for pure neon increases with increase of pressure, that is, in the range where the breakdown is determined by the energy which an electron gains between collisions with gas atoms. Under these conditions, the presence of positive ions causes a redistribution of the electric field between the electrodes, which increases the voltage gradient at the cathode, thus permitting an electron to gain sufficient energy between collisions to ionize, although the voltage impressed between the electrodes

[14] Penning, *Zeits. f. Physik* **45**, 335 (1928).
[15] Penning, *Zeits. f. Physik* **57**, 723 (1929).

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is lower than for break-down in pure neon. In other words, the presence of positive ions is equivalent to a reduction of distance between the electrodes. At low pressures where the break-down voltage increases with decrease of pressure, the electrons gain sufficient energy between collisions to ionize, but ionization is limited by the frequency of collision of the electrons with gas atoms. Consequently, the presence of positive ions will have practically no effect on the starting voltage, as is shown by Penning's curves at these pressures.

Since the function of the positive ions is to change the field distribution, it is evident that a slight change in the positive ion concentration will result in a comparatively large change in break-down voltage. Such an amplified action is necessary to account for the large increase in starting voltage when the tube is irradiated by outside neon light.

The strong lines in the visible radiation of neon arise from transitions between ten closely grouped levels of the neon atom and four lower levels, two of which are resonance levels and two metastable levels. If metastable neon atoms are exposed to radiation from a second neon tube, some will be raised to one of the ten higher levels, from which they cannot return directly to the normal state, but must return first to one of the four intermediate levels unless they absorb radiation of a second frequency which will raise them to a still higher level, from which it may be possible to return directly to the normal state. However, the probability of such a double absorption is extremely small and very few metastable atoms will be destroyed by this process. Most of the metastable atoms which have been raised, by absorption of radiation, to a higher level will return to a metastable level but some will go to a resonance level, from which they will return to normal with emission of resonance radiation. This does not indicate that atoms returning to normal in this manner cause an appreciable decrease in the concentration of metastable atoms, for, as pointed out above, the resonance radiation is strongly absorbed and by collisions of the second kind, metastable atoms are again formed. From these considerations it is quite evident that radiation from a second neon light cannot ordinarily be very effective in destroying metastable atoms.

In view of Penning's results, volt-ampere characteristics of a collector were taken when radiation from an external neon light fell on the un-ionized neon between the positive column and a collector placed 25 cm from the discharge. Such characteristics are given in Fig. 9 and show that although the collector currents at low voltages are only slightly decreased by the irradiation, at higher negative voltages, where the electric field is important, the decrease in current with radiation is much larger.

Moreover, not only the negative currents, but also the positive currents are decreased, proving that both are due to, or at least influenced by, the concentration of metastable neon atoms. By confining the external radiation to small regions of the space between the collector and the discharge, it was shown

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that the decrease in collector current took place when the irradiated region was close to the collector. Thus, the collector currents are caused chiefly by metastable atoms, formed close to collector, which reach it through diffusion.

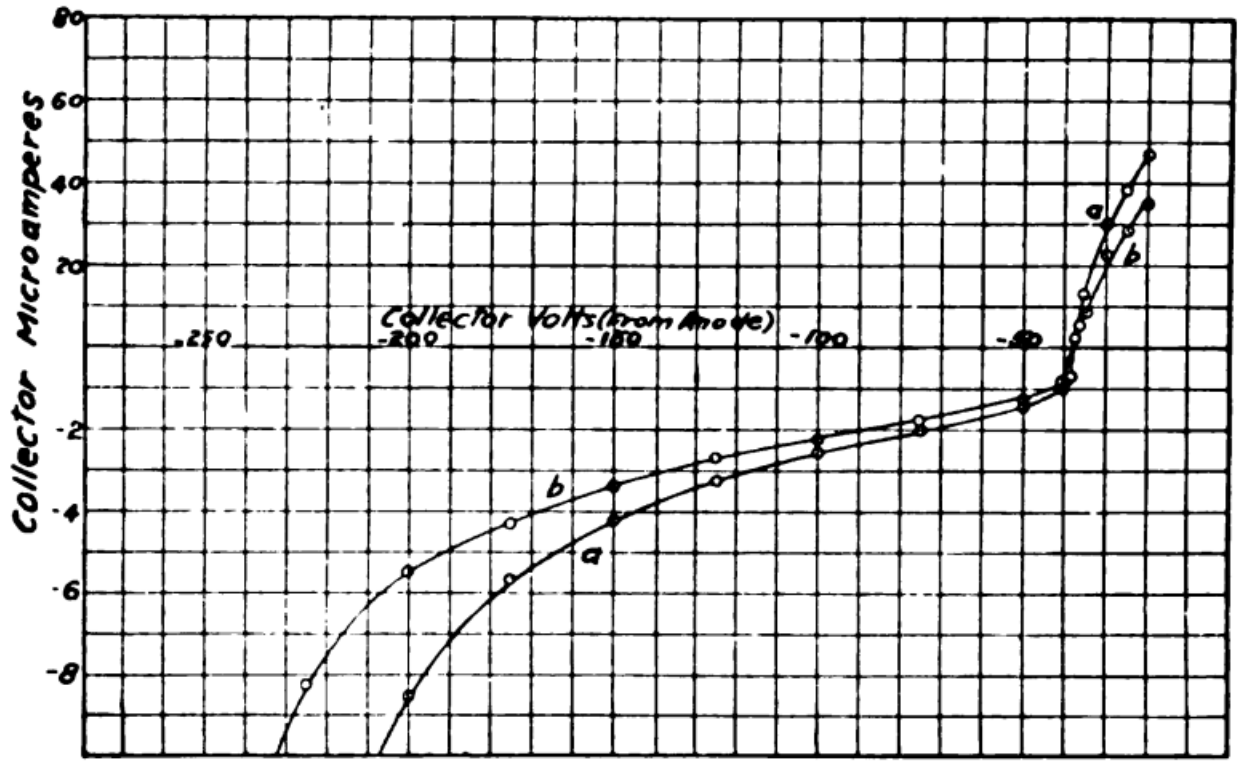

FIG. 9. Effect of radiation from external neon lamp operating at 3 amp on collector currents. Tube No. 20,091, diameter = 5 cm. Discharge current = 0.5 amp. Neon pressure = 1.75 mm. Collector 25 cm from end of positive column (a) without and (b) with external radiation.

No decrease in collector current was observed when the source of external radiation was other than a neon tube. Irradiation by helium, mercury and argon discharges, as well as by an incandescent lamp caused no change in collector current.

Effect of Orientation of Collector

The conclusions reached above as to the origin of the collector currents are confirmed by the results of measurements in which the collectors were two disks of exactly the same size, placed with their planes perpendicular to the axis of the tube in a 2.5 cm diameter discharge tube at a distance of 10 cm from the positive column. One of the disks *b* was covered with mica on the side facing the source of resonance radiation so that the exposed surface could receive no direct radiation. The second disk *d* was covered with mica on the opposite side. From the curves of Fig. 10 it is seen that although *b* could receive no direct radiation from the discharge, its currents were of the same order of magnitude as those to *d*. If radiation is the cause of secondary electron currents, one would expect the current from *d*, which is exposed to all the radiations reaching that cross-section of the tube to be much greater than the currents from *b* which is exposed to only that radiation absorbed, and subsequently re-emitted in the region immediately back of the collectors. On the other hand, on the assumption that the secondary emission is due to metastable atoms formed by the resonance

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radiation, the currents from both electrodes would be proportional, not to the resonance radiation reaching them, but to the resonance radiation absorbed close to them, which would not differ greatly for the two cases.

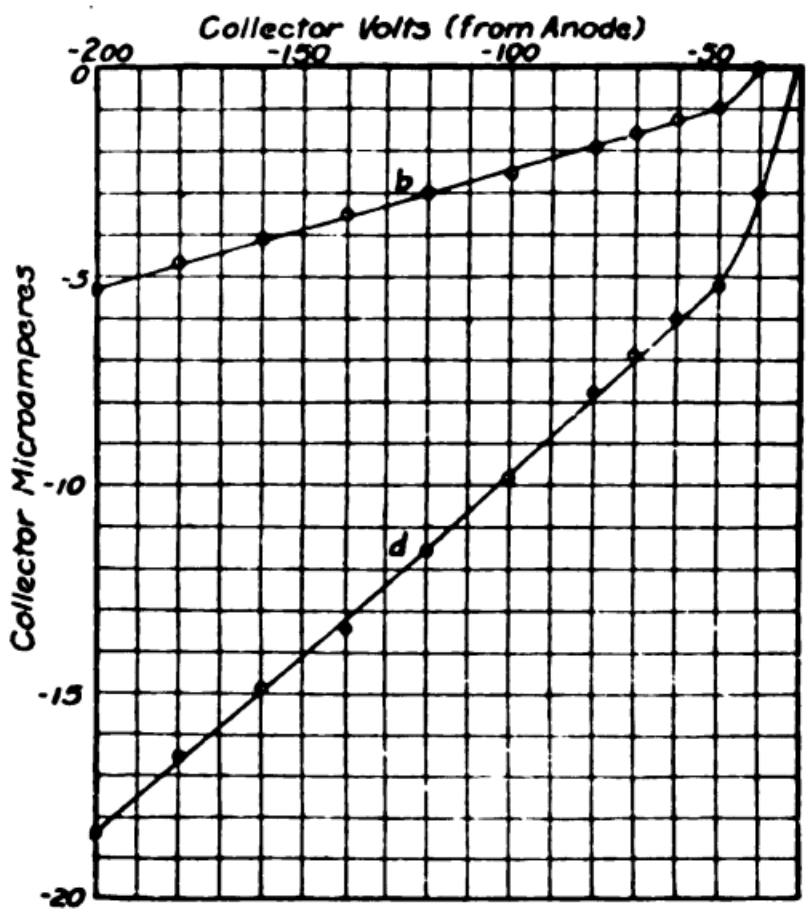

FIG. 10. Collector currents to electrode facing toward and away from discharge. Tube No. 23,459, diameter 5 cm. Discharge current = 1.0 amps. Neon pressure = 0.37 mm. Collectors 10 cm from end of positive column. *d* faces discharge, *b* away from discharge.

Preparation of Tubes and Purity of Neon

The tubes used were baked out at high temperature and all metal parts were thoroughly degassed both by high frequency and discharge bombardment. The tubes were also operated for long periods of time with a discharge at low pressure. This was found a very effective method of freeing the glass walls and metal parts of gases which might be left after the heat treatment. Moreover, the high amperage discharges, which were used, clean up impurities very rapidly. After the above treatment, neon was admitted into the tube. Before being admitted to the tube, the neon was purified by means of a magnesium arc and after admission was purified by running a discharge at high current for several hours.

C. Kenty[16] has attributed results with similar apparatus to a photoelectric effect of the resonance radiation and ionization of traces of impurities by metastable neon atoms. However, we believe that the results reported in this paper are characteristic of pure neon. We are led to this belief by the change in volt-ampere characteristics of a collector in un-ionized neon, when a minute amount (0.10 per cent) of an impurity, such as argon, is added. When the

[16] C. Kenty, *Phys. Rev.* **38**, 377–8 (1931).

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collector is negative, the addition of argon causes very little change either in the shape of the curve or the order of magnitude of the currents. However, in the range of voltage where the current passes from negative to positive, the transition is very much steeper for the admixture and on the positive side the rate of current increase is from ten to one hundred times that for pure neon. If the original characteristics are due to impurities, an increase of impurity might be expected to change the magnitude of the effect, but the change in form of the volt-ampere characteristics indicates that the addition of argon brings in a new phenomenon. Moreover, when the admixed argon is absorbed in charcoal immersed in liquid air, the volt-ampere characteristic obtained is the same as before the argon was admitted. A more complete account of the effect of addition of gases will be reserved for a future publication.

Summary

From the experimental evidence described, we have been led to the conclusion that the resonance radiation from a neon arc can travel distances as great as 20 to 30 cm through un-ionized neon. The absorption of this radiation by the normal neon atoms results in the formation of excited neon atoms, which, by subsequent collisions of the second kind, arrive in the metastable state. The metastable atoms thus produced diffuse to the walls and metal electrodes and there liberate secondary electrons from both types of surfaces.

The writers are pleased to have this opportunity to express their appreciation for the assistance of Dr. P. Lowe in making many of the measurements.

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APPENDIX I

IRVING LANGMUIR — CURRICULUM VITAE

Born

January 31, 1881

Parents

Charles and Sadie Comings Langmuir

Attended

Public Schools, Brooklyn	1887—1892
Boarding School, Suburban Paris	1892—1895
Chestnut Hill Academy, Philadelphia	1895—1896

Graduated

Manual Training High School, Pratt Institute	1898

Received degree

Metallurgical Engineer, Columbia University	1903
Doctor of Philosophy, University of Göttingen	1906

Taught

Stevens Institute of Technology	1906—1909

Married

Marian Mersereau	1912

Children

Son, Kenneth
Daughter, Mrs. H. R. Summerhayes

Research Scientist

General Electric Research Laboratory	1909—1950
Associate Director	1929—1950
Consultant	1950—1957

Honorary Degrees Awarded

D.Sc. Northwestern University	1921
LL.D. Edinburgh University	1921
D.Sc. Union University	1923
D.Sc. Columbia University	1925
D.Sc. Kenyon College	1927
D.Sc. Princeton University	1929
D.Ing. Tech. Hochschule, Berlin	1929
D.Sc. Lehigh University	1934

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LL.D.	Union University	1934
LL.D.	Johns Hopkins University	1936
D.Sc.	Harvard University	1938
D.Sc.	Oxford University	1938
D.Sc.	Rutgers University	1941
D.Sc.	Queen's College (Canada)	1941
LL.D.	University of California	1946

Medals Awarded

Nichols Medal	American Chemical Society	1915
Hughes Medal	Royal Society of London	1918
Rumford Medal	American Association for the Advancement of Science	1920
Nichols Medal	American Chemical Society	1920
Stanislao Cannizzaro Award	Royal National Academy of Linnaeus (Rome)	1925
Perkins Medal	Society Chemical Industry (American Section)	1928
Chandler Medal	Columbia University	1929
Willard Gibbs Medal	American Chemical Society	1930
NOBEL PRIZE	Swedish Academy	1932
Popular Science	Popular Science Monthly	1933
Franklin Medal	Franklin Institute	1934
Holley Medal	American Society of Mechanical Engineers	1934
John Scott Medal	Philadelphia Board of Directors of City Trusts	1937
Faraday Medal	Chemical Society (London)	1938
Egleston Medal	Columbia University Engineering Schools Alumni Association	1939
Modern Pioneer of Industry	National Association of Manufacturers	1940
Silver Beaver	Boy Scouts of America	1942
Faraday Medal	Institution of Electrical Engineers	1943
Medal of Merit	United States Army and Navy	1948
Mascart Medal	Academy des Sciences, Societé Française des Electriciens	1948
John J. Carty Medal	National Academy of Sciences	1949
Silver Buffalo	Boy Scouts of America	1950

Died

August 16, 1957

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APPENDIX II

COMPLETE LIST OF PAPERS OF DR. IRVING LANGMUIR AS THEY APPEAR IN THE LANGMUIR MEMORIAL VOLUMES*

VOLUME 1

Low-Pressure Phenomena

Mechanism of Chemical Reactions

1. The Partial Recombination of Dissociated Gases During Cooling. Inaugural dissertation for Doctor's Degree, Göttingen. (1906).
2. The Dissociation of Water Vapor and Carbon Dioxide at High Temperatures. JACS, **28**, 1357 (1906).
3. The Velocity of Reactions in Gases Moving Through Heated Vessels and the Effect of Convection and Diffusion. JACS, **30**, 1742 (1908).
8. A Chemically Active Modification of Hydrogen. JACS, **34**, 1310 (1912).
7. The Dissociation of Hydrogen into Atoms. JACS, **34**, 860 (1912).
18. Note on the Heat of Formation of Hydrogen from Hydrogen Atoms. Phil. Mag., **27**, 188 (1914).
22. The Dissociation of Hydrogen into Atoms. (With G. M. J. Mackay). JACS, **36**, 1708 (1914).
25. The Dissociation of Hydrogen into Atoms. II. Calcu lation of the Degre of Dissociation and the Heat of Formation. JACS, **37**, 417 (1915); Zeit. f. Electrochemie, **23**, 217 (1917).
37. Dissociation of Hydrogen into Atoms. III. JACS, **38**, 1145 (1916).
9. Chemical Reactions at Very Low Pressures. I. The Clean-up of Oxygen in a Tungsten Lamp. JACS, **35**, 105 (1913).
13. Chemical Reactions at Very Low Pressures. II. The Chemical Clean-up of Nitrogen in a Tungsten Lamp. JACS, **35**, 931 (1913).
27. Chemical Reactions at Low Pressures. JACS, **37**, 1139 (1915).
52. Chemical Reactions at Low Pressures. IV. The Clean-up of Nitrogen by a Heated Molybdenum Filament. JACS, **41**, 167 (1919).
66. Radiation as a Factor in Chemical Action. JACS, **42**, 2190 (1920).

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* **Editor's Note: The numbers represent the order in which the papers were published.**

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72. Chemical Reaction on Surfaces. G. E. Rev., **25**, 445 (1922); Trans. Faraday Soc., **17**, 607 (1921).
73. The Mechanism of the Catalytic Action of Platinum in the Reactions $2CO + O_2 = 2CO_2$ and $2H_2 + O_2 = 2HO$. Trans. Faraday Soc., **17**, 621 (1921).

VOLUME 2

Heat Transfer — Incandescent Tungsten

Part 1 — *Transfer of Heat*

4. Thermal Conduction and Convection in Gases at Extremely High Temperatures. Trans. Amer. Electrochem. Soc., **20**, 225 (1911).
5. Convection and Conduction of Heat in Gases. Phys. Rev., **34**, 401 (1912).
6. Convection and Conduction of Heat in Gases. Proc. AIEE, **31**, 1011 (1912).
10. Laws of Heat Transmission in Electrical Machinery. Proc. AIEE, **32**, 391 (1913).
11. Convection and Radiation of Heat. Trans. Amer. Electrochem. Soc., **23**, 299 (1913).
12. Flow of Heat Through Furnace Walls: the Shape Factor. (With E. Q. Adams and G. S. Meikle). Trans. Amer. Electrochem. Soc., **24**, 53 (1913).
103. Flames of Atomic Hydrogen. Science, **62**, 463 (1925).
104. Flames of Atomic Hydrogen. G. E. Rev., **29**, 153 (1926).
105. Atomic Hydrogen Arc Welding. (With R. A. Weinman). G. E. Rev., **29**, 160 (1926).
110. Flames of Atomic Hydrogen. Ind. and Engrg. Chem., **19**, 667 (1927).

VOLUME 2

Heat Transfer — Incandescent Tungsten

Part 2 — *Incandescent Lamps and Tungsten*

14. Tungsten Lamps of High-efficiency. I. Blackening of Tungsten Lamps and Methods of Preventing it. II. Nitrogen-filled Lamps. (With J. A. Orange). Proc. AIEE, **32**, 1894 (1913); Trans. AIEE, **32**, 1913 (1913); G. E. Rev., **16**, 956 (1913).
20. The Flicker of Incandescent Lamps on Alternating Current Circuits and Stroboscopic Effects. G. E. Rev., **17**, 294 (1914).
26. The Melting Point of Tungsten. Phys. Rev., **6**, 138 (1915).
34. The Characteristics of Tungsten Filaments as Functions of Temperature. Phys. Rev., **7**, 302 (1916).
111. The Characteristics of Tungsten Filaments as Functions of Temperature. (With H. A. Jones). G. E. Rev., **30**, 408; 310; 354 (1927).

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126. Effect of End Losses on the Characteristics of Filaments of Tungsten and other Materials. (With S. MacLane and K. B. Blodgett). Phys. Rev., **35**, 478 (1930).
151. The Design of Tungsten Springs to Hold Tungsten Filaments Taut. (With K. B. Blodgett). Rev. Sci. Instru., **5**, 321 (1934).
156. Radiation and Absorption of Energy by Tungsten Filaments at Low Temperatures. (With J. B. Taylor), Jour. Opt. Soc. Amer., **25**, 321 (1935).
159. The Heat Conductivity of Tungsten and the Cooling Effects of Leads upon Filaments at Low Temperatures. (With J. B. Taylor). Phys. Rev., **50**, 68 (1936).

VOLUME 3

Thermionic Phenomena

Part 1 — *Thermionics — Vacuum Pumps*

17. The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum. Phys. Rev., **2**, 450 (1913).
28. The Pure Electron Discharge and its Applications in Radio Telegraphy and Telephony. G. E. Rev., **18**, 327 (1915); Trans. AIRE, **3**, 261 (1915).
65. Fundamental Phenomena in Electron Tubes Having Tungsten Cathodes. (Part I and Part II). G. E. Rev., **23**, 503, 589 (1920).
79. Use of High-power Vacuum Tubes. Electrical World, **80**, 881 (1922).
81. The Effect of Space Charge and Initial Velocities on the Potential Distribution and Thermionic Current between Parallel Plane Electrodes. Phys. Rev., **21**, 419 (1923).
85. A New Photo-electric Effect Reflection of Electrons Induced by Light. Science, **58**, 398 (1923).
91. Currents Limited by Space Charge between Coaxial Cylinders. (With K. B. Blodgett). Phys. Rev., **22**, 347 (1923).
93. Currents Limited by Space Charge between Concentric Spheres. (With K. B. Blodgett). Phys. Rev. **23**, 49 (1924).
133. Diffusion of Electrons back to an Emitting Electrode in a Gas. Phys. Rev., **38**, 1656 (1931).
33. A New Vacuum Gage of Extreme Sensitiveness. (Abstract). Phys. Rev., **1**, 337 (1913).
38. A High Vacuum Mercury Vapor Pump of Extreme Speed. Phys. Rev., **8**, 48 (1916).
44. The Condensation Pump: an Improved Form of High Vacuum Pump. G. E. Rev., **19**, 1060 (1916); Jour. Franklin Inst., **182**, 719 (1916).

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VOLUME 3

THERMIONIC PHENOMENA

Part 2 — Electron Emission and Adsorbed Films

36. The Relation Between Contact Potentials and Electrochemical Action. Trans. Amer. Electrochem. Soc., **29**, 125 (1916).
80. Thermionic Effects Caused by Alkali Vapors in Vacuum Tubes. (With K. H. Kingdon). Science, **57**, 58 (1923).
83. Removal of Thorium by Positive Ion Bombardment. (With K. H. Kingdon). Phys. Rev., **22**, 148 (1923).
87. Electron Emission From Caesium-covered Filaments. (With K. H. Kingdon). Phys. Rev., **23**, 112 (1923).
90. The Electron Emission From Thoriated Tungsten Filaments. Phys. Rev., **22**, 357 (1923).
100. Thermionic Effects Caused by Vapors of Alkali Metals. (With K. H Kingdon). Proc. Roy. Soc., A, **107**, 61 (1925).
109. On the Surface Heat of Charging. (With L. Tonks). Phys. Rev., **29**, 524 (1927).
121. Contact Potential Measurements With Adsorbed Films. (With K. H. Kingdon). Phys. Rev., **34**, 129 (1929).
127. Electrochemical Interactions of Tungsten, Thorium, Caesium, and Oxygen. Ind. and Engrg. Chem., **22**, 390 (1930).
129. Oxygen Films on Tungsten. I. A Study of Stability by Means of Electron Emission in Presence of Cesium Vapor. (With D. S. Villars). JACS, **53**, 486 (1931).
130. The Alleged Production of Adsorbed Films on Tungsten by Active Nitrogen. Phys. Rev., **37**, 1006, (1931).
137. Cesium Films on Tungsten. JACS, **54**, 1252 (1932).
139. The Mobility of Caesium Atoms Adsorbed on Tungsten. (With J. B. Taylor). Phys. Rev., **40**, 463 (1932).
144. The Nature of Adsorbed Films of Caesium on Tungsten. I. The Space Charge Sheath and the Image Force. Phys. Rev., **43**, 224 (1933).
145. The Evaporation of Atoms, Ions, and Electrons from Caesium Films on Tungsten. (With J. B. Taylor). Phys. Rev., **44**, 423 (1933).
148. Thoriated Tungsten Filaments. Jour. Franklin Inst., **217**, 543 (1934).
164. Vapor Pressure of Caesium by the Positive Ion Method. (With J. B. Taylor). Phys. Rev., **51**, 753 (1937).

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VOLUME 4

Electrical Discharge

Fundamental Phenomena in Electrical Discharges

82. Positive Ion Currents from the Positive Column of Mercury Arcs. Science, **58**, 290 (1923).
84. Positive Ion Currents from the Positive Column of the Mercury Arc. G. E. Rev., **26**, 731 (1923).
92. A Simple Method for Quantitative Studies of Ionization Phenomena in Gases. Science, **59**, 380 (1924).
97. A New Type of Electric Discharge: the Streamer Discharge. Science, **60**, 392 (1924).

94., 95., 96., 98., 99. Studies of Electric Discharges in Gases at Low Pressures. (With H. Mott-Smith). G. E. Rev., **27**, 449; 538; 616; 762; (1924).

108. The Theory of Collectors in Gaseous Discharges. (With H. Mott-Smith). Phys. Rev., **28**, 727 (1926).
114a. Electric Discharges in Gases at Low Pressures. Estratto Dagli Atti del Congresso Internazionals del Fisici Como (Sept. 1927). English translation.
119. Control of an Arc Discharge by Means of a Grid. (With A. W. Hull). Proc. NAS, **51**, 218 (1929).
141a. Electric Discharges in Gases at Low Pressures. Jour. Franklin Inst., **214**, 275 (1932).
154. Electric Discharges in Vacuum and in Gases at Low Pressures. The Denki-Gakkwai, Iwadare Foundation, Lecture III, Japan (1934).
128. Electrical Discharges in Gases. I. Survey of Fundamental Processes. (With K. T. Compton). Rev. Mod. Phys., **2**, 123 (1930).
132. Electrical Discharges in Gases. II. Fundamental Phenomena in Electrical Discharges. (With K. T. Compton). Rev. Mod. Phys., **3**, 191 (1931).

VOLUME 5

Plasma and Oscillations

Ionized Gases

86. The Pressure Effect and Other Phenomena in Gaseous Discharges. Jour. Franklin Inst., **196**, 751 (1923).
101. Scattering of Electrons in Ionized Gases. Phys. Rev., **26**, 585 (1925).
107. The Flow of Ions Through a Small Orifice in a Charged Plate. (With L. Tonks and H. Mott-Smith). Phys. Rev., **28**, 104 (1926).

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116. Collisions Between Electrons and Gas Molecules. (With H. A. Jones). Phys. Rev., **31**, 357 (1928).
117. Oscillations in Ionized Gases. Proc. NAS, **14**, 627 (1928).
117a. Oscillations in an Ionized Gas. (With L. Tonks). Science, **68**, 598 (1928).
118. Oscillations in Ionized Gases. (With L. Tonks). Phys. Rev., **33**, 195 (1929).
120. The Interaction of Electron and Positive Ion Space Charges in Cathode Sheaths. Phys. Rev., **33**, 954 (1929).
122. General Theory of the Plasma of an Arc. (With L. Tonks). Phys. Rev., **34**, 876 (1929).
128a. Metastable Atoms and Electrons Produced by Resonance Radiation in Neon. (With C. G. Found). Phys. Rev., **36**, 604 (1930).
136. Study of a Neon Discharge by Use of Collectors. (With C. G. Found). Phys. Rev., 39, 237 (1932).

VOLUME 6

Structure of Matter

Part 1 — *Atomic Structure*

59. The Structure of Atoms and the Octet Theory of Valence. Proc. NAS, **5**, 252 (1919).
56. The Arrangement of Electrons in Atoms and Molecules. G. E. Rev., **22**, 505, 587, 789 (1919); JACS, **41**, 868 (1919).
60. The Octet Theory of Valence and its Applications With Special Reference to Organic Nitrogen Compounds. JACS, **42**, 274 (1920).
61. The Structure of Atoms and its Bearing on Chemical Valence. Jour. Ind. and Engrg. Chem., **12**, 386 (1920).
62. The Charge on the Electron and the Value of Planck's Constant **h**. Jour. Franklin Inst., **189**, 603 (1920).
63. Theories of Atomic Structure. Nature, **105**, 261 (1920).
64. The Structure of the Helium Atom. Science, **51**, 604 (1920).
68. The Structure of the Helium Atom. Phys. Rev., **17**, 339 (1921).
69. The Structure of the Static Atom. Science, **53**, 290 (1921).
71. Types of Valence. Science, **54**, 59 (1921).

VOLUME 6

Structure of Matter

Part 2 — *Molecules and Crystalline Structure*

58. Isomorphism, Isosterism, and Covalence. JACS, **41**, 1543 (1919).
64a. The Structures of the Hydrogen Molecule and the Hydrogen Ion. Science, **52**, 433 (1920).

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76. The Diffusion Coefficient in Solids and its Temperature Coefficient. (With S. Dushman). Phys. Rev., **20**, 113 (1922).
102. The Distribution and Orientation of Molecules. Colloid Symposium Monograph, **3**, 48 (1925).
106. The Effects of Molecular Dissymmetry on Properties of Matter. Colloid Chemistry, **1**, 525 (1926).
125. Forces Near the Surfaces of Molecules. Chem. Rev., **6**, 451 (1929).
67. The Crystal Structure of the Ammonium Halides Above and Below the Transition Temperatures. (With Guy Barlett). JACS, **43**, 84 (1921).
184. Vector Maps and Crystal Analysis. (With D. M. Wrinch). Nature, **142**, 581 (1938).
186. The Role of Attractive and Repulsive Forces in the Formation of Tactoids, Thixotropic Gels, Protein Crystals, and Coacervates. Jour. Chem. Phys., **6**, 873 (1938).
134. Regions of Reversed Magnetization in Strained Wires. (With K. J. Sixtus) Phys. Rev., **38**, 2072 (1931).

VOLUME 7

Protein Structures

Molecular Structure

163. Built-up Films of Proteins and Their Properties. (With V. J. Schaefer and D. M. Wrinch). Science, **85**, 76 (1937).
168. Multilayers of Sterols and the Adsorption of Digitonin by Deposited Monolayers. (With V. J. Schaefer and H. Sobotka). JACS, **59**, 1751 (1937).
169. Improved Methods of Conditioning Surfaces for Adsorption. (With V. J. Schaefer). JACS, **59**, 1762 (1937).
170. Monolayers and Multilayers of Chlorophyll. (With V. J. Schaefer). JACS. **59**, 2075 (1937).
177. The Adsorption of Proteins at Oil-water Interfaces and Artificial Protein-lipoid Membranes. (With D. F. Waugh). Jour. Gen. Physio., **21**, 745 (1938).
178. Activities of Urease and Pepsin Monolayers. (With V. J. Schaefer). JACS, **60**, 1351 (1938).
183. Salted-out Protein Films. (With V. J. Schaefer). JACS, **60**, 2803 (1938).
182a. The Structure of the Insulin Molecule. (With D. M. Wrinch). JACS, **60**, 2247 (1938).
188. Nature of the Cyclol Bond. (With D. M. Wrinch). Nature, **143**, 49 (1939).
189. The Properties and Structure of Protein Films. Proc. Roy. Inst., **30**, 483 (1938).

Digitized by Google Original from UNIVERSITY OF CALIFORNIA

191. Properties and Structure of Protein Monolayers. (With V. J. Schaefer). Chem. Rev., **24**, 181 (1939).
192. The Structure of Proteins. Proc. Phys. Soc., London, **51**, 592 (1939).
193. Letter to the Editor on Structure of Proteins. Nature, **143**, 280 (1939).
194. A Note on the Structure of Insulin. (With D. M. Wrinch). Proc. Phys. Soc., London, **51**, 613 (1939).
187. Protein Monolayers. Cold Spring Harbor Symposia on Quantitative Biology, **6**, 171 (1938).
196. Pressure-soluble and Pressure-displaceable Components of Monolayers of Native and Denatured Proteins. (With D. F. Waugh). JACS, **62**, 2771 (1940).

VOLUME 8

Properties of Matter

Part 1 — *Fundamental Properties of Solids and Liquids*

42. The Constitution and Fundamental Properties of Solids and Liquids. JACS, **38**, 2221 (1916).
49. The Constitution and Fundamental Properties of Solids and Liquids. II. Liquids. JACS, **39**, 1848 (1917).

VOLUME 8

Properties of Matter

Part 2 — *Interfacial Phenomena*

43. The Constitution of Liquids with Especial Reference to Surface Tension Phenomena. Met. and Chem. Engrg., **15**, 468 (1916).
48. The Shapes of Group Molecules Forming the Surfaces of Liquids. Proc. NAS, **3**, 251 (1917).
54. The Mechanism of the Surface Phenomena of Flotation. Trans. Faraday Soc., **15**, 62 (1920); G. E. Rev., **24**, 1025 (1921).
146. Surface Chemistry. Nobel Lecture Presented in Stockholm on December 14, 1932. Kungl. Boktryckeriet. P. A. Norstedt and Söner, Stockholm (1933).
153. Surface Chemistry. The Denki-Gakkwai, Iwadare Foundation, Lecture II, Japan (1934).
162. Two-dimensional Gases, Liquids and Solids. Science, **84**, 379 (1936).
176. Surface Electrification due to the Recession of Aqueous Solution from Hydrophobic Surfaces. JACS, **60**, 1190 (1938).
185. Repulsive Forces Between Charged Surfaces in Water and the Cause of the Jones-Ray Effect. Science, **88**, 430 (1938).

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VOLUME 9

SURFACE PHENOMENA

Part 1 — *Evaporation, Condensation, Adsorption*

16. The Vapor Pressure of Metallic Tungsten. Phys. Rev., **2**, 329 (1913).
23. The Vapor Pressure of the Metals Platinum and Molybdenum. (With G. M. J. Mackay). Phys. Rev., **4**, 377 (1914).
112. The Rates of Evaporation and the Vapor Pressures of Tungsten, Molybdenum, Platinum, Nickel, Iron, Copper, and Silver. (With G. M. J. Mackay and H. A. Jones). Phys. Rev., **30**, 201 (1927).
41. The Evaporation, Condensation, and Reflection of Molecules and the Mechanism of Adsorption. Phys. Rev., **8**, 149 (1916).
47. The Condensation and Evaporation of Gas Molecules. Proc. NAS, **3**, 141 (1917).
50. The Adsorption of Gases on Plane Surfaces of Glass, Mica, and Platinum. JACS, **40**, 1361 (1918).
138. Accommodation Coefficient of Hydrogen: a Sensitive Detector of Surface Films. (With K. B. Blodgett). Phys. Rev., **40**, 78 (1932).
140. Vapor Pressures, Evaporation, Condensation, and Adsorption, JACS, **54**, 2798 (1932).
142. A Film Which Adsorbs Atomic H and Does Not Adsorb H_2. (With K. B. Blodgett). JACS, **54**, 3781 (1932).
143. An Extension of the Phase Rule for Adsorption Under Equilibrium and Non-equilibrium Conditions. Jour. Chem. Phys., **1**, 3 (1933).

VOLUME 9

SURFACE PHENOMENA

Part 2 — *Monomolecular Films*

131. Experiments With Oil on Water. Jour. Chem. Education, **8**, 850 (1931).
147. Oil Lenses on Water and the Nature of Monomolecular Expanded Films. Jour Chem. Phys., **1**, 756 (1933).
150. Mechanical Properties of Monomolecular Films. Jour. Franklin Inst., **218**, 143 (1934). Franklin Medal Speech on May 16, 1934.
157. A New Method of Investigating Monomolecular Films. (With K. B. Blodgett). Kolloid-Zeitschrift, **73**, 257 (1935).
158. Composition of Fatty Acid Films on Water Containing Calcium or Barium Salts. (With V. J. Schaefer). JACS, **58**, 284 (1936).
165. Built-up Films of Barium Stearate and Their Optical Properties. (With K. B. Blodgett). Phys. Rev., **51**, 964 (1937).

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167. Optical Measurement of the Thickness of a Film Absorbed from a Solution. (With V. J. Schaefer). JACS, **59**, 1406 (1937).
179a. Overturning and Anchoring of Monolayers. Science, **87**, 493 (1938).
172. The Effect of Dissolved Salts on Insoluble Monolayers. (With V. J. Schaefer). JACS, **59**, 2400 (1937).
181. Effect of X-rays on Surface Potentials of Multilayers. (With F. J. Norton). JACS, **60**, 1513 (1938).
190. Molecular Layers. Pilgrim Trust Lecture. Proc. Roy. Soc., London, A, **170**, 1 (1939).
195. Monolayers on Solids. (Seventeenth Faraday Lecture). Jour. Chem. Soc., London, 511 (1940).
113. The Effect of Monomolecular Films on the Evaporation of Ether Solutions. (With D. B. Langmuir). Jour. Phys. Chem., **31**, 1719 (1927).
207. Molecular Films in Chemistry and Biology. Rutgers University Press, 27 (1942).

VOLUME 10

Atmospheric Phenomena

Part 1 — *Aviation, Weather, Cloud Seeding*

51. The Evaporation of Small Spheres. Phys. Rev., **12**, 368 (1918).
77. Radial Flow in Rotating Liquids. (With H. Mott-Smith). [Abstract]. Phys. Rev., **20**, 95 (1922).
135. A Study of Light Signals in Aviation and Navigation. (With W. F. Westendorp). Physics, **1**, 273 (1931).
160. Airplane Tracks in the Surface of Stratus Clouds. (With A. Forbes). Jour. Aeronautical Sci., **3**, 385 (1936).
173. Air Traffic Regulations as Applied to Private Aviation. The Sportsman Pilot, **18**, 8 (1937).
175. Surface Motion of Water Induced by Wind. Science, **87**, 119 (1938).
198. Rates of Evaporation of Water Through Compressed Monolayers on Water. (With V. J. Schaefer). Jour. Franklin Inst., **235**, 119 (1943).
209. The Production of Rain by a Chain Reaction in Cumulus Clouds at Temperatures Above Freezing. Jour. Met., **5**, 175 (1948).
210. The Growth of Particles in Smokes and Clouds and the Production of Snow from Supercooled Clouds. Proc. Amer. Phil. Soc., **92**, 167 (1948).
213. Control of Precipitation from Cumulus Clouds by Various Seeding Techniques. Science, **112**, 35 (1950).
219. A Seven-day Periodicity in Weather in United States during April, 1950. Bull. Amer. Met. Soc., **31**, 386 (1950).
220. Cloud Seeding by Means of Dry Ice, Silver Iodide, and Sodium Chloride. Trans. N.Y. Acad. Sci., **14**, 40 (1951).

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VOLUME 10

Atmospheric Phenomena

Part 2 — *Smoke Filters — Cloud Droplets* (*Unpublished*)

225. Super-cooled Water Droplets in Rising Currents of Cold Saturated Air. (Report No. RL-223). Parts I and II. October 1943-August 1944. (Re-issued June 1949).
226. Mathematical Investigation of Water Droplet Trajectories. Report No. RL-224. January 1945. (Re-issued June 1949).
227. Mathematical Investigation of Water Droplet Trajectories. Report No. RL-225, December .1944-July 1945. (Re-issued June 1949).
228. Report on Smokes and Filters.

VOLUME 11

Cloud Nucleation

Part 1 — *Nucleation of Clouds with Dry Ice* (*Unpublished*)

211. Summary of Results Thus Far Obtained in Artificial Nucleation of Clouds. Project Cirrus Report on Meteorological Research (July, 1947).
212. Studies of the Effects Produced by Dry Ice Seeding of Stratus Clouds. Project Cirrus Final Report (December 31, 1948).
214. Progress in Cloud Modification by Project Cirrus. Occasional Report No. 21, Project Cirrus. (April 15, 1950).
215. Cause and Effect Versus Probability in Shower Production. Occasional Report No. 22, Project Cirrus. (July 15, 1950).
216. A Gamma Pattern Seeding of Stratus Clouds, Flight 52, and a Racetrack Pattern Seeding of Stratus Clouds, Flight 53. (With C. A. Woodman). Occasional Report No. 23, Project Cirrus. (June 1, 1950).
217. Results of the Seeding of Cumulus Clouds in New Mexico. Occasional Report No. 24, Project Cirrus. (June 1, 1950).
218. Studies of Tropical Clouds. Occasional Report No. 25, Project Cirrus. (July 1, 1950).

VOLUME 11

Cloud Nucleation

Part 2 — *Nucleation of Clouds with Silver Iodide* (*Unpublished*)

222. Final Report, Project Cirrus. Part II. Analysis of the Effects of Periodic Seeding of the Atmosphere with Silver Iodide. 340 pages. (May 1953).
229. Widespread Control of Weather by Silver Iodide Seeding. 55-RL-1263. July 1955.

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VOLUME 12

LANGMUIR, THE MAN AND THE SCIENTIST

Part **1** — *Biography*

VOLUME 12

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Part **2** — *Philosophy of Science*

70. Future Developments of Theoretical Chemistry. Chem. and Met. Engrg., **24**, 533 (1921).
115. Atomic Hydrogen as an Aid to Industrial Research. Science, **57**, (February 24, 1928). Printed again in Scientific Monthly, **70**, 3 (1950).
123. Address of Presentation of Priestley Medal Award. Ind. and Engrg. Chem. **21**, 896 (1929).
124. Modern Concepts in Physics and Their Relation to Chemistry. JACS, **51**, 2847 (1929). [President's Address].
126a. Selecting the Chemist-elect. Jour. Chem. Education, **7**, (March, 1930). [Address delivered at dedication of Francis P. Garvan Chair of Chemical Education at Johns Hopkins, October 11, 1929].
149. Science as a Guide in Life. G.E. Rev., **37**, 312 (1934).
155. Mechanical Properties of Matter. Mch. Engrg., **57**, 486 (1935).
166. Chemical Research. Ind. and Engrg. Chem., **15**, 188 (1937). [Address delivered at the dedication of the new building of Mellon Institute, May 5–9, 1937].
171a. Fundamental Research and its Human Value. G.E. Rev., **40**, 569 (1937).
180. The Speed of the Deer Fly. Science, **87**, 233 (1938).
190a. A chapter "Simple Experiments in Science" on page 3 of book: "Excursions in Science", edited by Neil G. Reynolds and Ellis L. Manning, McGraw-Hill Book Company (1939).
197. Science, Common Sense, and Decency. Science, **97**, 1 (1943). [Retiring President's Address from AAAS].
199. Unforeseeable Results of Research. U.S. Rubber Company Broadcast. (January 21, 1945).
200. Electronics of Tomorrow. N.Y. Herald Tribune Forum. (November 17, 1943).
201. Discussion on Science Legislation. Joint Hearings on Science Bills before Senate Military Affairs and Commerce Committees. (October 1945).
202. Testimony on Atomic Energy Control. Senate Hearings on Atomic Energy. (November 30, 1945).

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203. World Control of Atomic Energy. Proc. Amer. Phil. Soc., **90**, No. 1 (1946). [Read November 16, 1945 in the Symposium on Atomic Energy and Its Implications].
204. My Trip to Russia. Chem. and Engrg. News, **24**, 759 (1946).
205. Science and Incentives in Russia. Scientific Monthly, **63**, 85 (1946).
206. Faraday Medalist: Response by I. Langmuir to Presentation Address by A. P. M. Fleming. Elec. Engrg., **63**, 463 (1944).
221. Planning for Progress: Two Alternatives. Columbia University Engineering Center News, **1**, 2 (1951). [Talk presented at Columbia University Engineering Center Dinner on November 7, 1951].
223. Saul Dushman — A Human Catalyst. Vacuum, **3**, 112 (1954).
224. Freedom — The Opportunity to Profit from the Unexpected. Res. Lab. Bulletin, Fall 1956.

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